ULTRA-LOW TEMPERATURES NUCLEAR ADIABATIC DEMAGNETIZATION

European Advanced Cryogenics School

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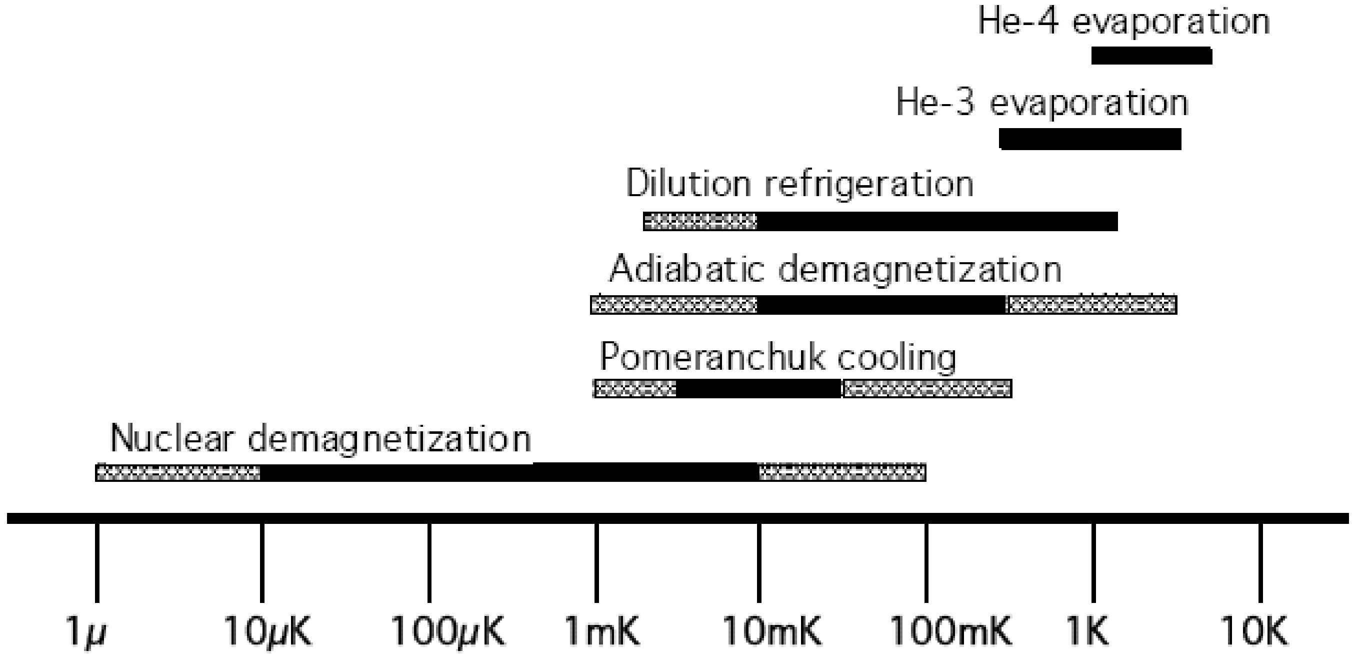
Refrigeration below 1K

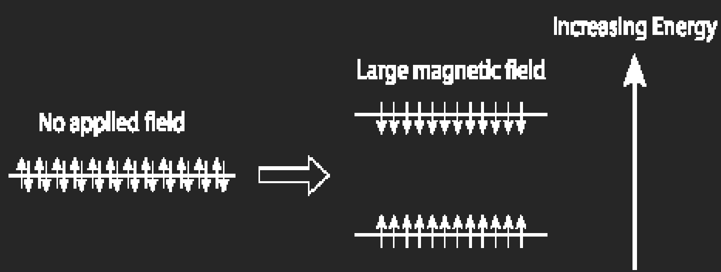
1. Dilution refrigerator
2. Adiabatic demagnetization
3. Pomeranchuk cooling

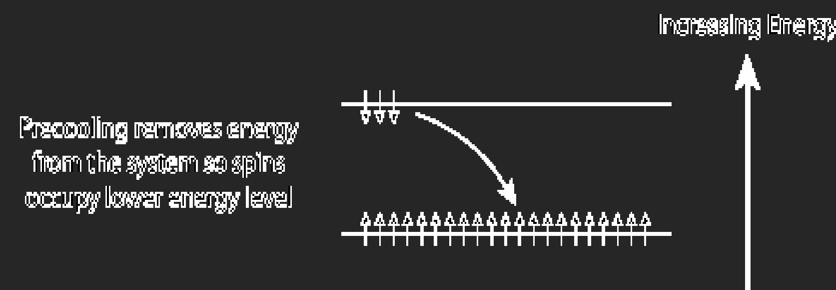


Nuclear demagnetization

Suggested by Gorter (1934) and Kurti and Simon (1935)
Realized by Kurti, Robinson, Simon and Spohr (1956)
Cooled Cu nuclear spins to about 1 μK, while lattice and electrons remains in 12 mK.

$$\exp \frac{\mu H}{kT}$$

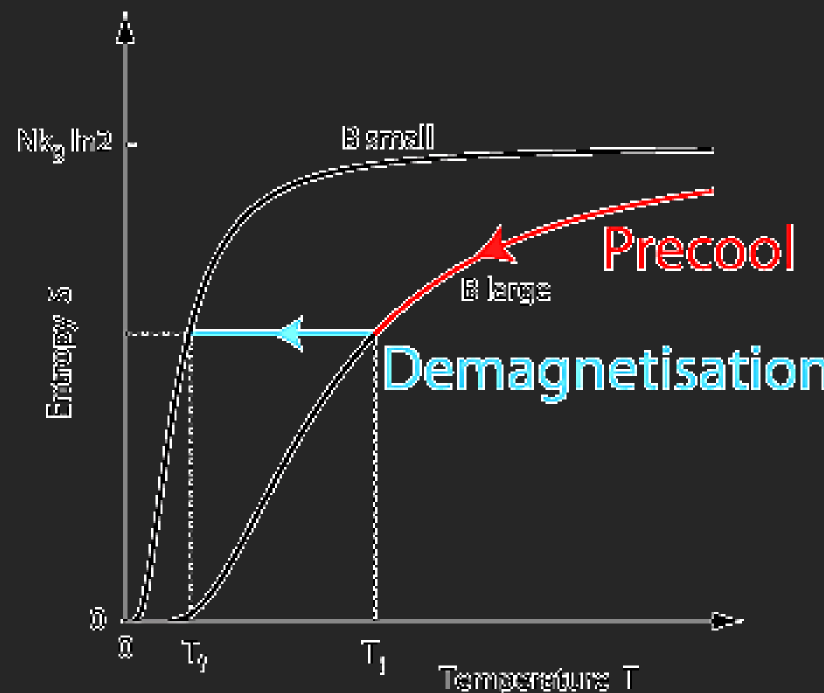




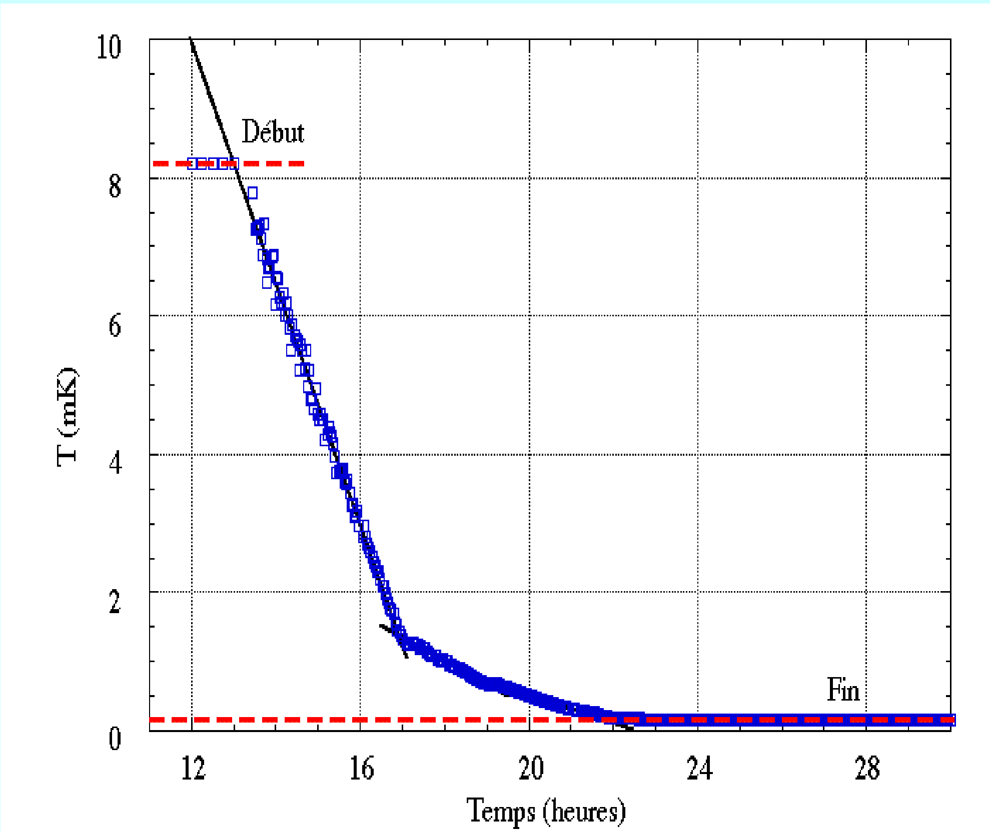
$$C_b = \frac{A(B^2+b^2)}{T^2}$$

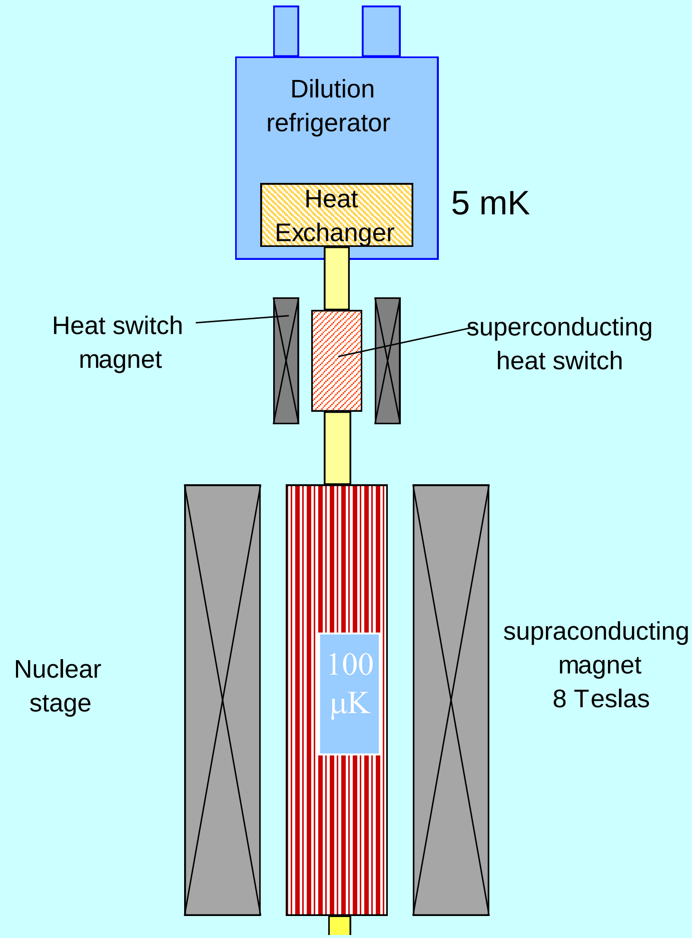
$$S = R \ln (2J + 1) - \frac{A(B^2+b^2)}{2T^2}$$

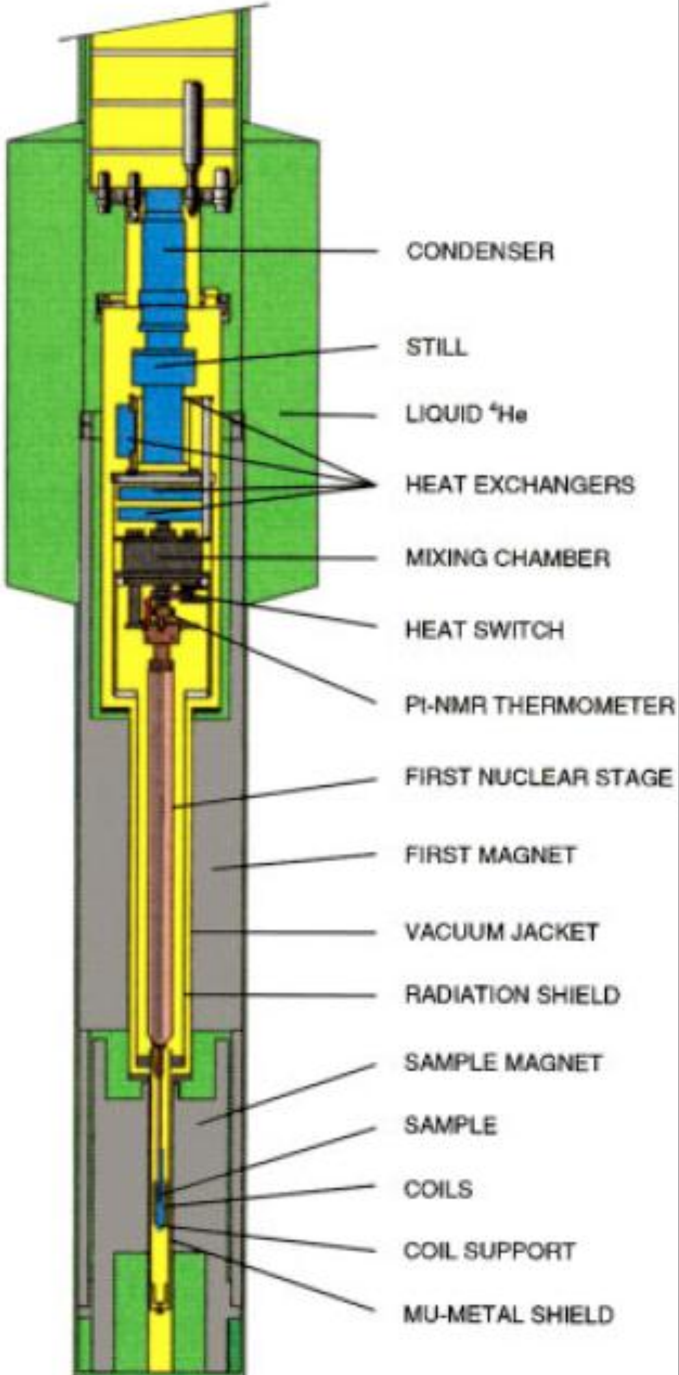
$$T_f = \mathrm{SQRT}\left(\frac{B_f^2 + b^2}{B_i^2 + b^2}\right) T_i$$

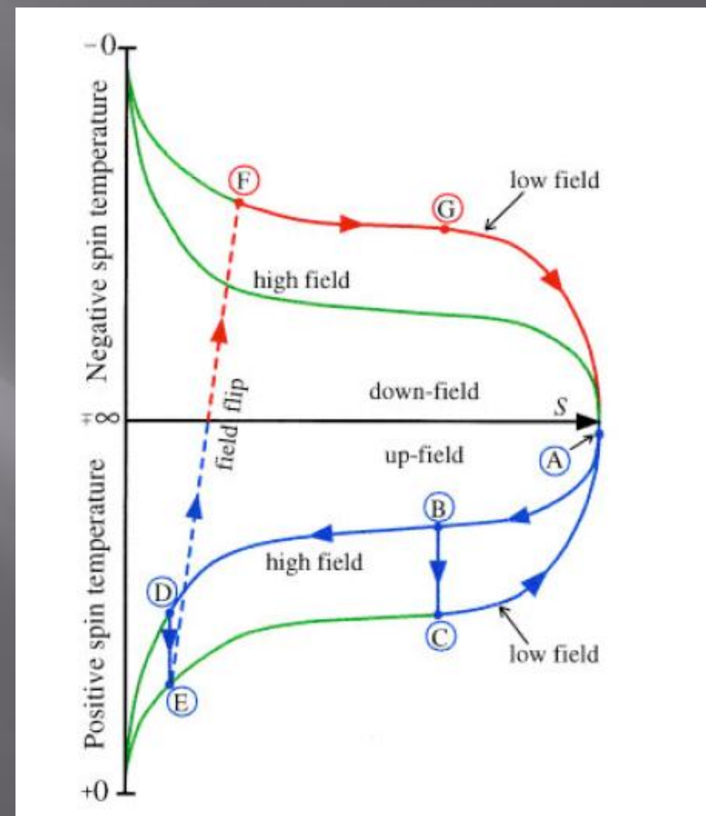


Nuclear demagnetization refrigerator






CONDENSER
STILL
LIQUID ⁴He
HEAT EXCHANGERS
MIXING CHAMBER
HEAT SWITCH
Pt-NMR THERMOMETER
FIRST NUCLEAR STAGE
FIRST MAGNET
VACUUM JACKET
RADIATION SHIELD
SAMPLE MAGNET
SAMPLE
COILS
COIL SUPPORT
MU-METAL SHIELD


−0
∓∞
+0
Negative spin temperature
Positive spin temperature
S
low field
high field
down-field
up-field
field flip
high field
low field
A
B
C
D
E
F
G

Material

Cu

$PrNi_5$

Geometry

Nuclear stage can be done from wires, bulk road or plates.

Heat switch

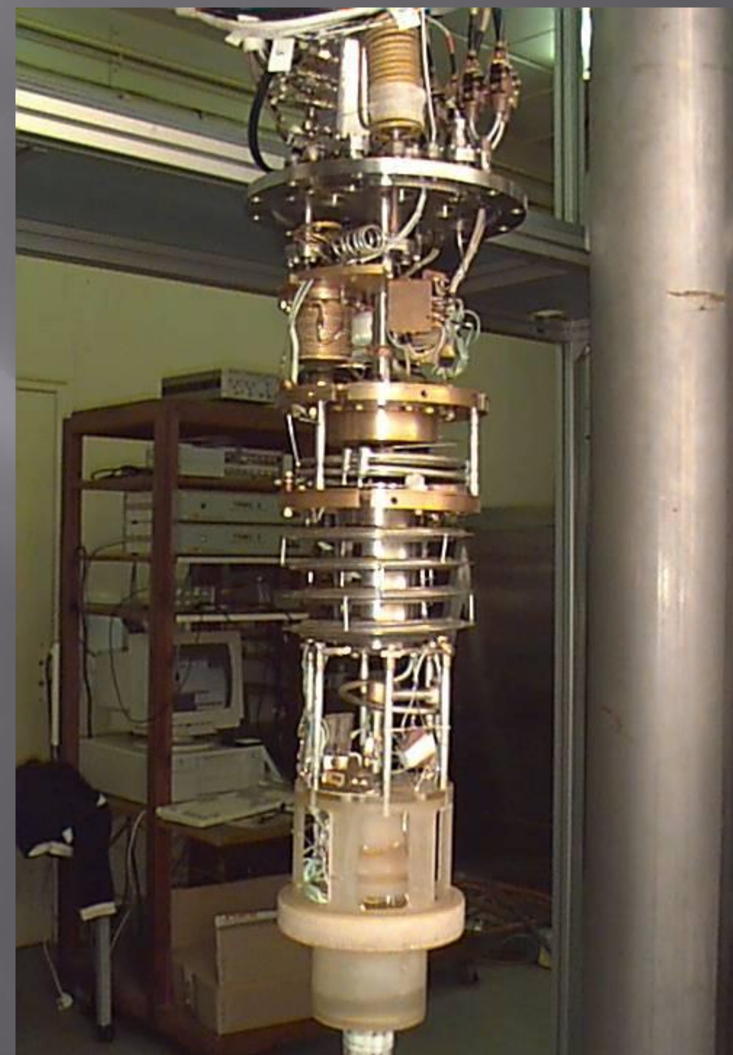
Sn

Zn

Pb

Al

Cooling superfluid ^{3}He down to 100 μK



Adiabatic demagnetization cooling

- Paramagnetic salts
- Hyperfine enhanced systems
- Nuclear demagnetization

! Spin entropy S_J depends just on the ratio of the magnetic field B and the temperature T

$$S_J = S_J(B/T)$$

Compare:
- magnetic energy μB
- thermal energy $k_B T$

when $k_B T \gg \mu B$

$$S_J \to S_{J,max} = R \ln(2J+1)$$

① Prepare the initial conditions

$$\begin{cases} B = B_i \\ T = T_i \end{cases} \Rightarrow S_J = S_{J,max} - \Delta S_J$$

② Isolate the system
- no changes in entropy ≡ adiabatic system

③ Reduce the magnetic field (adiabatically)

$$B_i \to B_f$$

The same entropy is obtained when

$$T_f = \frac{B_f}{B_i} T_i < T_i \quad !$$

What if $B_f = 0$?
At that limit effective fields other than the external applied field come into play:

Spin-spin interactions contibute by

$$b \sim \frac{k_B T_c}{\mu} \ ; \ B_{tot} \sim \sqrt{B^2 + b^2}$$

where T_c is the magnetic ordering temperature

Thermodynamics

Dimensionless variable

$$\underline{\underline{x = \mu B / k_B T}} \quad (\text{neglect } b \ll B)$$

Zeeman states give the partition function

$$Z = \left[\sum_{m=-J}^{+J} e^{-\frac{mx}{J}} \right]^{nN_A} = \left[\frac{\sinh \frac{(2J+1)x}{2J}}{\sinh \frac{x}{2J}} \right]^{nN_A}$$

You get:

$$\begin{cases} \text{Entropy} \quad S = k_B \partial (T \ln Z)/\partial T \\ \text{Heat capacity} \quad C_B = T \left(\frac{\partial S}{\partial T} \right)_B \\ \text{Magnetization} \quad M = k_B T (\partial \ln Z / \partial B)_T \end{cases}$$

... worked out for you

$$\begin{cases} \frac{S}{nR} = \frac{x}{2J} \coth \frac{x}{2J} - \frac{(2J+1)x}{2J} \coth \frac{(2J+1)x}{2J} + \ln \left[\sinh \frac{(2J+1)x}{2J} \Big/ \sinh \frac{x}{2J} \right] \\ \frac{M}{nN_A \mu} = \frac{M}{M_{sat}} = B_J(x) = \frac{2J+1}{2J} \coth \frac{(2J+1)x}{2J} - \frac{1}{2J} \coth \frac{x}{2J} \\ \frac{C_B}{nR} = \frac{x^2}{4J^2} \sinh^{-2} \frac{x}{2J} - \frac{(2J+1)^2 x^2}{4J^2} \sinh^{-2} \frac{(2J+1)x}{2J} \end{cases}$$

Often $x \ll 1$ $(k_B T \gg \mu B)$ so that high-T approximations hold

$$\begin{cases} \frac{S}{nR} \approx \ln(2J+1) - \frac{J+1}{6J} x^2 \\ \frac{M}{M_{sat}} \approx \frac{J+1}{3J} x \\ \frac{C_B}{nR} \approx \frac{J+1}{3J} x^2 \end{cases}$$

Other useful quantities:

- susceptibility χ

$$\chi = \frac{\mu_0 M}{VB} \approx \frac{J+1}{3J} \frac{\mu_0 M_{sat}}{VB} x = \frac{\lambda}{T}$$

where $\lambda = \frac{J+1}{3J} \frac{nR\mu_0}{V} \frac{\mu^2}{k_B^2}$

is the <u>Curie constant</u>

$([\lambda] = K)$

Now, e.g.

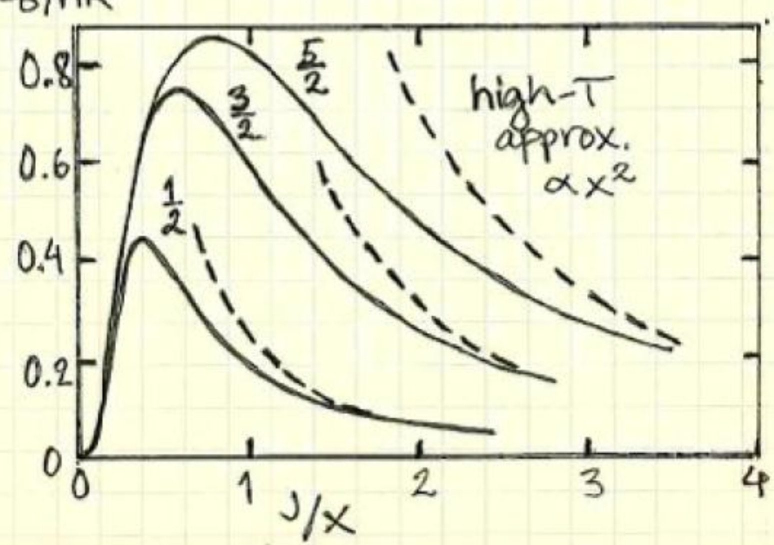
$$c_B = \frac{J+1}{3J} nRx^2 = \frac{V\lambda}{\mu_0}\left(\frac{B}{T}\right)^2$$

We observe that any magnetic system has a <u>heat capacity</u> peaking up to a maximum, whose magnitude

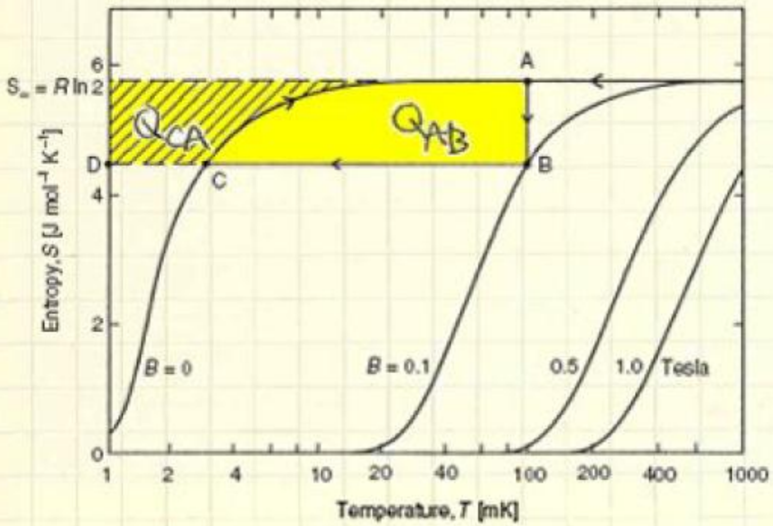
DOES NOT depend on the size of the magnetic moment, but only on its spin J.

This maximum occurs at

$$\frac{x}{J} = \frac{\mu B}{J k_B T} \approx 1 \ldots 2$$



Magnetic Cooling Cycle



① Precool to point A
$B = 0$, $T = T_i$

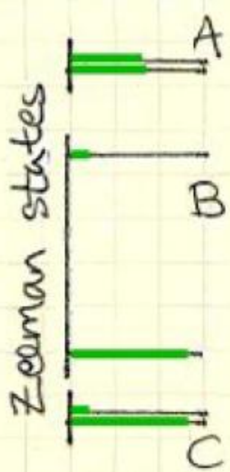
② Magnetize $A \to B$
heat removed : $Q_{AB} = T\Delta S$

③ Isolate $\Rightarrow \Delta S = 0$

④ Demagnetize $B \to C$

⑤ Use the cooling capacity $C \to A$

$$Q_{CA} = \int_{T_f}^{T} T\left(\partial S/\partial T\right)_B dT = \int_{T_f}^{T} c_B \, dT$$



$Q_{AB} \gg Q_{CA}$

To succeed:

- magnetic entropy must dominate
  $T_i \ll T_D$ (Debye, phonons)
  $T_i \ll T_F$ (Fermi, electrons)

  any other entropy in the system causes nonadiabaticity

- magnetic ordering temperature T_c must be lower than the target T
  - T_c due to dipole-dipole interactions scales as μ^2/r^3
  - metals exhibit exchange interaction also; can be large

- initial condition (B_i, T_i) should be such that the spin entropy has been reduced as much as possible. This gives cooling capacity.

  In practice $\Delta S_i \sim 5 \ldots 50\%\ S_{max}$

Paramagnetic salts

(magnetic metals are ruled out because of the strong exchange interaction ⇒ T_c is high)

We play with the electronic moment

$$\mu = g \mu_B J$$

with the Landé factor

$$g = \frac{3}{2} + \frac{1}{2}\,\frac{S(S+1) - L(L+1)}{J(J+1)}$$

and the Bohr magneton

$$\mu_B = \frac{e\hbar}{2m_e} \approx 9.27 \cdot 10^{-24}\,J/T$$

Elements with unpaired electrons at inner shells produce local magnetic moments in the solid state

- 3d transition elements
- 4f rare earth metals

Often their magnetism is too strong (high T_c) so that they are diluted by forming an ionic compound (salt) with loads of crystal water

$T_c \propto \frac{\mu^2}{r^3}$ goes down

Ion to be used can be

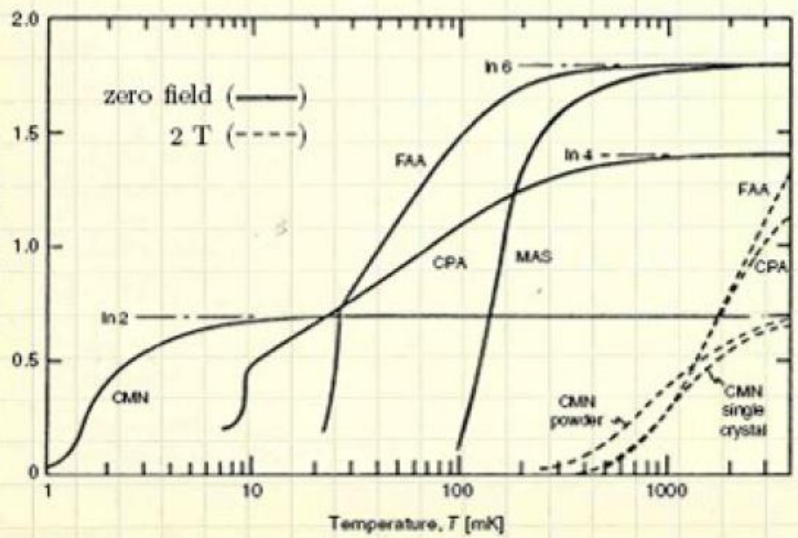
Mn^{2+}, Fe^{3+}, Cr^{3+}, Ce^{3+}, ...

Suitable compounds, e.g.

		T_c/K
"MAS"	$Mn^{2+}SO_4(NH_4)_2SO_4 \cdot 6H_2O$	0.17
"FAA"	$Fe_2^{3+}(SO_4)_3 ; (NH_4)_2SO_4 \cdot 24H_2O$	0.03
"CPA"	$Cr_2^{3+}(SO_4)_3 ; K_2SO_4 \cdot 24H_2O$	0.01
"CMN"	$2Ce^{3+}(NO_3)_3 ; 3Mg(NO_3)_2 \cdot 24H_2O$	0.002

Cerium can be diluted by Lantanum ⇒ "LCMN"

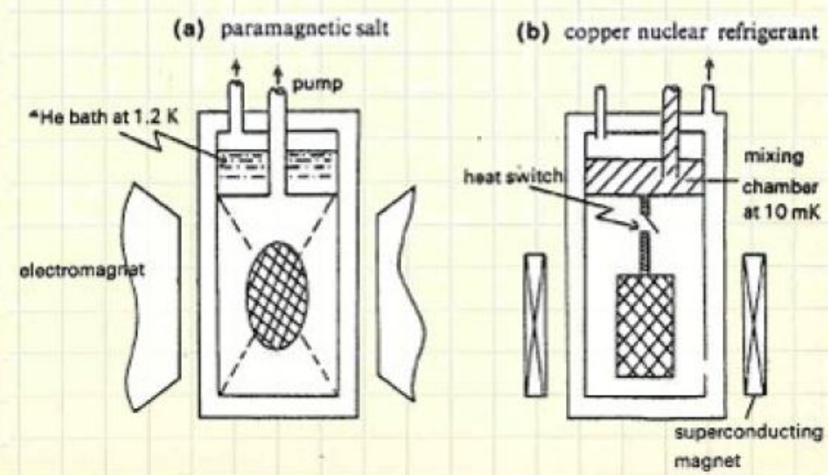
Initial condition $\begin{cases} T \sim 1K \\ B \sim 1T \end{cases}$ is fine



Benefits:
- easy initial condition
- huge heat capacity (also at $B=0$)
- "built in" thermometer ($\chi = \lambda/T$)
- fast (demag in a few minutes)

Drawbacks:
- poor thermal conductivity (these salts are insulators)
- difficult thermally connect (exception: ^3He in powdered salt CMN + ^3He → 2mK)
- single shot cooling

This is why dilution refrigerators have replaced these



Nuclear demagnetization

Often the nucleus carries a magnetic moment – all you need is an unpaired proton or neutron

$$\mu \sim \mu_N \approx 5.05 \cdot 10^{-27} \, J/T$$

$$\left(\frac{\mu_N}{\mu_B} = \frac{m_e}{m_n} \sim \frac{1}{1840} \right)$$

$$\Rightarrow \quad T_c \lesssim 0.1 \, \mu K$$

Therefore:

- one can reach conciderably lower temperatures
- no need to dilute the moments
  $\Rightarrow$ good moment density & more heat capacity
- metals can be used
  - good thermal conductivity
  - easy thermal contacts

The challenge:

- small μ $\Rightarrow$ need large B_i/T_i
  e.g. $\begin{cases} T_i = 10 mK \\ B_i = 6T \end{cases}$
  $\Rightarrow \Delta S = 5\% \, S_{max}$ for Cu
- to maintain heat capacity one cannot demag to zero or even close to b
  (typically $b \sim 0.1$ mT)
  Often $B_f \sim 10 \dots 100$ mT

Material candidates?

A) Pure metal for sufficient thermal conductivity

B) Reasonable moment, abundance (maybe large spin)

e.g.	μ/μ_N	I
^{27}Al	3.64	5/2
^{63,65}Cu	2.3	3/2
^{113,115}In	5.5	9/2

^{55}Mn & ^{59}Co are very nice nuclei, too, but the host metal is ferromagnet
$\Rightarrow$ these cannot be demagnetized

^{93}Nb & ^{51}V are also very good isotopes, but they are in superconductors, $B_c = 0.2/0.1\,T$
$\Rightarrow$ demag only that far

Al & In are superconductors, too, but $B_c \sim 10/30\,mT$ only

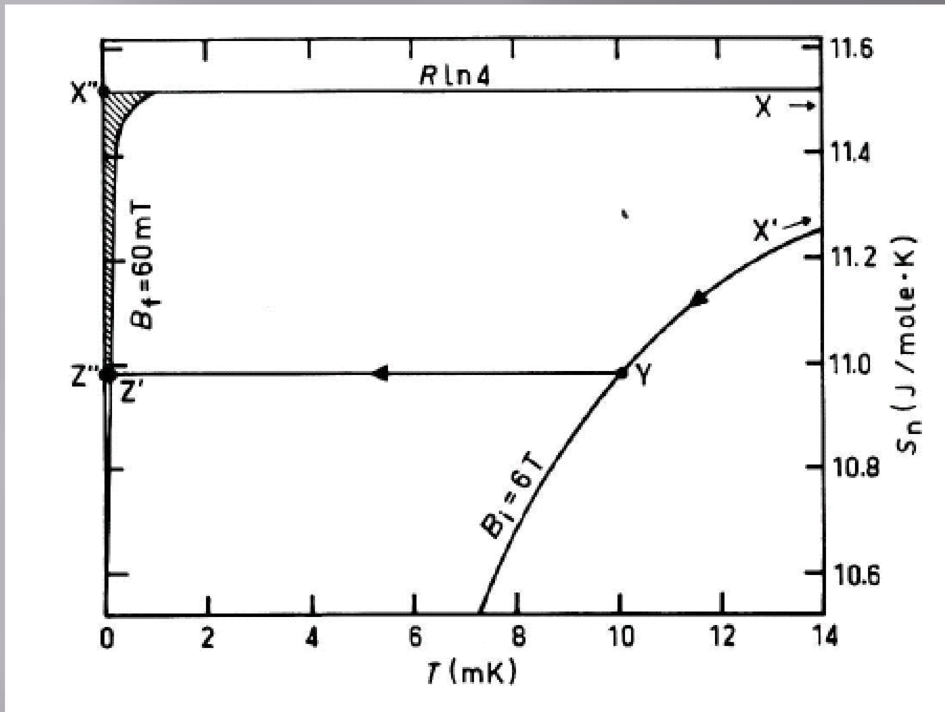
C) Cubic lattice or spin $I = \frac{1}{2}$

in other cases electric quadrupole interaction may be strong and contributes to the effective field

- In (TETR) $b_q \sim 0.25T$ is thus ruled out

We are left with

(Al might do in some cases)



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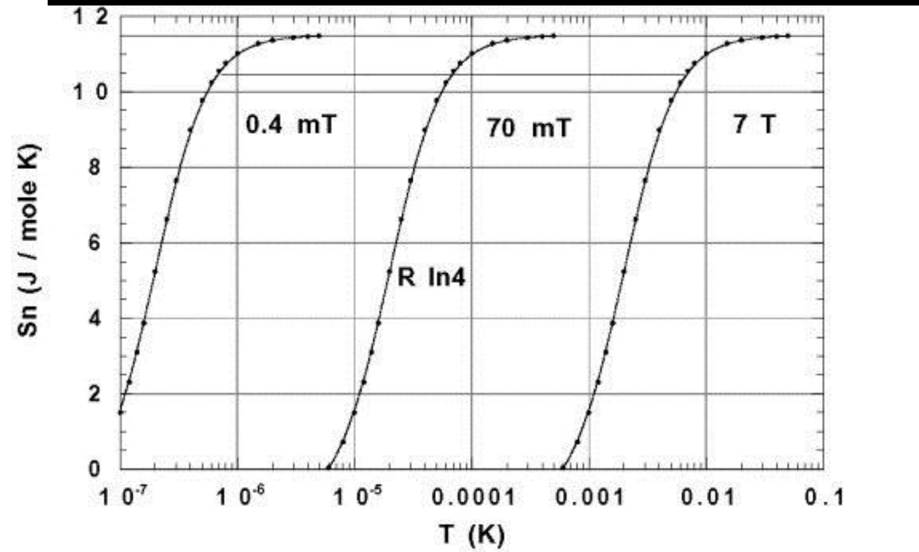


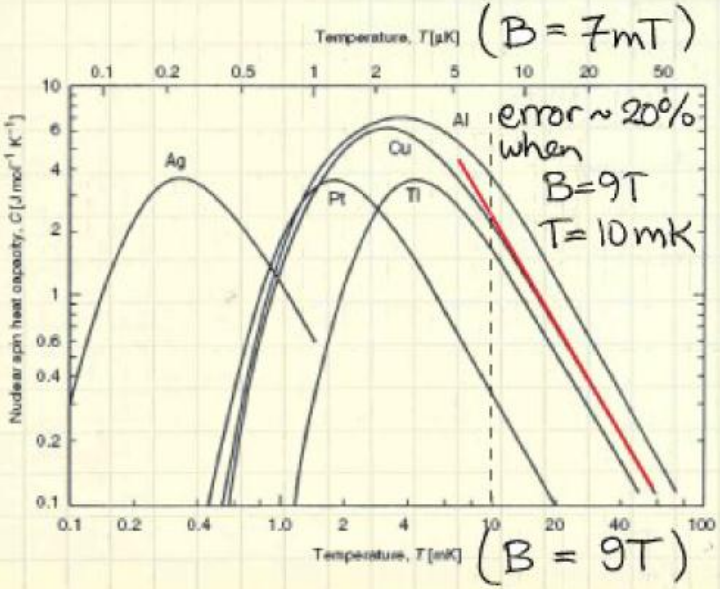
Figure 3. Entropy-temperature diagram for copper demagnetization process.

1. **Adiabatic**
2. **Large Curie constant Cu 0.57 μK (Pt 0.019, In 1.11); $PrNi_5$ x 17**
3. **In equilibrium with electrons (small Korringa constant) Cu 1.1 secK (Pt 0.03)**
4. **No superconducting transition**
5. **Small residual field (PrNi5 ~ 1mK)**
6. **High thermal conductivity**
7. **Desirable mechanical and metallurgical properties (heat leak)**

When working with nuclear spins it is usually sufficient to use the simple high-T expansions

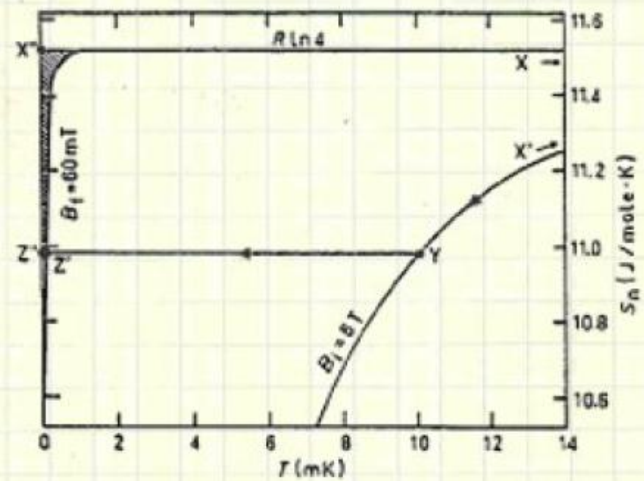
to recapitulate:

$$\begin{cases} \Delta S = \ln(2I+1) - S/nR \propto \left(\frac{B}{T}\right)^2 \\ p = \frac{M}{M_{sat}} \propto \left(\frac{B}{T}\right) \\ \frac{c_B}{nR} \approx 2\Delta S \propto \left(\frac{B}{T}\right)^2 \end{cases}$$



To meet the requirements for the initial conditions you need

- a dilution fridge $T_i \sim 10$ mK
- superconducting magnet $B_i \sim 10$T



The nuclear stage is usually magnetized fast (~1h) i.e. not isothermally in order to save time. The heat of magnetization is then larger, but dilution fridges remove that quicker at higher T ~ 50..100 mK

A copper nuclear stage typically has $n \approx 10 \ldots 100$ mol ($\hat{=}$ several kg of Cu)

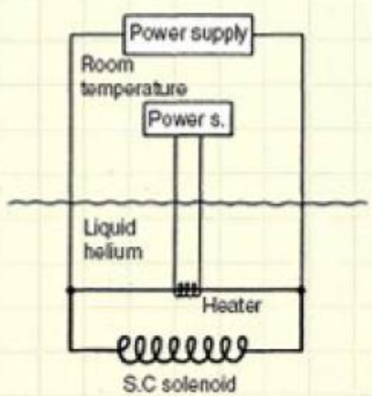
1) Precooling with a dilution fridge in $B_i \simeq 6 \ldots 9$ T to $T_i \sim 10$ mK takes some 1…5 days

2) Thermal isolation is controlled by a <u>superconducting heat switch</u>

3) Demagnetization to $B_f \sim 10 \ldots 100$ mT takes ~ 10 h (1 day); speed limited by eddy current heating

4) Depending on the heat load one can maintain $T \lesssim 100\,\mu$K for 1 day … over a month

The superconducting magnet must be persistable. During phase 1) there is a current of order 100 A. During phase 4) the greatest stability is required.



Phase 1) Precooling

- power of the dilution fridge

$$\dot{Q}_d = a T_d^2 - \dot{q}_d$$

where $a = 84\,\dot{n}_3\ \mathrm{J/(mol\,K^2)}$ and $\dot{q}_d$ is the background load of the fridge (depends on $\dot{n}_3$ too)

Base temperature $T_0 = \sqrt{\dot{q}_d / a}$

- thermal resistance of the heat switch (normal state)

$$R \sim 2 \ldots 20 \frac{\mathrm{K}^2}{\mathrm{W}} \quad \text{(typically)}$$

whereby

$$T_{NS}^2 - T_d^2 = 2R\,\dot{Q}_{NS\text{-}d}$$

- heat of magnetization

$$\dot{Q}_{NS} = -C_B \frac{dT_{NS}}{dt},$$

$$C_B = \frac{c}{T_{NS}^2}; \quad c = \frac{I+1}{3I}\left(\frac{\mu B}{k_B}\right)^2 nR = \frac{V\lambda}{\mu_0} B^2$$

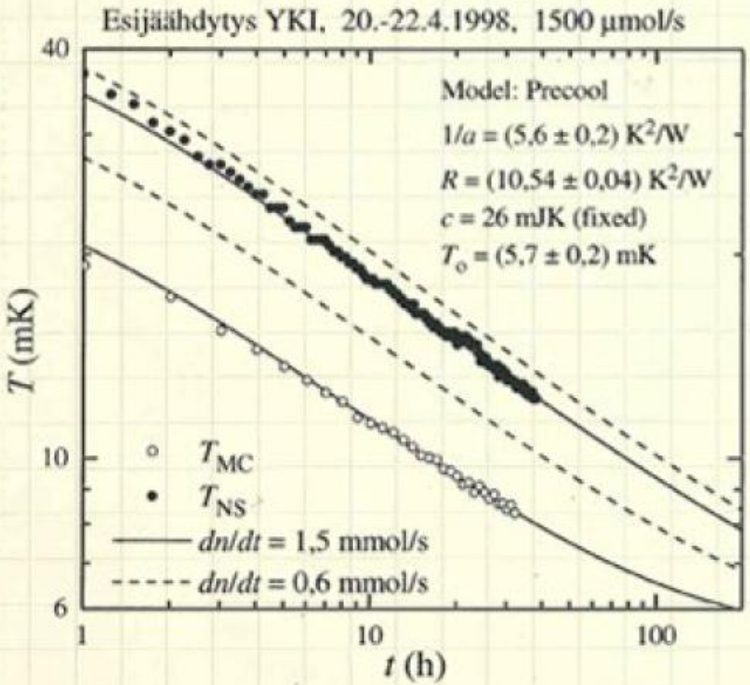
These determine the precooling time …

... Precooling time

$$t(T) = \left(\frac{1}{a} + 2R\right)\frac{c}{T_0^2}\left(\frac{1}{2T_0}\ln\frac{T+T_0}{T-T_0} - \frac{1}{T}\right)$$

$$\underset{T \gg T_0}{\approx} \left(\frac{1}{a} + 2R\right)\frac{c}{3T^3}$$

$B = 9T \quad n_{Cu} \approx 100\,\text{mol}$



For example:

$\dot{n}_3 = 1\,\frac{\text{mmol}}{\text{s}} \Rightarrow \frac{1}{a} = 12\,\frac{K^2}{W}$

$R = 10\,\frac{K^2}{W}\ ; \quad T_0 = 3\,\text{mK}$

$n_{NS} = 100\ \text{mol} \quad (\text{Cu})$

$B_i = 9T \quad \Rightarrow \quad c = 26{,}5\ \text{mJK}$

results in $t(10\,\text{mK}) = 3\cdot 10^5\,\text{s} \sim 3.5$ days

Phase 2) heat switch

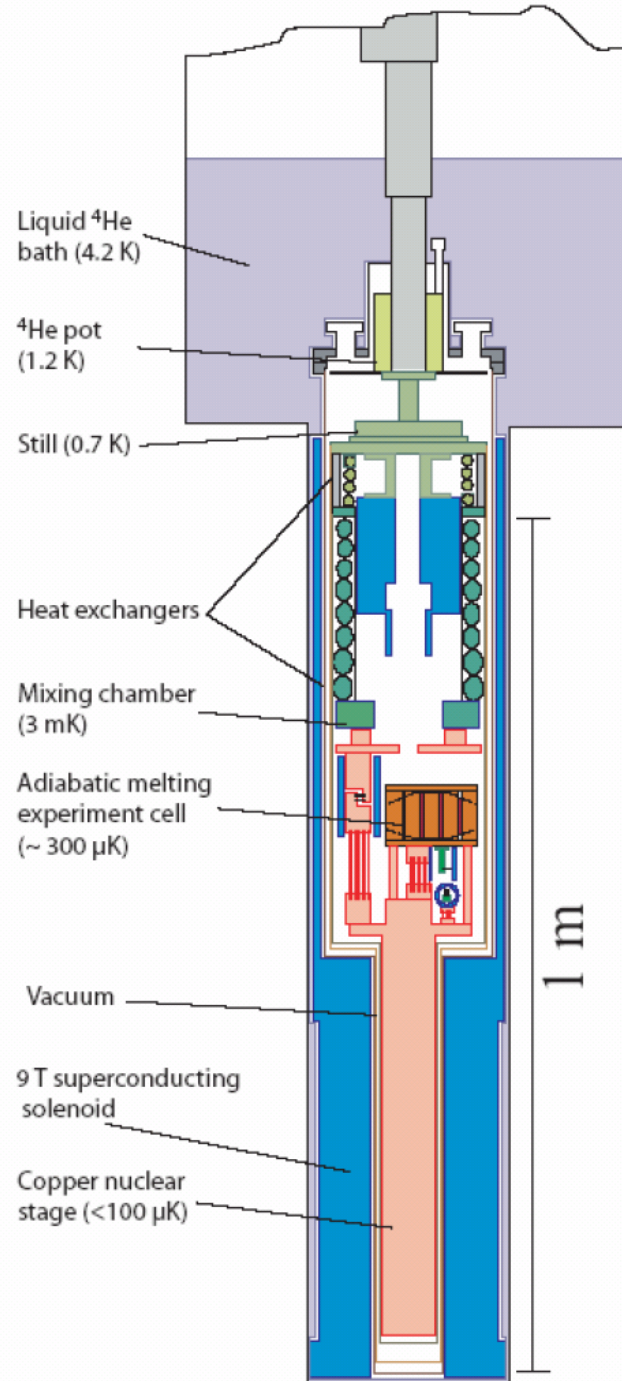
We assumed $R \sim 10\,\frac{K^2}{W}$ in normal state

A good switch has a switching ratio $> 10^5$ $(T \approx T_0)$

We can then estimate the leakage through the SC switch

$$\dot{Q} \sim \frac{T_0^2}{2\cdot 10^5 R} \simeq 4.5\ \text{pW}$$

This is perfectly OK



Liquid ^{4}He bath (4.2 K)
^{4}He pot (1.2 K)
Still (0.7 K)
Heat exchangers
Mixing chamber (3 mK)
Adiabatic melting experiment cell (~ 300 μK)
Vacuum
9 T superconducting solenoid
Copper nuclear stage (<100 μK)
1 m

superconducting Heat switch

Good electrical conductivity in normal state (Wiedemann-Franz → thermal conductivity)

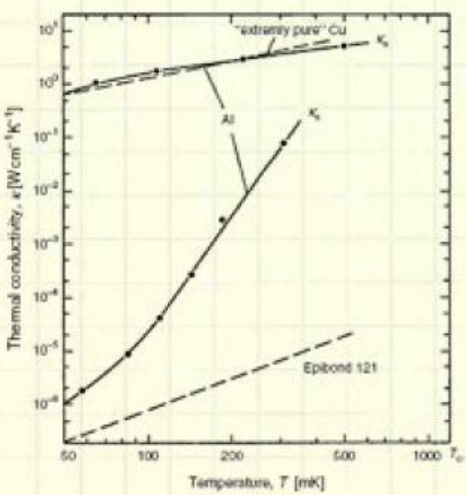
Poor phonon conductivity i.e high T_D (Debye temperature)

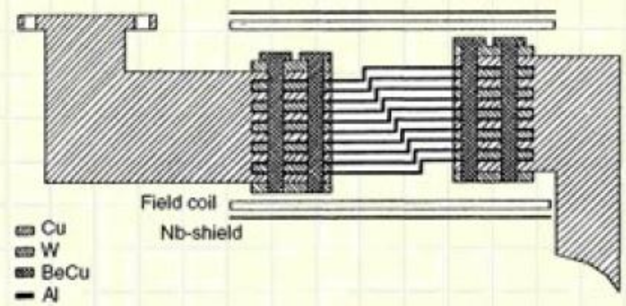
Possible materials:

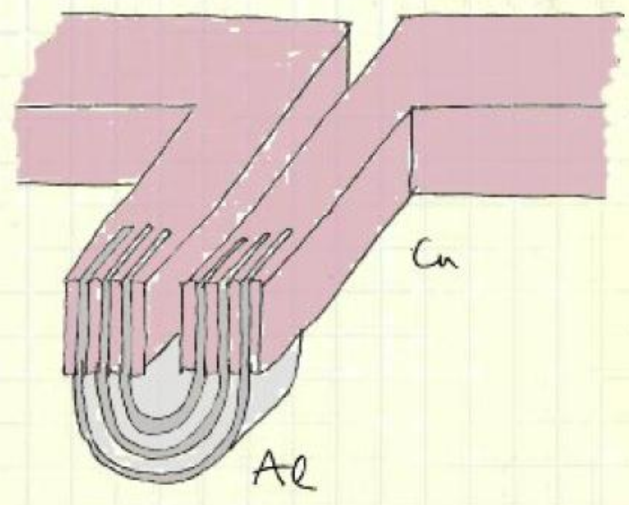
	T_c	B_c	T_D
Al	1.2 K	10 mT	394 K
Zn	0.9	5	234
Sn	3.7	30	~200

Aluminum has exellent material properties but one must overcome the oxide problem on the surface

- diffusion welding Al-Cu @ 500°C for ~15 min in vacuum under compression







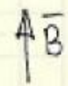
Phase 3) demagnetization

Optimal demag profile?

a simple model:

- constant background heat leak $\dot{q}_{NS}$
- eddy current heating $\gamma \dot{B}^2$

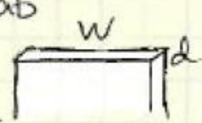
$$\Rightarrow \quad \dot{Q}_{NS} = \dot{q}_{NS} + \gamma \dot{B}^2$$



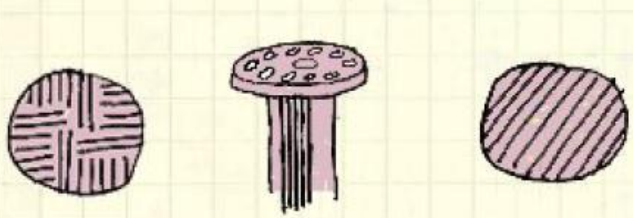
where

$$\gamma = \begin{cases} \frac{r^2}{8} V \frac{RRR}{\rho_0} \; ; & \text{for a cylinder} \\ \frac{d^2}{16} V \frac{k^2}{1+k^2} \frac{RRR}{\rho_0} \; , & \text{for a slab} \end{cases}$$

$k = w/d$



So: typically a cylindrical block is sliced into vertical plates or it is bundled from plates ~1mm thick



in an optimal demag $\dot{q}_{NS} = \gamma \dot{B}^2$

if $\dot{q}_{NS}$ is const.

$\Rightarrow$ linear profile $\dot{B} = \sqrt{\dot{q}_{NS}/\gamma}$

; $t_{opt} = \sqrt{\gamma/\dot{q}_{NS}}\,(B_i - B_f)$

The measure for nonadiabaticity:

$$\Delta\left(\frac{B}{T}\right) = 2\,\frac{\sqrt{\gamma \dot{q}_{NS}}}{\sqrt{\lambda/\mu_0}} \ln \frac{B_i}{B_f}$$

However, this is usually unacceptably slow - several days for reasonable $\dot{q}_{NS}$

To save time ($t_{dm} < t_{opt}$) the best demag profile is parabolic

$$B(t) = a\left(t/t_{dm}\right)^2 - b\left(t/t_{dm}\right) + B_i$$

with

$$\begin{cases} a = B_i + B_f - \sqrt{4B_iB_f + \frac{\dot{q}_{NS}}{\gamma} t_{dm}^2} \\ b = 2B_i - \sqrt{4B_iB_f + \frac{\dot{q}_{NS}}{\gamma} t_{dm}^2} \end{cases}$$

Losses for parbolic demag:

$$\Delta\left(\frac{B}{T}\right) = \frac{4\mu_0}{\sqrt{\lambda}}\left[a\frac{\gamma}{t_{dm}} + \sqrt{\dot{q}_{NS}\gamma}\ \operatorname{arcsinh}\left(\sqrt{\frac{\dot{q}_{NS}}{\gamma}}\frac{t_{dm}}{2\sqrt{B_iB_f}}\right)\right]$$

For example:

$$\left.\begin{cases} n_{Cu} = 100\ \text{mol} \\ d = 2\text{mm} \\ w \gg d \\ RRR = 1000 \end{cases}\right\} \Rightarrow \gamma = 10{,}6\ \frac{Ws^2}{T^2}$$

$$\left.\begin{array}{l} \dot{q}_{NS} = 10\text{nW} \\ B_i = 9T \\ T_i = 10\text{mK} \\ B_f = 50\ \text{mT} \end{array}\right\} \Rightarrow \begin{array}{l} t_{opt} \approx 3\text{days}\ (!) \\ \Delta\left(\frac{B}{T}\right)_{opt} / \frac{B_i}{T_i} \approx 1.2\% \end{array}$$

vs. parabolic $t_{dm} = 10h \Rightarrow$ losses $\sim 3\%$ (OK)

linear demag in time t_{dm}

$$\Rightarrow \Delta\left(\frac{B}{T}\right) = \frac{\mu_0}{\sqrt{\lambda}}\ln\left(\frac{B_i}{B_f}\right)\left[\frac{\dot{q}_{NS}}{B_i - B_f}t_{dm} + \frac{\gamma(B_i - B_f)}{t_{dm}}\right]$$

with $t_{dm} = 10h \Rightarrow$ losses $\sim 5\%$ (not bad)

$9T \to 50mT$

linear in 10h $\sim 15\ mT/min$

best parabolic in 10 h begin $\sim 27\ mT/min$, end $\sim 3\ mT/min$

Take into account also:

- possible vibrational heating $\dot{Q} \sim B^2$
  $\Rightarrow$ sweep faster at beginning
- magnetoresistance
  $\rho = \rho(B) \Rightarrow \gamma = \gamma(B)$
  $\Rightarrow$ sweep faster at beginning

($\frac{\Delta\rho}{\rho_0}$ can be > 10 in $10T$)

Phase 4) $T \leq 100\,\mu K$

Heat capacity to spend

$$Q = \int_{T_f}^{\infty} c_B dT = \frac{I+1}{3I} \frac{nR}{T_f} \left(\frac{\mu B_f}{k_B} \right)^2$$

$$\sim 10\, mJ$$

Question: the cooling process operates on the nuclear spins, how is that connected to your sample?

The nuclear spins thermalize among themselves due to spin-spin interactions in a time scale τ_2 (spin-spin relaxation time, remember NMR)

τ_2 is typically $\sim 0.1 \cdots 10$ ms (fast)

$\Rightarrow$ spin system can be concisidered to be in thermal equilibrium distribution

Energy exchange between the nuclear spins and conduction electrons (or lattice : phonons) is characterized by spin-lattice relaxation time τ_1

For metals

Korringa law: $\boxed{\tau_1 = \frac{\kappa}{T_e}}$

typically $\kappa \sim 0.01 \cdots 10$ sK

when $T_e \sim 50\,\mu K \Rightarrow \tau_1 \sim 200 \cdots 2 \cdot 10^5$ s $\gg \tau_2$

THEREFORE:

nuclear spin temperature $T_n \neq T_e$ electron temperature

Electrons respond fairly quickly:

$$\tau_e \approx \frac{\ell}{A} \frac{c_e}{\kappa_e} \sim 1 ms$$

$(c_e \propto T,\ \kappa_e \propto T)$

Phonon specific heat is negligible

$(c_{ph} \propto T^3) \Rightarrow T_{ph} \approx T_e$

The thermal load (or heat leak) usually directs to the electron system

$\Rightarrow T_e > T_n$

Lets analyze this problem:

Nuclear relaxation is governed by

$$\frac{dM}{dt} = -\frac{1}{\tau_1}(M - M_0) \quad \binom{\text{NMR}}{\text{Bloch}}$$

where $M \approx \frac{I+1}{3I} M_{sat} \frac{\mu B}{k_B T_n} \propto \frac{1}{T_n}$

and the equilibrium value

$$M_0 \approx \frac{I+1}{3I} M_{sat} \frac{\mu B}{k_B T_e} \propto \frac{1}{T_e}$$

$$\Rightarrow \frac{d}{dt}\left(\frac{1}{T_n}\right) = -\frac{1}{\tau_1}\left(\frac{1}{T_n} - \frac{1}{T_e}\right)$$

Since

$$\frac{d(1/T)}{dt} = -\frac{1}{T^2}\frac{dT}{dt} \quad \text{and} \quad \tau_1 = \frac{\kappa}{T_e}$$

$$\Rightarrow \frac{dT_n}{dt} = (T_e - T_n)\frac{T_n}{\kappa}$$

If there is a heat load $\dot{Q}$
($\dot{Q} \rightarrow$ electrons
$\rightarrow$ nuclei ; $c_e \ll c_n$)

we get

$$\frac{dT_n}{dt} = \frac{\dot{Q}}{c_B} = \frac{\mu_0 T_n^2}{V\lambda B_f^2}\dot{Q}$$

$$\Rightarrow \frac{T_e}{T_n} = 1 + \frac{\mu_0 \kappa \dot{Q}}{V\lambda B_f^2} = 1 + \frac{\dot{Q}}{\dot{Q}_n}$$

We shall call

$$\dot{Q}_n = \frac{V\lambda B_f^2}{\mu_0 \kappa} \sim 1\,\mu W \quad \left\{ \begin{array}{l} B_f = 50\,mT \\ n_{Cu} = 100\,mol \\ \kappa_{Cu} = 1{,}2\,sK \end{array} \right.$$

the "normative load"
depending on the
- material (λ, κ)
- conditions (B_f)
- size (V)

T_e deviates from T_n severely
when $\dot{Q}$ approaches $\dot{Q}_n$

(or $\dot{Q}_n$ approaches $\dot{Q}$)

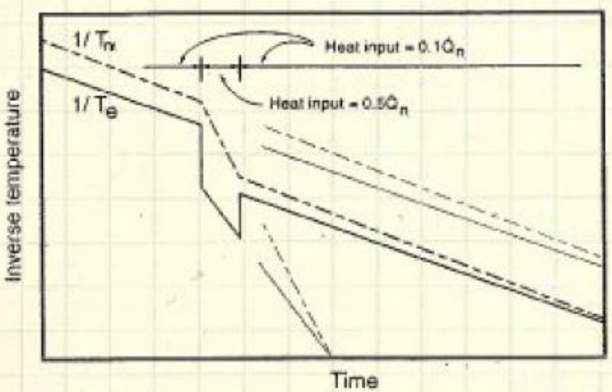
If $\dot{Q}$ is constant, then T_n^{-1} decreases linearly:

$$\frac{dT_n^{-1}}{dt} = -\frac{\mu_0 \dot{Q}}{V\lambda B_f^2} = -\frac{\dot{Q}}{\kappa \dot{Q}_n}$$

So does T_e^{-1}:

$$\frac{1}{T_n} = \frac{\dot{Q}_n + \dot{Q}}{\dot{Q}_n} \frac{1}{T_e}$$

$$\Rightarrow \frac{dT_e^{-1}}{dt} = \frac{\dot{Q}_n}{\dot{Q}+\dot{Q}_n} \frac{dT_n^{-1}}{dt} = -\frac{\dot{Q}}{\kappa(\dot{Q}+\dot{Q}_n)}$$



To not have $T_e \gg T_n$ we want small κ (and OF COURSE smallest $\dot{Q}$) ‖ $\kappa_{Cu} = 1.2\,sK$, $\kappa_{Pt} = 0.03\,sK$

Good advice: mount a heater on your fridge
It helps calibrating many things

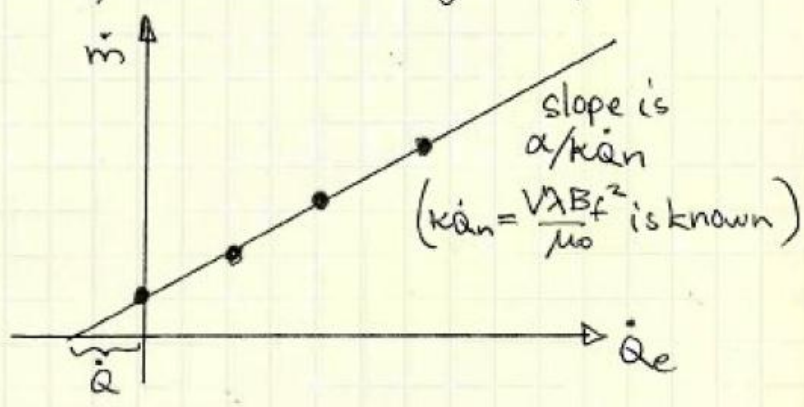
We may have an uncalibrated thermometer following a know dependence, e.g. measured reading

$$m(T_e) = a\chi_m(T_e) = a\frac{\lambda_m}{T_e} = \frac{\alpha}{T_e}$$

One takes readings with and without additional heating to obtain

$$\begin{cases} \dot{m}_1 = -\dfrac{\alpha \dot{Q}}{\kappa(\dot{Q}+\dot{Q}_n)} \\ \dot{m}_2 = -\dfrac{\alpha(\dot{Q}+\dot{Q}_e)}{\kappa(\dot{Q}+\dot{Q}_e+\dot{Q}_n)} \end{cases} \Rightarrow \begin{cases} \alpha \\ \dot{Q} \end{cases}$$

If $\dot{Q}, \dot{Q}_e \ll \dot{Q}_n$ you get a plot



If you desire the lowest possible T_e there is an optimum B_f to stop the demag:

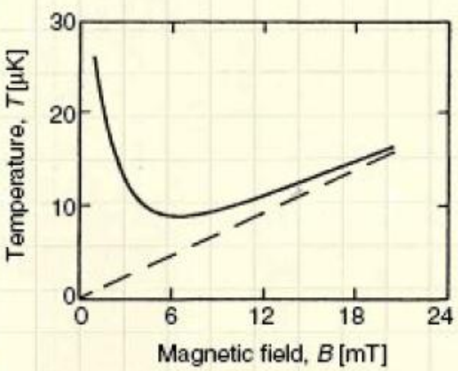
$$\begin{cases} \dfrac{T_{e,f}}{T_{n,f}} = 1 + \dfrac{\mu_0 \kappa \dot{Q}}{V\lambda B_f^2} \\ T_{n,f} = T_i \dfrac{B_f}{B_i} \end{cases}$$

$$\Rightarrow T_{e,f} = \frac{T_i}{B_i}\left(B_f + \frac{\mu_0 \kappa \dot{Q}}{V\lambda}\frac{1}{B_f}\right)$$

This has a minimum at

$$B_f = \sqrt{\frac{\mu_0 \kappa \dot{Q}}{V\lambda}} \quad \text{i.e. } \dot{Q}_n = \dot{Q}$$

At this point $T_{e,f} = 2T_{n,f}$



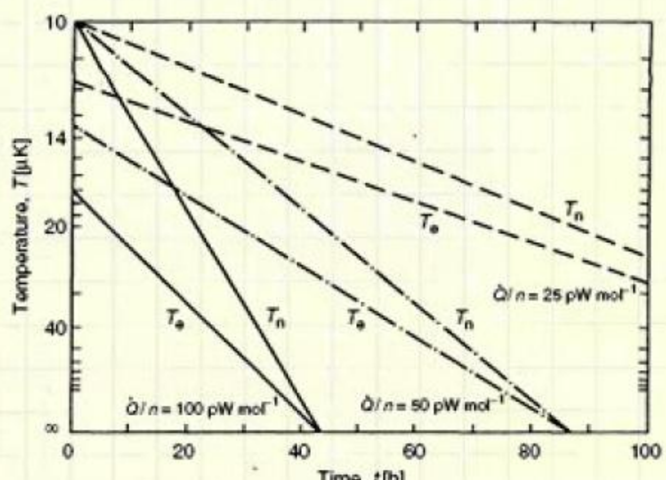
Usually, however this is unpractically low field. Often a compromise has to be made to sustain low T_e long enough ($C \propto B_f^2$)

(Remember: we assumed $B \gg b$; for Cu $b = 0.34\,mT$)

To keep cold you must eliminate all possible heat loads

- conduction (support, leads, heat switch, ...)
  - thermal anchoring is important
- thermal radiation (shield at $T < 1K$)
- remnant gas in vac. ($p_{He} < 10^{-10}$ Pa)
- vibrations! (building, LHe in dewar, pumps, ...)
  - big mass (tons)
  - flexible support (air springs, flexible tubes, ...)
  - $\Rightarrow$ eigenfrequencies < 1 Hz
- radioactivity & cosmic radiation
- electric and magnetic fields
  - shielding, filtering
- internal time dependent loads
  - H_2 ortho-para conversion
  - amorphous substances (plastics)

Typically 10–50 pW/mol (at best 5–10 pW/mol)



Hyperfine enhancement

In some metal compounds of rare-earth ions (like Pr^{3+} in $PrNi_5$) the electronic moment vanishes in zero field

- $J = 1$ (or 2, 3, ...)
- Lattice is non-cubic and the ground state is $m = 0$

Due to the quadrupole effect, any finite magnetic field can induce a nonzero effective moment (Van Vleck paramagnetism)

$\Rightarrow$ the nucleus feels a strong magnetic field $B_n = B(1+K)$

where $K \sim 10 \dots 100$ is the hyperfine enhancement factor ($K = 11.2$ for ^{141}Pr in $PrNi_5$)

Then it is easy to polarize the nuclei even at modest temperature and external magnetic field

For example: $PrNi_5$ $\left.\begin{array}{l} T_i = 25\,\text{mK} \\ B_i = 6\,\text{T} \end{array}\right\} \Rightarrow \frac{\Delta S}{S_{max}} \sim 70\%$

The nuclei can still be demagnetized because the electronic moment disappears in $B = 0$

+ Benefits:
  - easy initial conditions
  - great heat capacity / volume
  - fast spin-lattice relaxation τ_1

− Drawbacks:
  - difficult materials
  - bad conductivity
  - ordering temperature ~ 0.1 … 1 mK

This is an intermediate from paramagnetic salts to purely nuclear spin systems

In early days these vere often used between dilution fridge and a copper nuclear stage to assist in lowering T_i

Not much in use nowadays due to increased power of dilution fridges

In a rough envIronment this might be your choice if you only need $T < 1\,\text{mK}$

- combined with a cryocooler & DR ?

An example of heat load measurement:

Cu stage of the HMI-cryostat: 21 mol of Cu

The specimen ~ 2g of silver, which was exposed to a neutron beam ~ $2 \cdot 10^5 / cm^2 s$

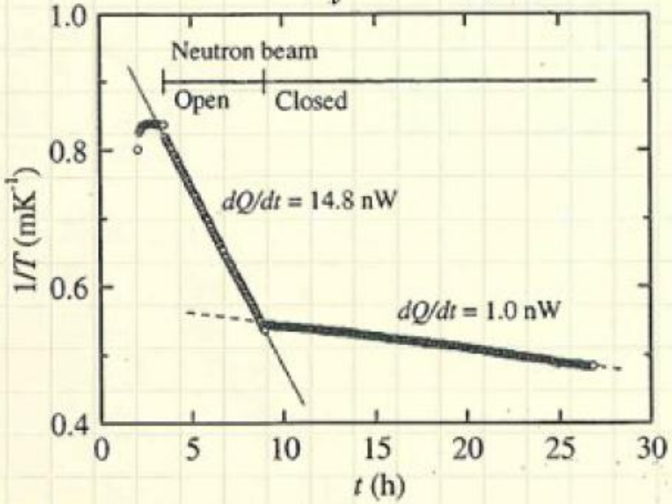
~ 80% of neutrons become absorbed

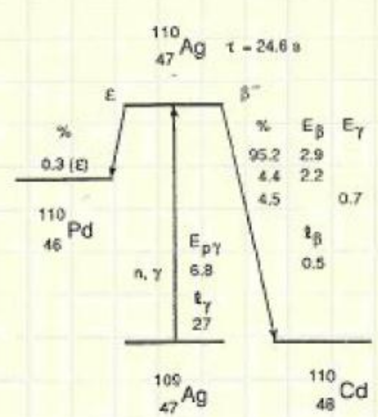
This produces a radioactive daughter nucleus wich decays emitting β & γ

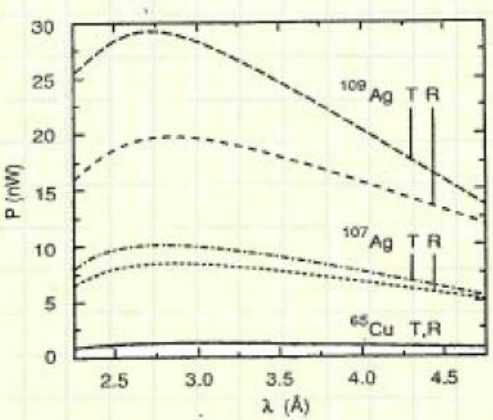
These deposit heat to Lattice & electrons

Background ~ 1 nW

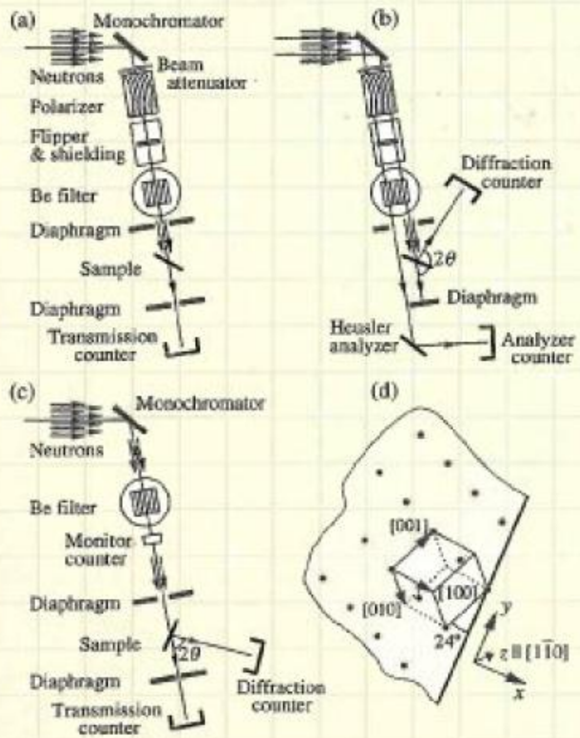
Radioactivity (~ 2µCi) gives 14 nW

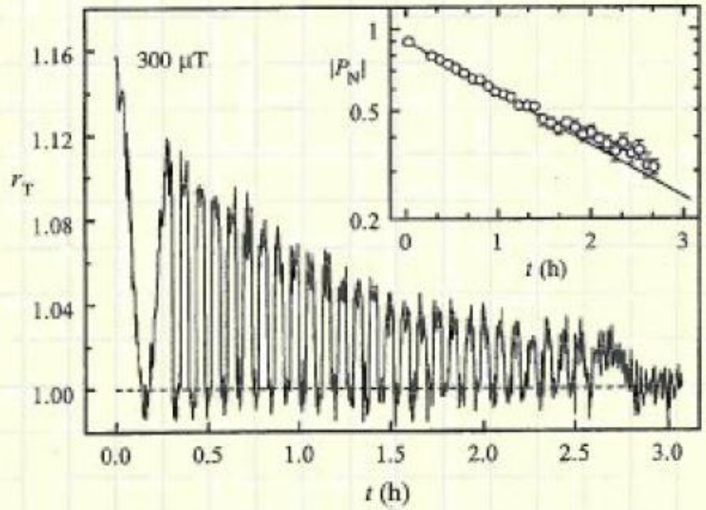






Special purpose thermometry by transmission of a neutron beam (T ~ nanoK!)

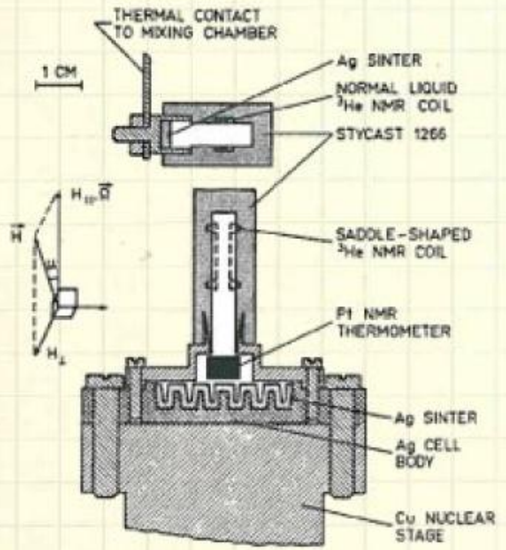


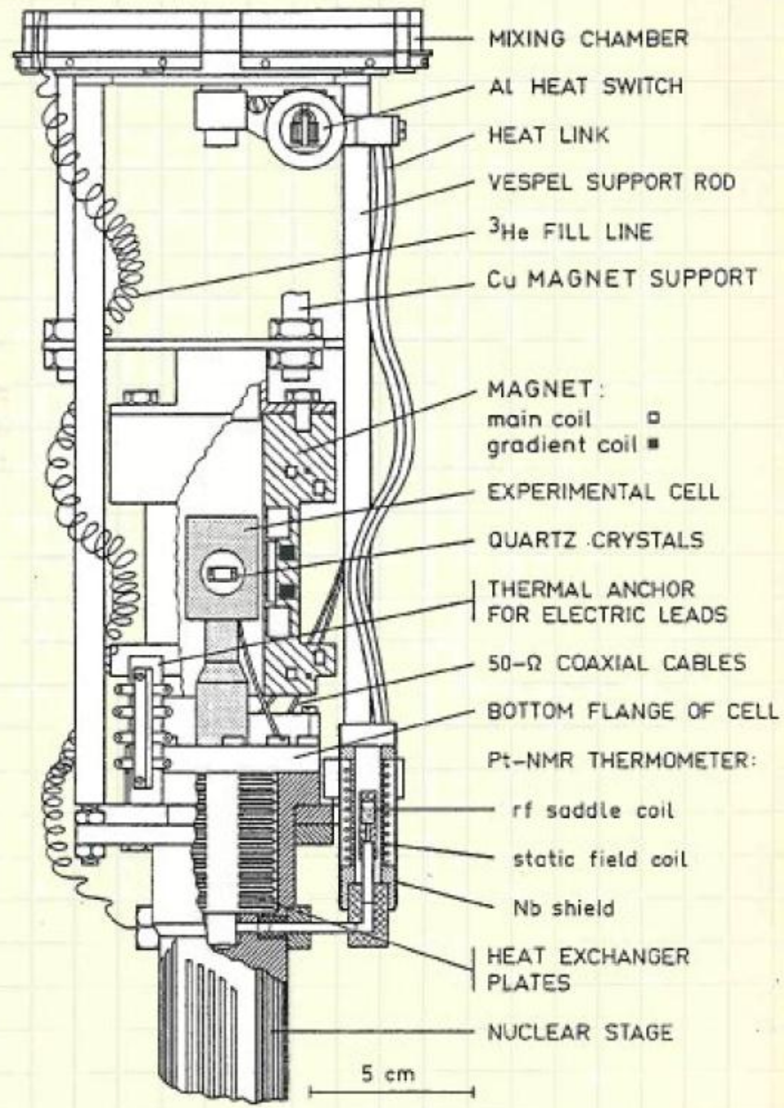


Cooling He liquid

Limited by the Kapitza resistance

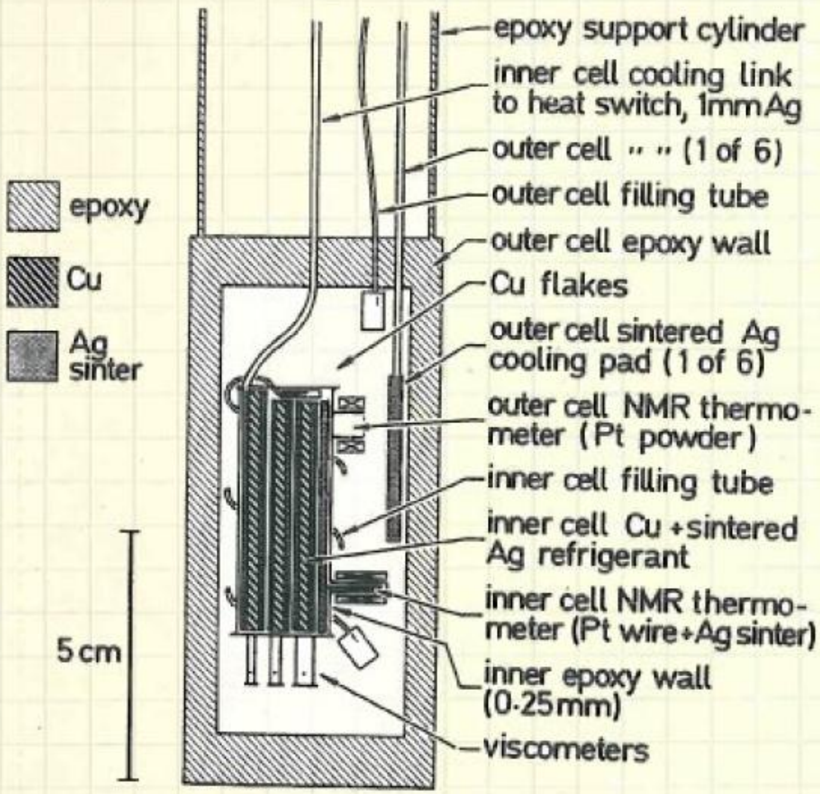
- minimized by means of metal sinter heat exchangers
  A tens of m^2
- no matter how cold your refrigerator is, $T_{He,min} \sim 0.1$ mK
- heat capacity is no problem
  $C_4 \sim 0$
  $C_3 \propto T$ (vs. nuclear spins T^{-2})





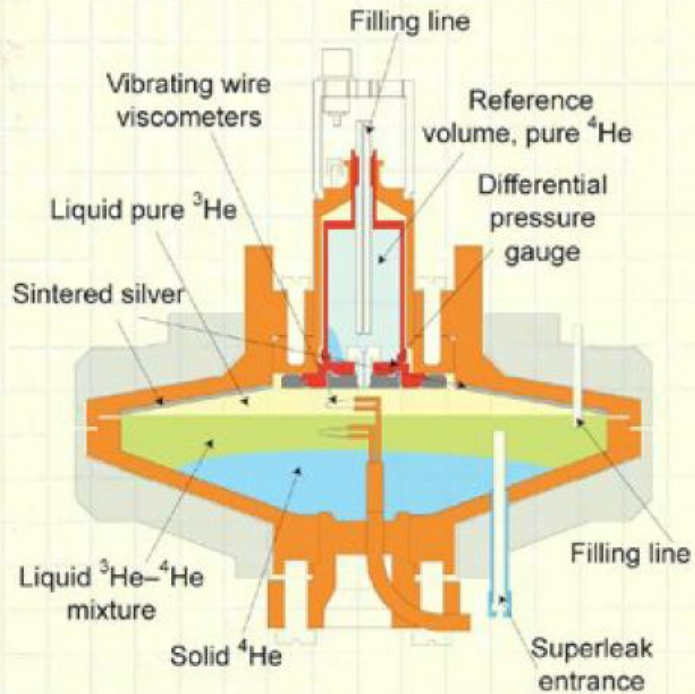
D. I. Bradley *et al.*

JLTP 57 359 (1984)



Lancaster design:
Cu nuclear refrigerant
immersed into He to be cooled

Helsinki experiment on
He mixtures: cooling by
adiabatic melting
of solid ^{4}He in superfluid ^{3}He



The record low temperatures have been achieved by operating two nuclear stages in cascade

Lowest nuclear spin temperatures:

Silver ~ 500 pK (LTL, TKK)

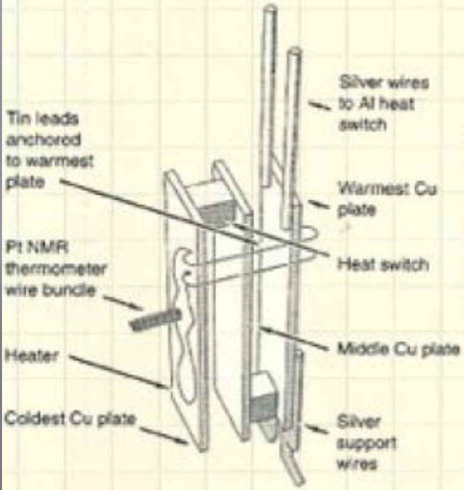
rhodium ~ 100 pK (LTL, TKK)

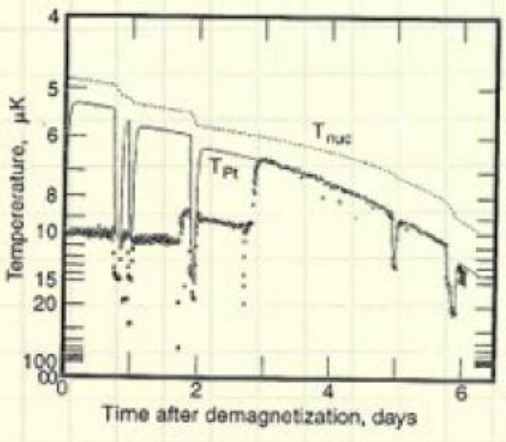
Good to have reasonably large κ, no heat switch between the stages

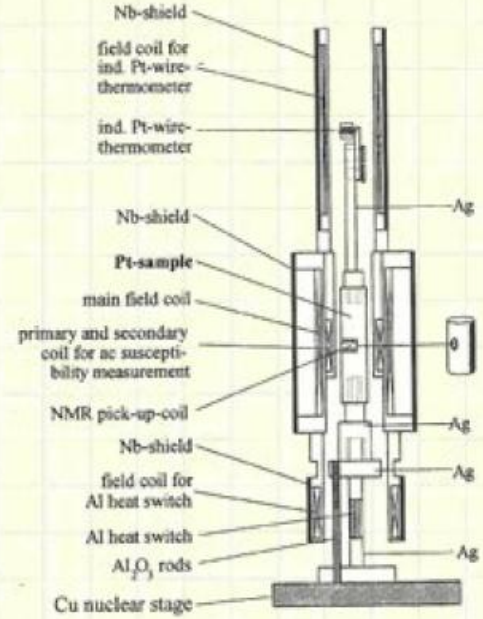
Lowest cond. electron temperatures:

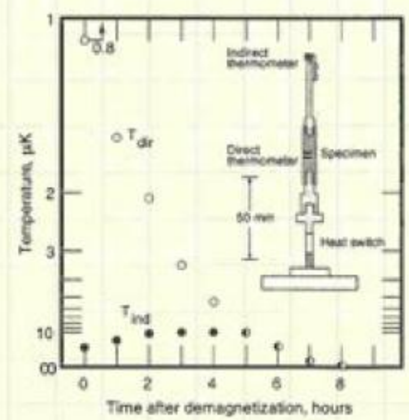
copper ~ 5 μK (Lancaster)

platinum ~ 1 μK (Bayreuth)









In cascade nuclear refrigeration the simple treatment above fails

- spin-spin interactions must not be omitted in low fields

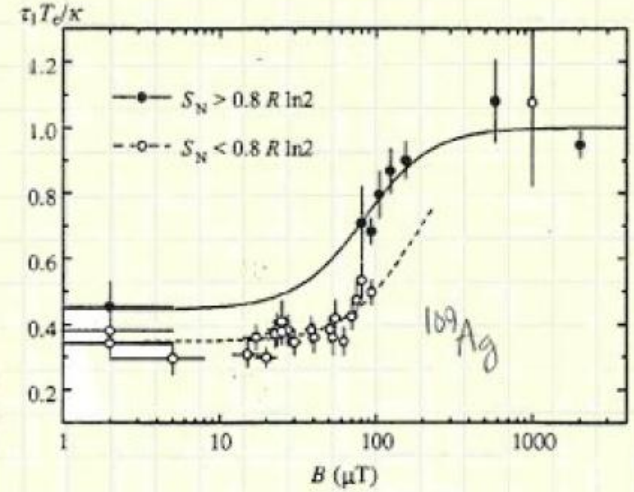
$$\left(\text{e.g.} \quad \tau_1 \approx \frac{\kappa}{T_e} \frac{B^2 + b^2}{B^2 + \alpha b^2} \; ; \; \alpha = 2 \ldots 3 \right)$$

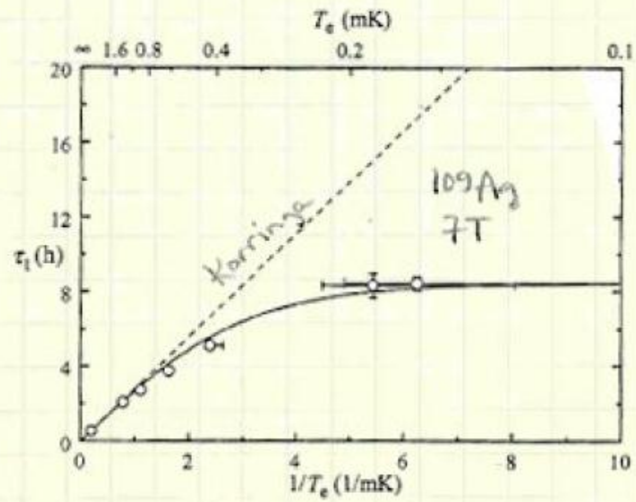
- μB is no longer small in comparison with $k_B T_e$
  $\Rightarrow$ Korringa law is modified

e.g. $I = \frac{1}{2}$

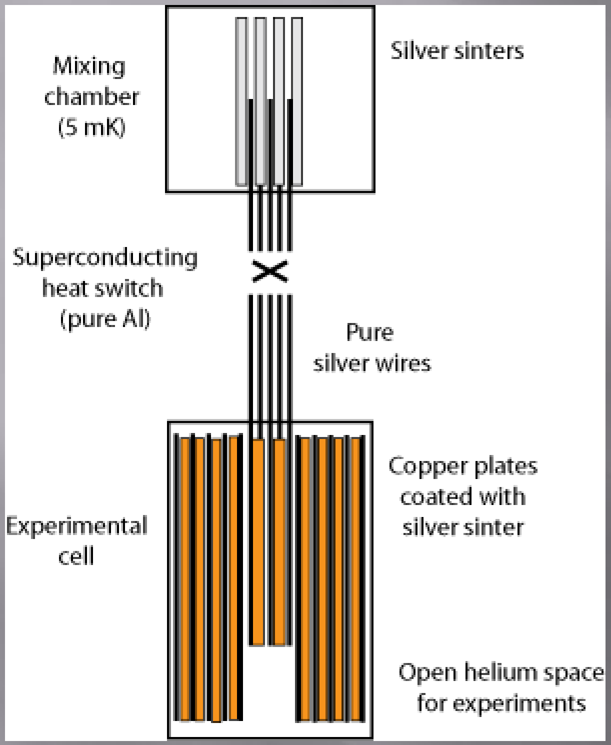
$$\tau_1 = \frac{\kappa k_B}{\mu B} \tanh \frac{\mu B}{k_B T_e}$$

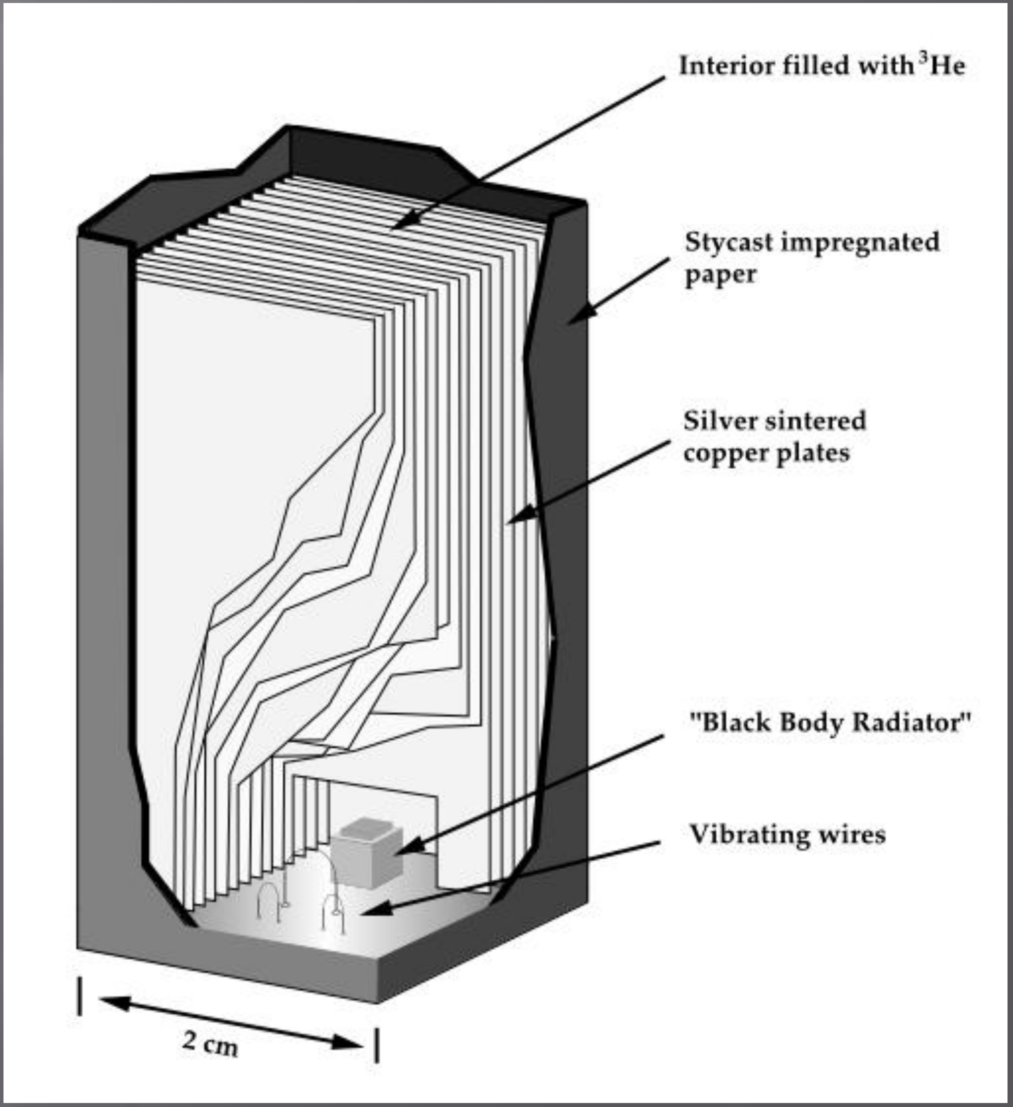
$$\rightarrow \begin{cases} \dfrac{\kappa k_B}{\mu B} & ; \; \mu B \gg k_B T_e \\ \dfrac{\kappa}{T_e} & ; \; \mu B \ll k_B T_e \end{cases}$$



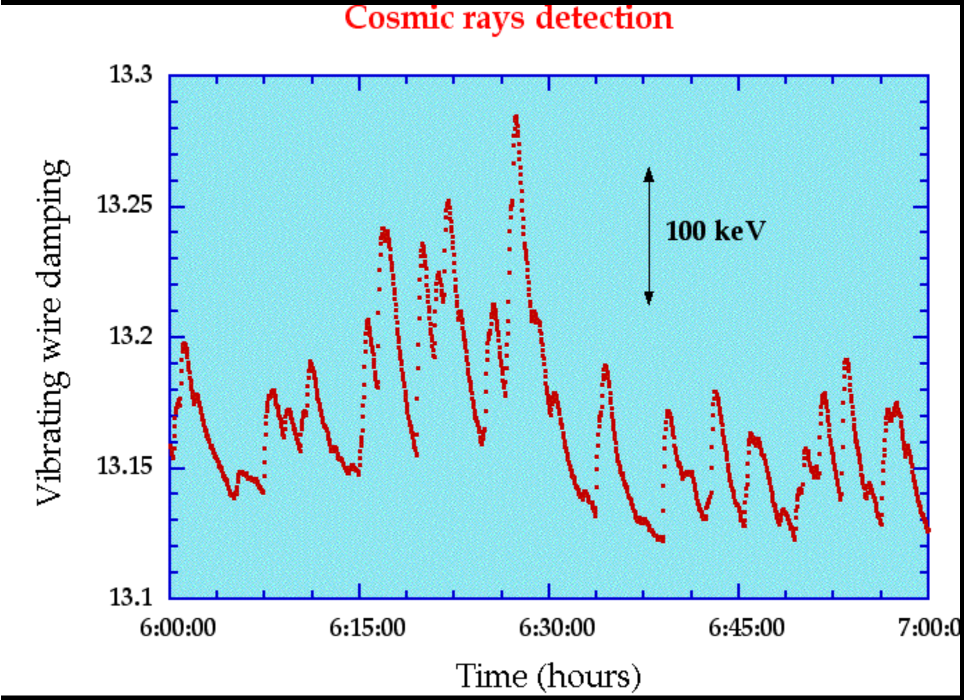


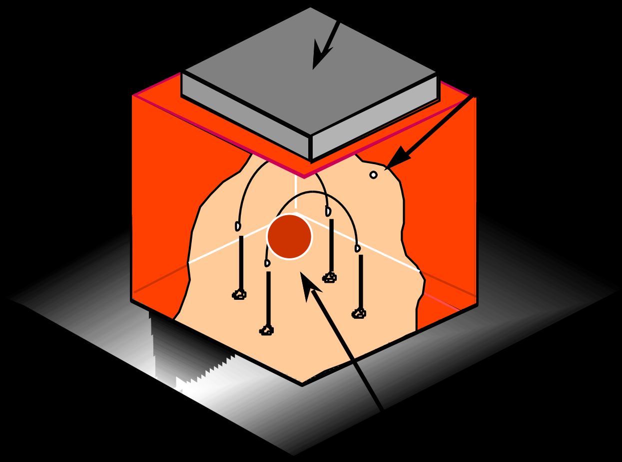
Lancaster - type



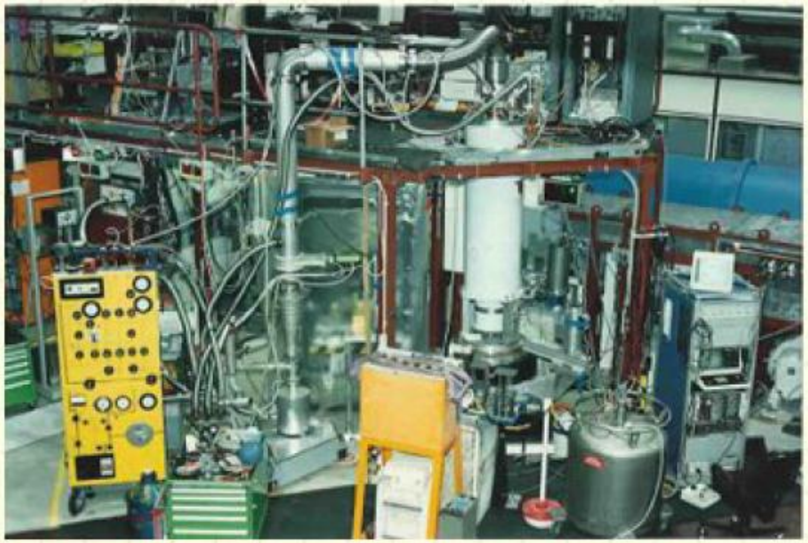


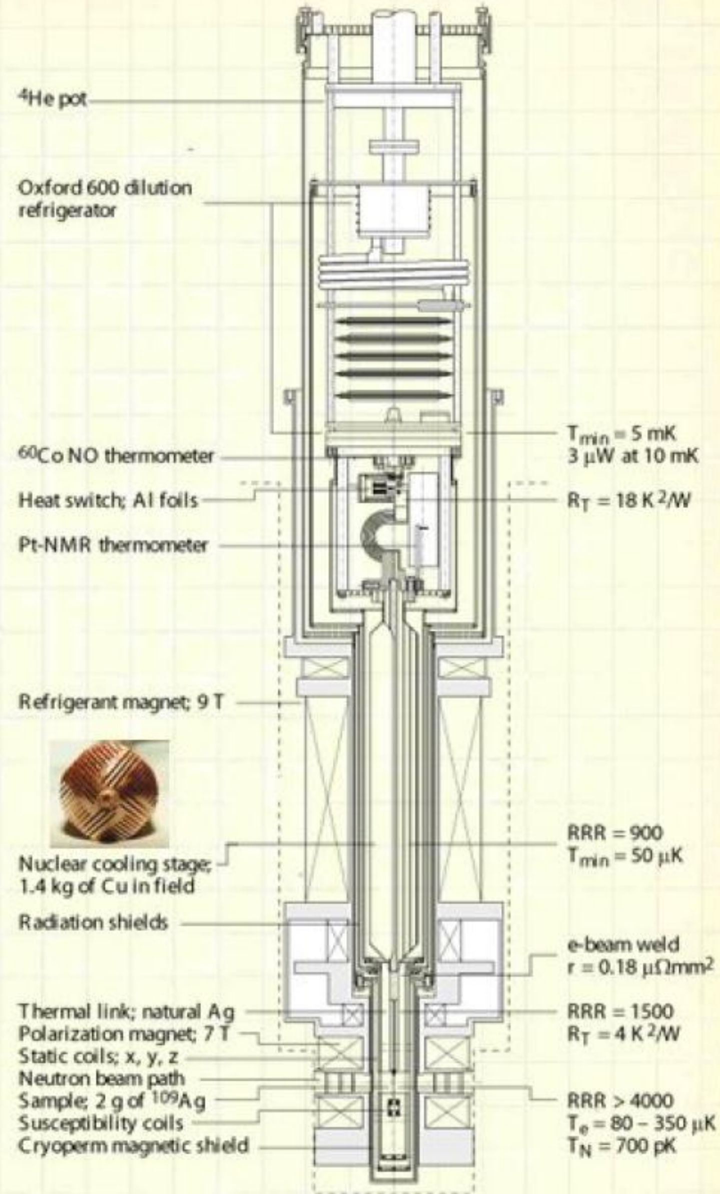
Superfluid ^{3}He bolometry



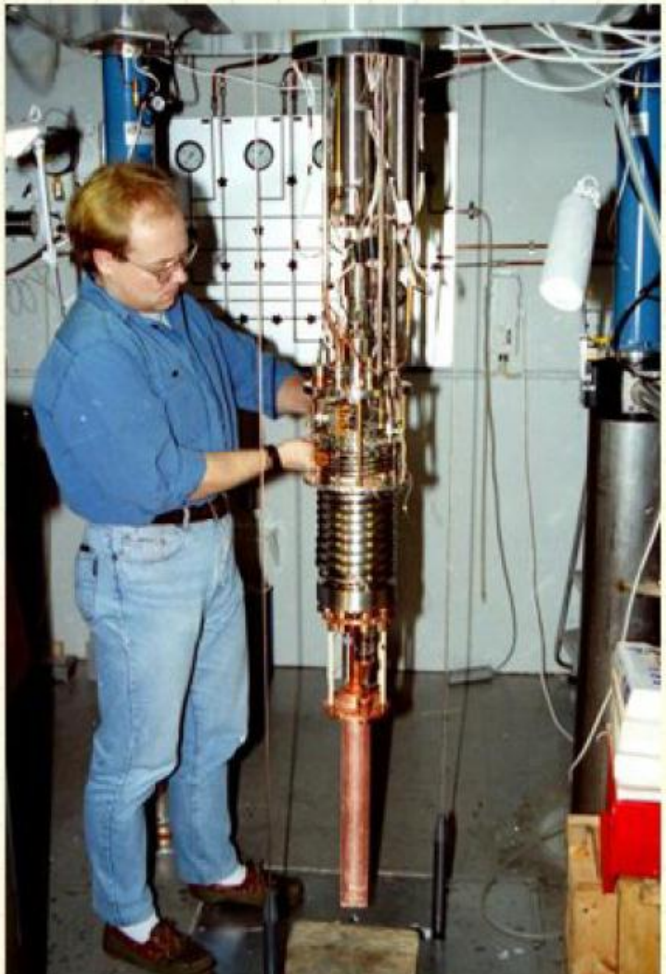


picokelvin installation at HMI in Berlin
(operational from 1992 to 1996)
Neutron diffraction on nuclear
spin ordering in silver



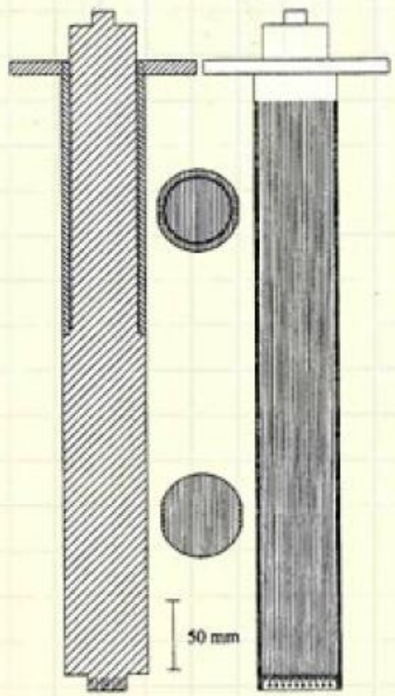


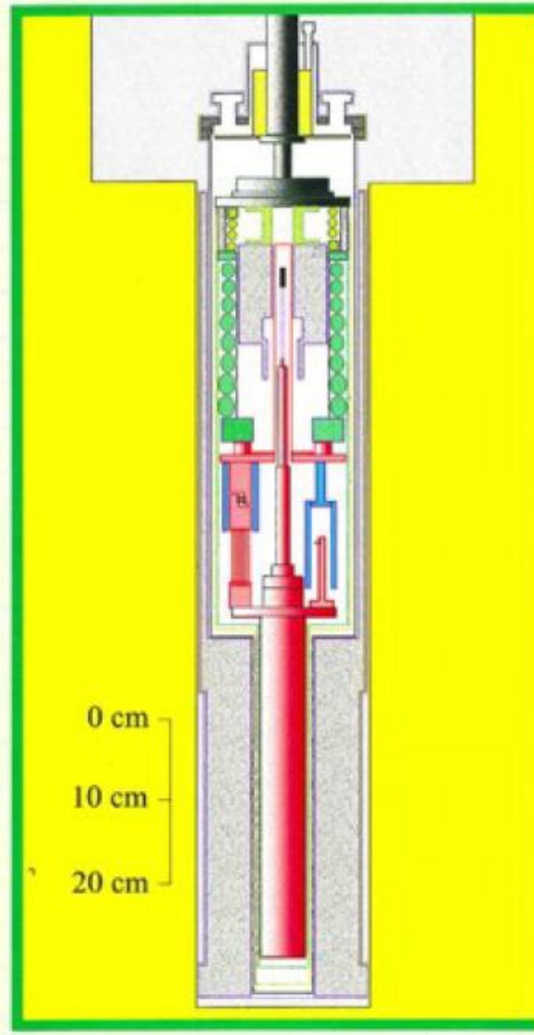
World record YKI cryostat
LTL, Helsinki: 100 pK in 1999
Nuclear magnetism in Rh, Li,
He mixtures, etc.



~2g of Rh, 7T

100 mol Cu, 9T





Reported minimum temperatures