Properties of Solids

Electrical Resistivity

Manuel Núñez Regueiro
Semiclassical theory of conduction in metals

Drude’s Law

\[ \rho = \frac{m^*}{ne^2 \tau} \]

\[ \tau = \frac{l}{v} \]

\[ l \approx T \text{ independent} \]

Residual resistivity, due to defects temperature independent

\[ \rho_0 = \frac{m^*}{ne^2 \tau} \neq f(T) \]
Electron-phonon scattering

The ideal resistance of a metal linear in $T$ and $T^5$ at low $T$

Carrier-phonon scattering

$R \propto T$

$R \propto T^5$
Landau theory of Fermi liquids: Quasiparticle-quasiparticle scattering

\[ \tau_{ee}(k_1) \propto \sum_{k_2,k_3,k_4} P(k_1,k_2;k_3,k_4) \]
\[ q = \vec{k}_3 - \vec{k}_2 \]
\[ \tau_{ee}(k_1) \propto \sum_{q,E(k_2),E(k_3)} P(q,E(k_2),E(k_3)) \]

\[ \tau_{ee}^{-1} \propto T^2 \]

very general result

delocalized s-electrons versus localized d-electrons

electron scattering against spin fluctuations of the d-electrons

then also \( T^2 \)
Quasiparticle-quasiparticle scattering

Rice PRL 1968

\[ \rho_{ee} \propto m^* \tau^{-1} \propto \langle W \rangle m^* T^2 \propto AT^2 \]

Kadowaki & Woods 1986

\[ C_v = \gamma T \propto m^* T \]
Quasiparticle-quasiparticle scattering

From Landau theory of Fermi liquids

\[ A \propto \lambda^2 \]

as

\[ T_c \propto e^{-\frac{1}{\lambda}} \]

then

\[ T_c \propto e^{-\frac{\zeta}{\sqrt{A}}} \]

There is a clear correlation between \( T_c \) and \( A \) as they vary simultaneously with an external parameter, e.g. pressure.
The same scattering that causes the resistance is the one responsible for superconductivity: the worst the conductance is the stronger the superconductivity.
Empirical relation between superconducting transition temperature and quadratic resistance temperature term

\[ T_c = f(A) \]

The relation can be explained by Landau theory of Fermi liquids

\[ T_c = \theta \cdot e^{-\zeta / \sqrt{A}} \]

Scaling yields coupling parameter \( \lambda \)
Inelastic impurity scattering

$R = AT^2$ can be also due to inelastic scattering against impurities

Koshino-Taylor

Changes in Electrical Resistance Caused by Incoherent Electron-Phonon Scattering

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(Received 13 March 1964)

A small proportion of the events in which a conduction electron is scattered by an impurity atom involve the emission or absorption of a phonon. An investigation is made of the suggestion that such incoherent electron-phonon interactions may lead to appreciable deviations from Matthiessen's rule. The effect of such processes on the electrical resistivity is found to be too small to be observable.

\[
\rho \approx 10^{-2} \left( \frac{T}{\Theta} \right)^2 \rho_0 + 500 \left( \frac{T}{\Theta} \right)^5 \rho_\Theta + \rho_0,
\]

Fig. 1. Inelastic scattering processes.

Fig. 2. Elastic scattering processes.

Fig. 3. The temperature variation of impurity resistance. The total resistance (a) is composed of a part due to elastic scattering (b) and a part due to inelastic scattering (c).

In fact, much more subtle

Reizer Sov. Phys. JETP 65 (1987) 1291

Mahan & Wang, PRB 39(1989)4926
Inelastic impurity scattering

Example: $\text{Nb}_{0.47}\text{Ti}_{0.53}$ superconducting alloy.

A proportional to $R_o$, not $R_o^2$

inelastic impurity scattering, expected due to disordered nature of sample

If $A \sim 10^{-5} R_o$ then $AT^2$ due to inelastic impurity scattering
Magnetic scattering and magnetic order

Mean field gap

$$\Delta = \sqrt{1 - \left( \frac{T}{T_c} \right)^2}$$

Nordheim

$$\tau \propto (1 - \Delta^2)^{-1}$$

De Gennes Friedel Model

Electrical Resistivity

$$\rho = \frac{m^*}{ne^2 \tau} \propto (1 - \Delta^2)$$
\[ \rho_{\text{CF}(T)} = \rho_{\text{CF}(\infty)} \frac{1}{\cosh^2 \left( \frac{\Delta}{2k_B T} \right)} \]

Temperature increases the accessible components of the localized magnetic moment.
Magnetic scattering and magnetic order

DeGennes-Friedel

Crystal Field

Electrical resistance

Temperature (K)

De Gennes Friedel Model

Electrical Resistivity

Temperature (K)
Magnetic scattering:
Kondo Effect

Mysterious minimum in resistivity 1950’s
Kondo 1964

First order scattering gives impurity scattering \(R_0\)

\[
\sum_{k''} J(k \downarrow, \uparrow \rightarrow k'' \downarrow, \uparrow) J(k'' \uparrow, \downarrow \rightarrow k' \downarrow, \uparrow) \frac{1 - f_{k''}}{\epsilon_k - \epsilon_{k''}}
\]

\[
J^2 \rho \int \frac{1 - f_{k''}}{\epsilon_k - \epsilon_{k''}} d\epsilon_{k''} = J^2 \rho \int_{\epsilon_F}^{\epsilon_D} \frac{1}{\epsilon_k - \epsilon_{k''}} d\epsilon_{k''}
\]

\[
J^2 \rho \log \left( \frac{\epsilon_k - \epsilon_F}{\epsilon_k - \epsilon_D} \right)
\]

\[
R(T) = R_0 \left[ 1 + 2J \rho \log \left( \frac{k_B T}{D - \epsilon_F} \right) \right]
\]
Magnetic scattering: Kondo Effect

\[ R(T) = R_0 \left[ 1 + 2J \rho \log \left( \left| \frac{k_B T}{D - \epsilon_F} \right| \right) \right] \]
Itinerant antiferromagnetism
Spin density waves
Charge density waves

\[ \chi_Q = \sum_k \frac{(f_k - f_{k+Q})}{(\varepsilon_k - \varepsilon_{k+Q})} \]

Nesting wavevector

portions of Fermi surface gapped & lost for conduction

SDW in Chromium
Itinerant antiferromagnetism
Spin density waves
Charge density waves

\[ \rho \propto \frac{m^*}{n\tau} \]

Nordheim \( \tau \propto (1 - \Delta(T)^2)^{-1} \)

\[ n_{\text{gapped}}(T) \propto \Delta(T) \]

\[ n = n - n_{\text{gapped}}(T) \]

\[ \Delta(T) = \Delta_0 \sqrt{1 - \left(\frac{T}{T_c}\right)^2} \]

\[ \rho \propto \frac{m^*}{n\tau} \propto \frac{m^* \cdot (1 - \Delta^2(T))}{n - n_{\text{gapped}} \cdot \Delta(T)} + \rho_{ph}T \]

**SDW in Chromium**
Examples with two DW

**CDW**

$1D - NbSe_3$

**SDW**

$2D - Na_{0.5}CoO_2$

From the ratio of the slopes we can estimate the percentage of the FS that disappears at each transition.
Semiconductors

The exponential carrier population controls the temperature resistivity dependance in intrinsic semiconductors

\[ n \propto e^{-\frac{\Delta}{k_B T}} \]

But impurities give extrinsic carriers

\[ \rho \propto \frac{m^*}{n \tau} \propto \left( \frac{1}{n_{\text{int}} e^{-\frac{\Delta}{k_B T}} + n_{\text{ext}}} \right) \cdot \frac{\Delta}{k_B T} \]

semiconducting gap \( \Delta \)
Localization

Defects cause localized states where effective masses are higher.

Interference through elastic scattering also causes localization.
Localization

1. Conduction by thermal activation above mobility edge $E_c$

$$\sigma_1 \propto e^{-\frac{(E_c - E_F)}{k_BT}}$$

2. Activation to a neighbouring localized state

average energy separation

$$\Delta_\xi \sim \left[ n(0) \xi^d \right]^{-1}$$

$$\sigma_2 \propto e\left(-\frac{\Delta_\xi}{k_BT}\right)$$

3. Variable range hopping i.e. between sites of similar energy

dependence of hopping with distance $\sim e^{\frac{2L}{\xi}}$

$$\Delta_L \sim \left[ n(0) L^d \right]^{-1} \sim \Delta_\xi \left(\frac{\xi}{L}\right)^d \quad (L \gg \xi)$$

Total dependence $\sim e^{-\frac{2L}{\xi} - \frac{\Delta_L}{k_BT}}$

optimizing

$$\sigma_3 \propto e^{-C\left(\frac{T_0}{T}\right)^{\frac{1}{d+1}}}$$

where $k_B T_0 \sim \Delta_\xi$
Localization

\[ 2D - (Bi, Pb)_2Ba_3Co_2O_y \]

In bulk materials only

3D behaviour

\[ \frac{1}{4} \rightarrow \frac{1}{3+1} \rightarrow 3D \]

\[ \frac{1}{2} \ not \ \frac{1}{1+1} \] but "Coulomb" Gap!
“Heavy Carriers” : Polarons

The carrier is dressed by a lattice deformation must jump with it

Thermal activated hopping \( \langle w \rangle \sim e^{-\hbar \omega^* / k_B T} \)

the mobility is \( \mu = \frac{2ea^2}{k_B T} \langle w \rangle \) (Einstein diffusion relation)

hence \( \rho \sim \frac{1}{\mu} \sim T \cdot e^{\hbar \omega^* / k_B T} \)
"Heavy Carriers": Polarons

Manganites

![Graphs showing electrical resistance and magnetic properties of Manganites](image-url)
“Heavy Carriers” : Polarons

Ln-1111

Also “polaron-like” behaviour in Iron superconductors

Æ-122
Cuprates

Underdoped $\rho \sim$ Crystal field type

Optimal doping $\rho \sim T$

Overdoped $\rho \sim T^2$

Free Fermions

SuperConductor
Conclusions

The behavior of the electrical resistivity can yield many hints into the understanding of the physics of materials.