Properties of Solids

Electrical Resistivity

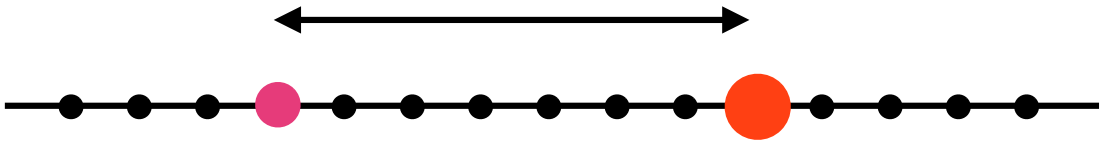
Manuel Núñez Regueiro

Semiclassical theory of conduction in metals

Drude's Law

$$\rho = \frac{m^*}{ne^2\tau} \qquad \tau = \frac{l}{v}$$

$$l \approx T \ \ independant$$



Residual resistivity,
due to defects
temperature independant

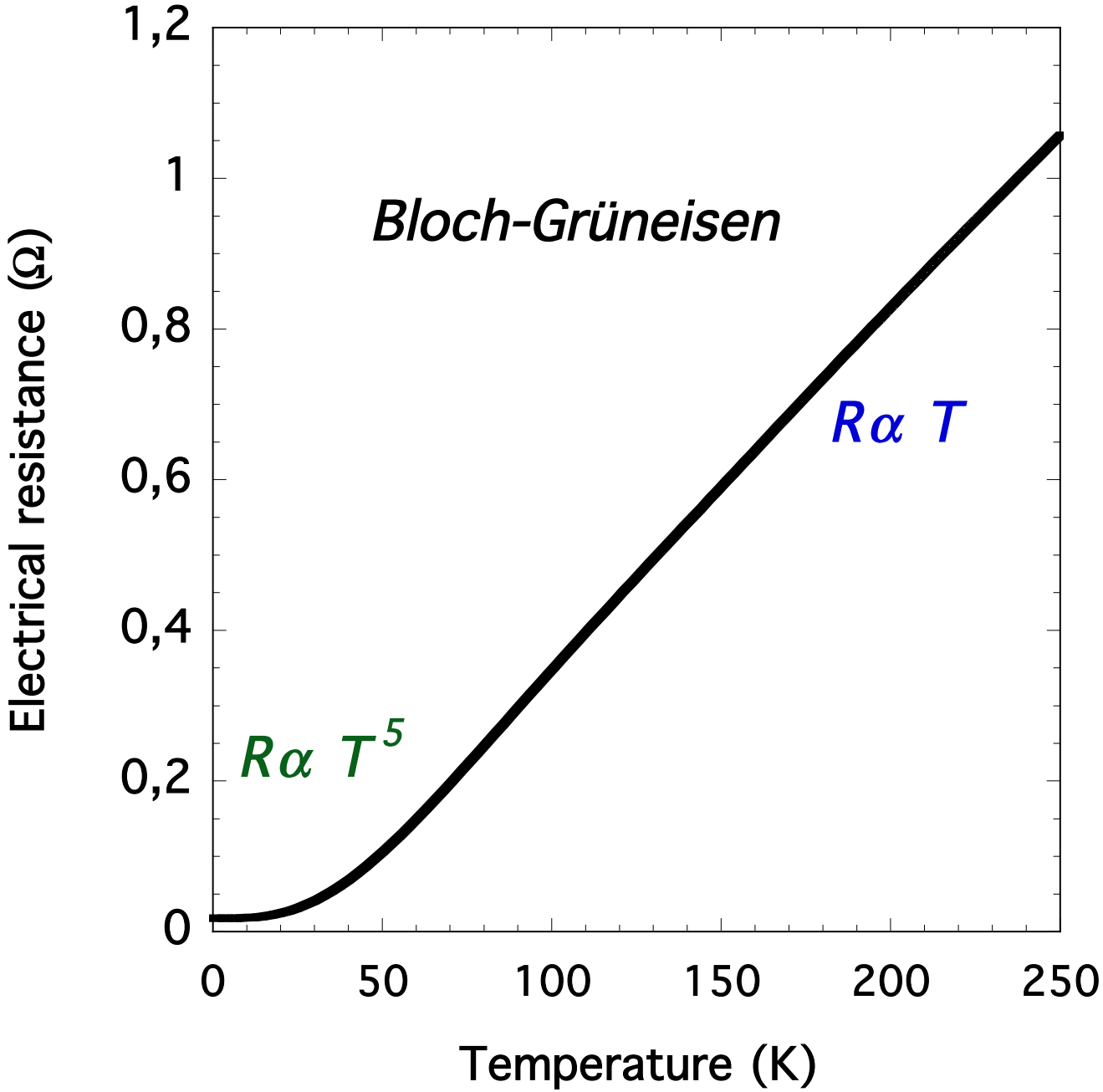
$$\rho_0 = \frac{m^*}{ne^2\tau} \neq f(T)$$

Electron-phonon scattering

The ideal resistance of a metal

linear in T and T^5 at low T

Carrier-phonon scattering

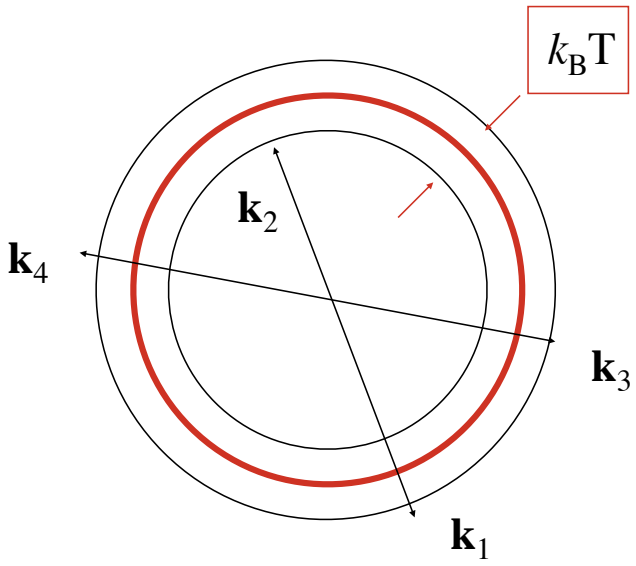


Landau theory of Fermi liquids : Quasiparticle-quasiparticle scattering

$$\tau_{ee}^{-1}(\vec{k}_1) \propto \sum_{k_2,k_3,k_4} P\left(\vec{k}_1,\vec{k}_2;\vec{k}_3,\vec{k}_4\right)$$

$$q = \vec{k}_3 - \vec{k}_2$$

$$\tau_{ee}^{-1}(\vec{k}_1) \propto \sum_{q\ ,E(k_2),E(k_3)} P\left(q,E(\vec{k}_2),E(\vec{k}_3)\right)$$



$$\boxed{\tau_{ee}^{-1} \propto T^2}$$

very general result

delocalized ***s***-electrons versus localized ***d***-electrons

electron scattering against spin fluctuations of the ***d***-electrons

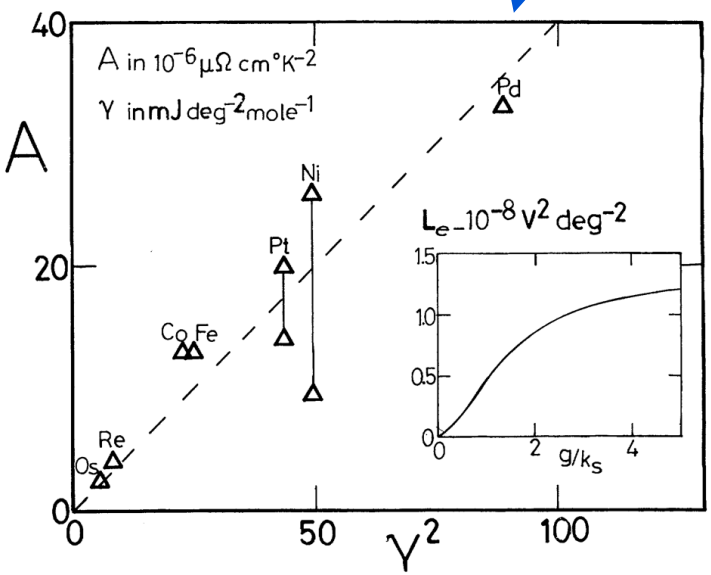
then also T^2

Quasiparticle-quasiparticle scattering

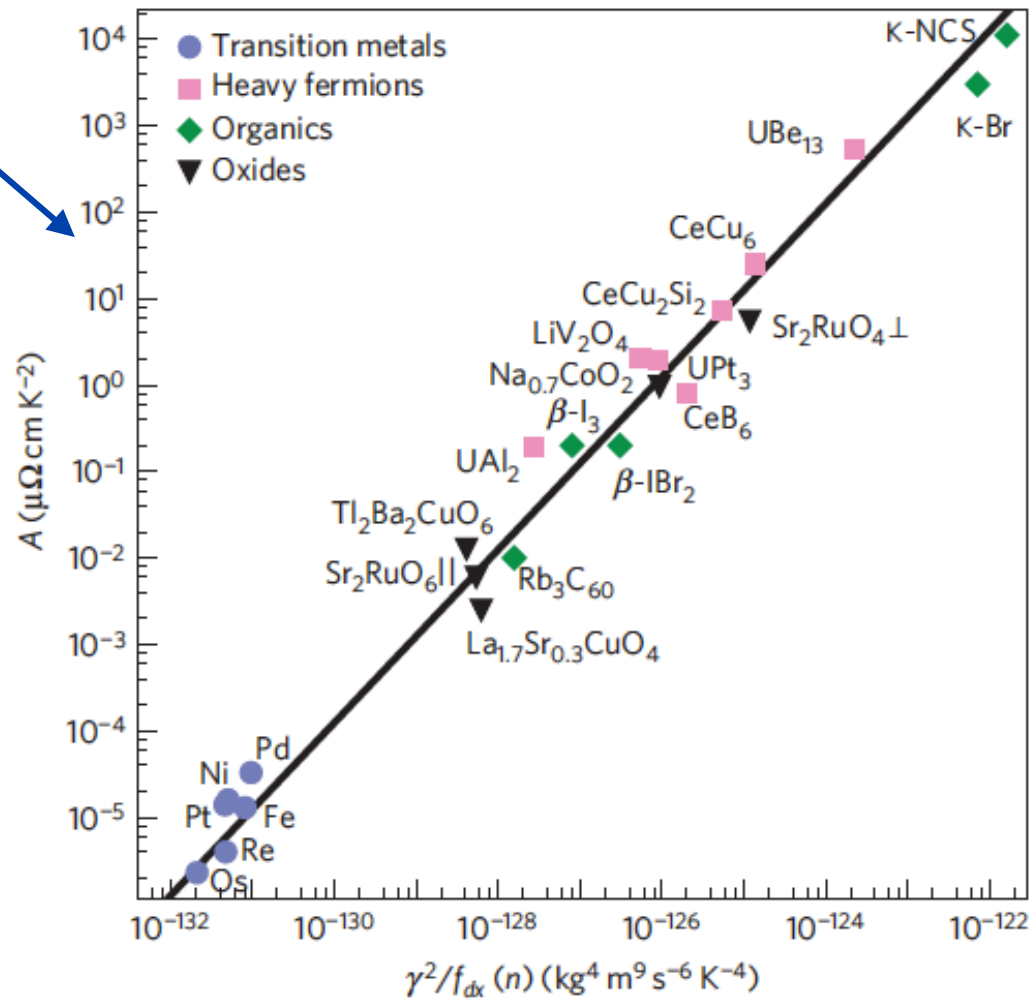
Rice PRL 1968

$$\rho_{ee} \propto m^* \tau^{-1} \propto \langle W \rangle m^{*2} T^2 \propto A T^2$$

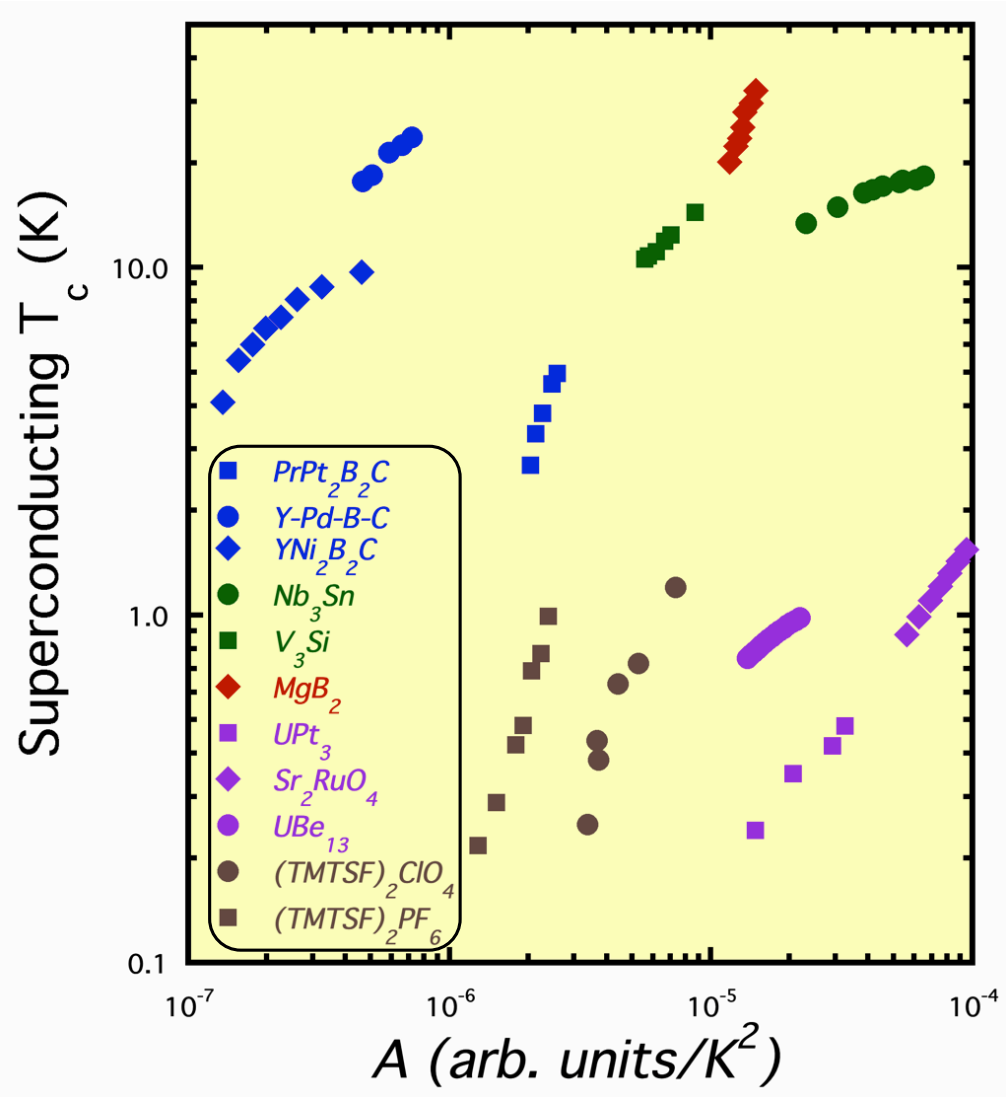
Kadowaki&Woods 1986



$$C_v = \gamma T \propto m^* T$$



Quasiparticle-quasiparticle scattering



There is a clear correlation between T_c and A as they vary simultaneously with an external parameter, e.g. pressure

From Landau theory of Fermi liquids

$$A \propto \lambda^2$$

as

$$T_c \propto e^{-\frac{1}{\lambda}}$$

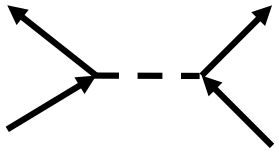
then

$$\boxed{T_c \propto e^{-\frac{\varsigma}{\sqrt{A}}}}$$

Quasiparticle-quasiparticle scattering

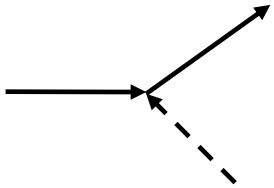
Superconductivity

$$T_c \propto e^{-\varsigma/\sqrt{A}} \propto e^{-1/N(0)V}$$



attractive **V**

Low Temperature
Phonon mediated
e-e scattering
AT^2 dependence



High Temperature
e-phonon scattering
T dependence

Tc

Temperature

The same scattering that causes the resistance is the one responsible for superconductivity :
the worst the conductance is the stronger the superconductivity

Electron-Phonon Enhancement of Electron-Electron Scattering in Al

A. H. MacDonald
Division of Physics, National Research Council of Canada, Ottawa, Ontario K1A 0R6, Canada
(Received 25 October 1979)

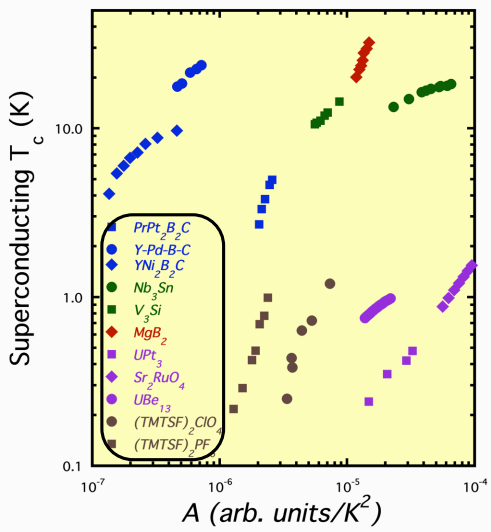
The influence of the electron-phonon interaction on electron-electron scattering in simple metals has been described within the framework of Landau Fermi-liquid theory. The predicted electron-electron scattering contribution to the low-temperature resistivity of Al is enhanced by a factor of ~20 by the electron-phonon interaction and is in excellent agree-

Aluminum!

Quasiparticle-quasiparticle scattering

Empirical relation between superconducting transition temperature and quadratic resistance temperature term

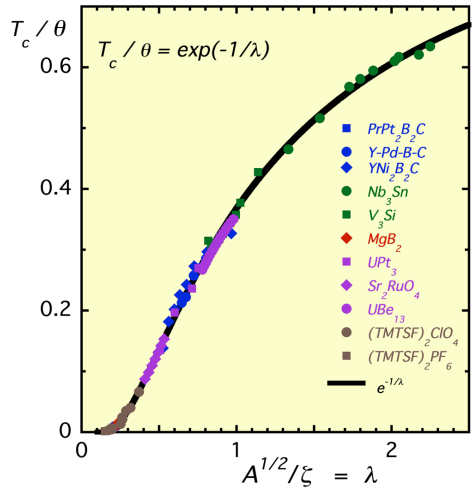
$$T_c = f(A)$$



The relation can be explained by Landau theory of Fermi liquids

$$T_c = \theta \cdot e^{-\varsigma/\sqrt{A}}$$

Scaling yields coupling parameter λ



Inelastic impurity scattering

$R = AT^2$ can be also due to inelastic scattering against impurities
Koshino-Taylor

Changes in Electrical Resistance Caused by Incoherent Electron-Phonon Scattering

P. L. Taylor*
Department of Physics, Case Institute of Technology, Cleveland, Ohio
(Received 13 March 1964)

A small proportion of the events in which a conduction electron is scattered by an impurity atom involve the emission or absorption of a phonon. An investigation is made of the suggestion that such incoherent electron-phonon interactions may lead to appreciable deviations from Matthiessen's rule. The effect of such processes on the electrical resistivity is found to be too small to be observable.

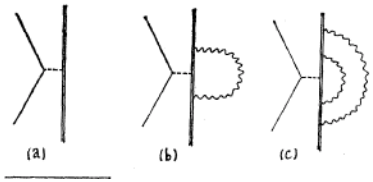


Fig. 2. Elastic scattering processes.

$$\rho \approx 10^{-2}\left(\frac{T}{\Theta}\right)^2 \rho_0 + 500\left(\frac{T}{\Theta}\right)^5 \rho_\Theta + \rho_0 ,$$

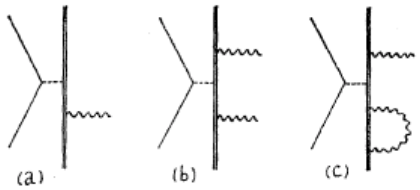
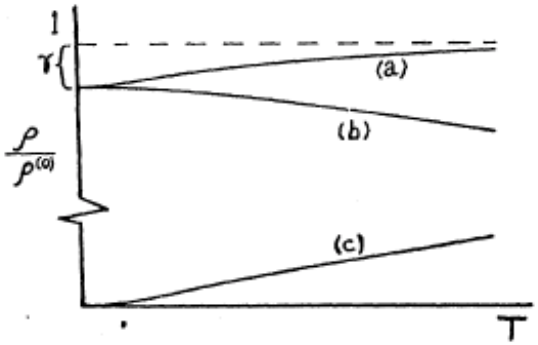


Fig. 1. Inelastic scattering processes.

Fig. 3. The temperature variation of impurity resistance. The total resistance (a) is composed of a part due to elastic scattering (b) and a part due to inelastic scattering (c).



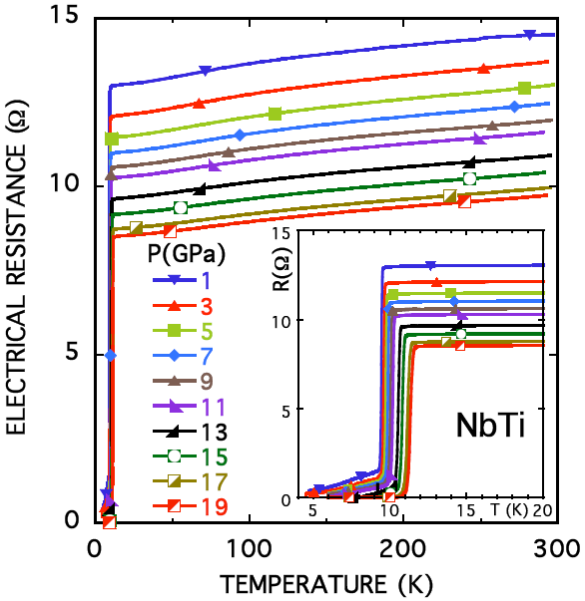
In fact, much more subtle

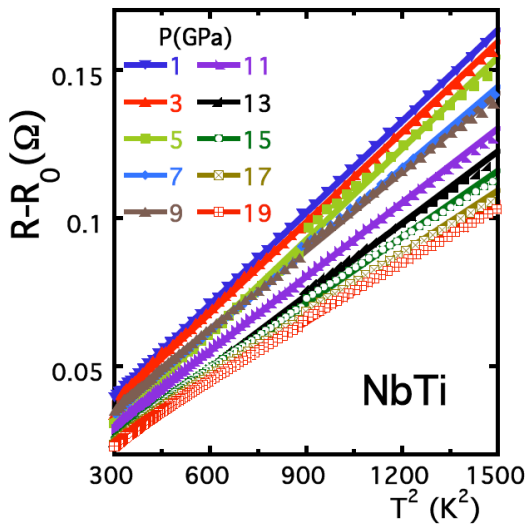
Reizer Sov. Phys. JETP ***65*** *(1987) 1291*

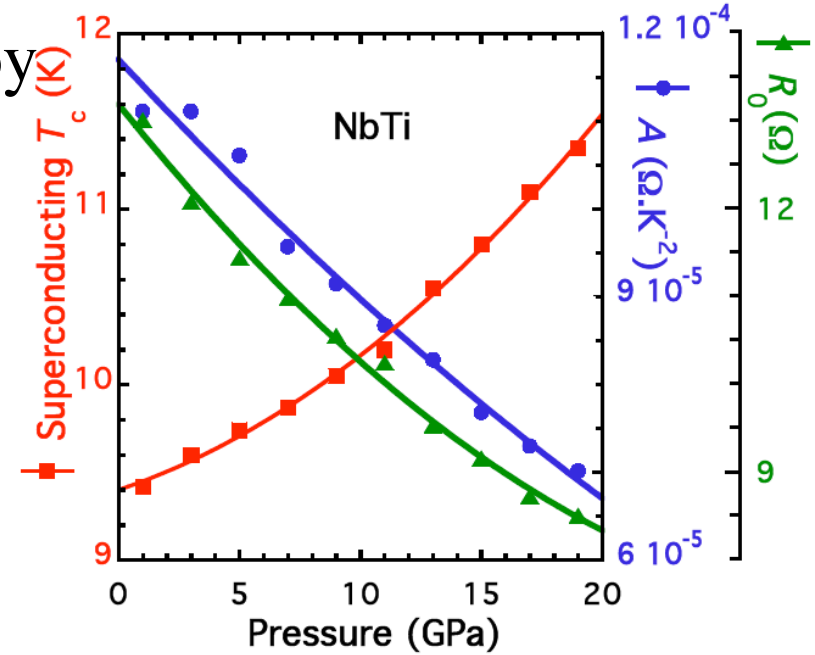
Mahan & Wang, PRB ***39****(1989)4926*

Inelastic impurity scattering

Example : $Nb_{0.47}Ti_{0.53}$ superconducting alloy

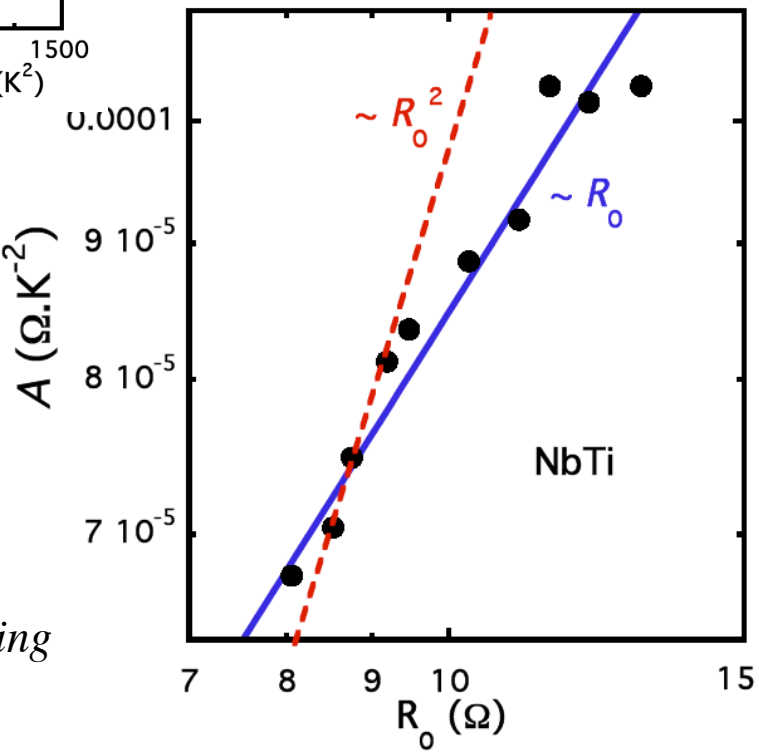






A proportional to $\boldsymbol{R_o}$, not $\boldsymbol{R_o^2}$

inelastic impurity scattering, expected due to disordered nature of sample



If $A \sim 10^{-5} R_0$ *then* AT^2 *due to inelastic impurity scattering*

Magnetic scattering and magnetic order

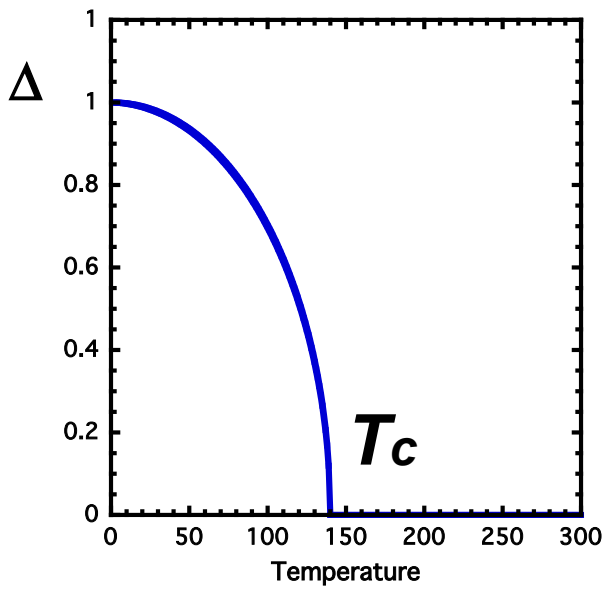
Mean field gap

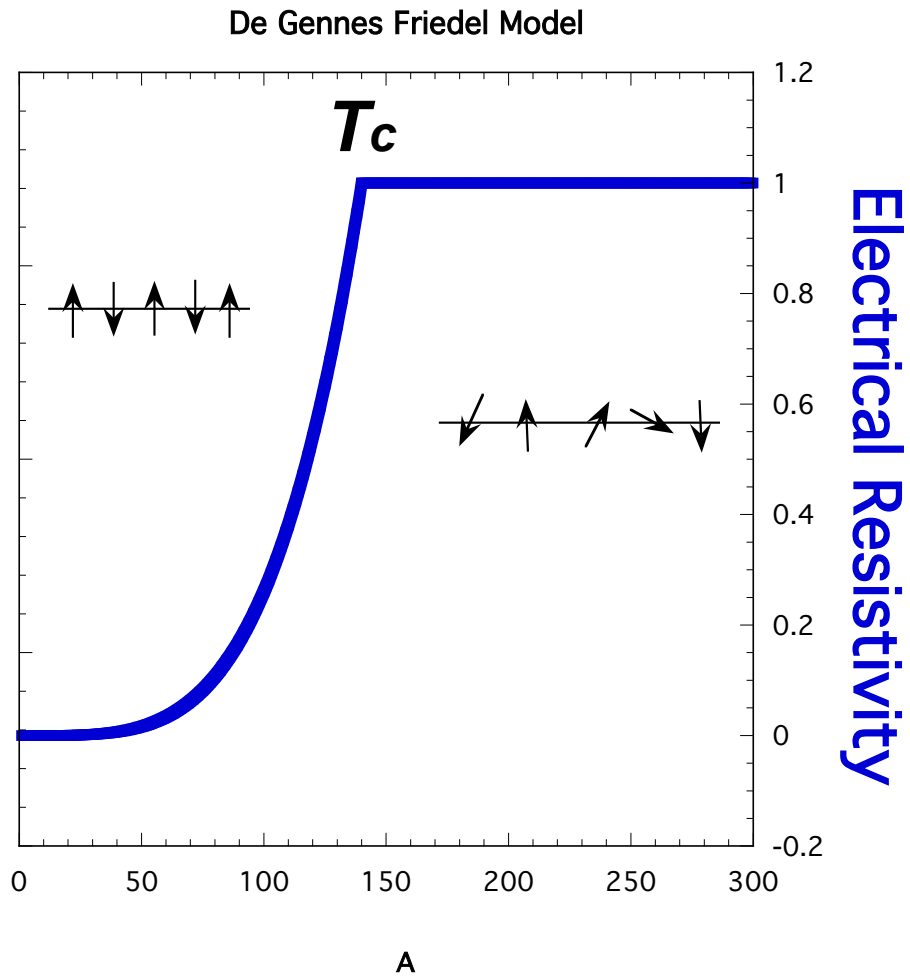
$$\Delta = \sqrt{1 - \left(\frac{T}{T_c}\right)^2}$$

Nordheim

$$\tau \propto (1 - \Delta^2)^{-1}$$

$$\rho = \frac{m^*}{ne^2\tau} \propto (1 - \Delta^2)$$

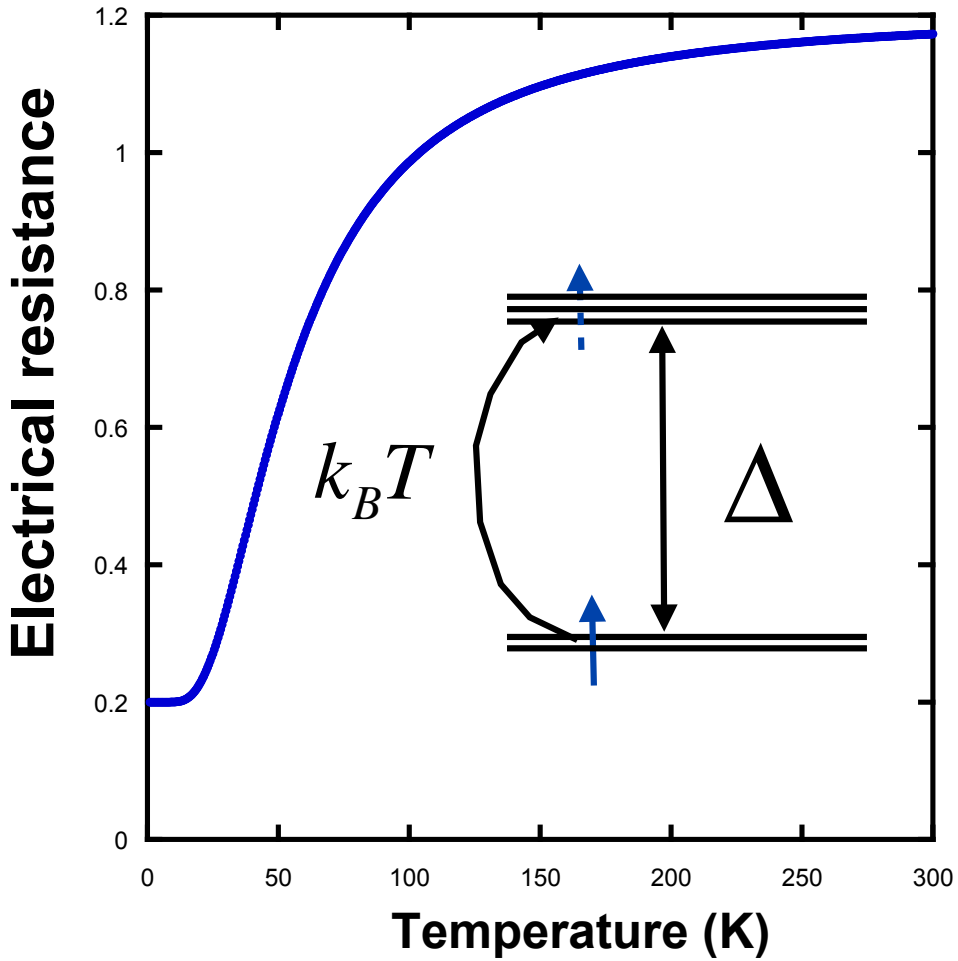




Magnetic Scattering
Crystal Field Resistivity

$$\rho_{CF(T)} = \rho_{CF(\infty)} \frac{1}{\cosh^2\left(\Delta / 2k_B T\right)}$$

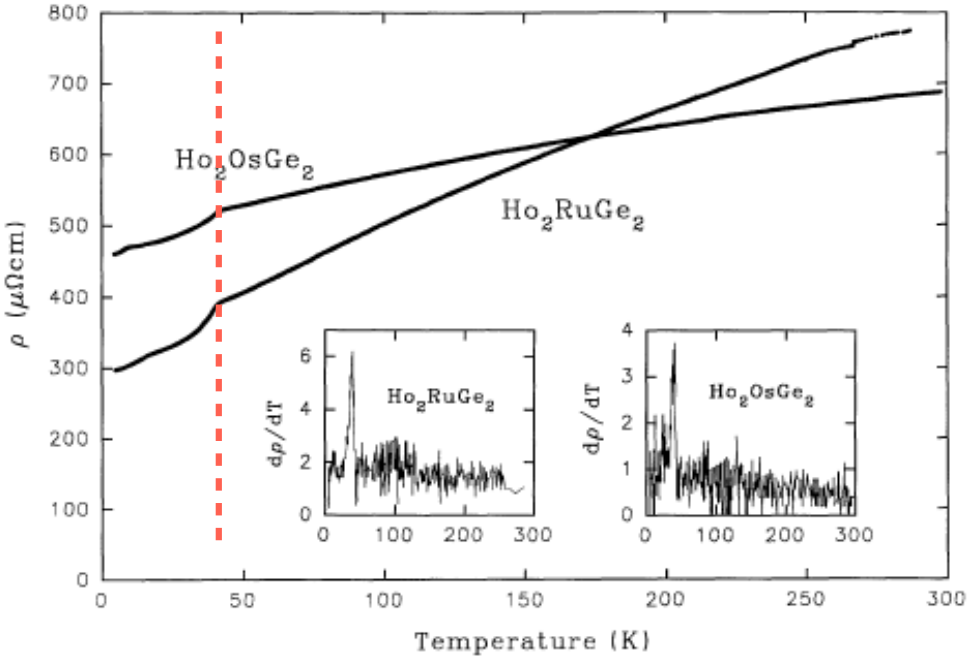
Temperature increases the accesible components of the localized magnetic moment

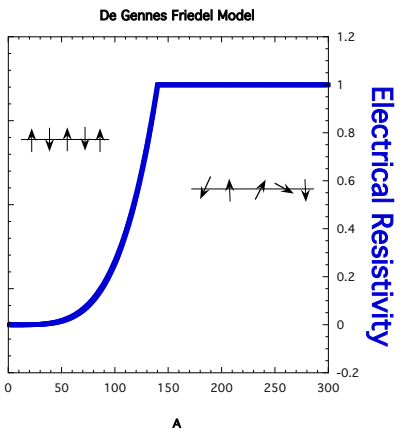


Magnetic scattering and magnetic order

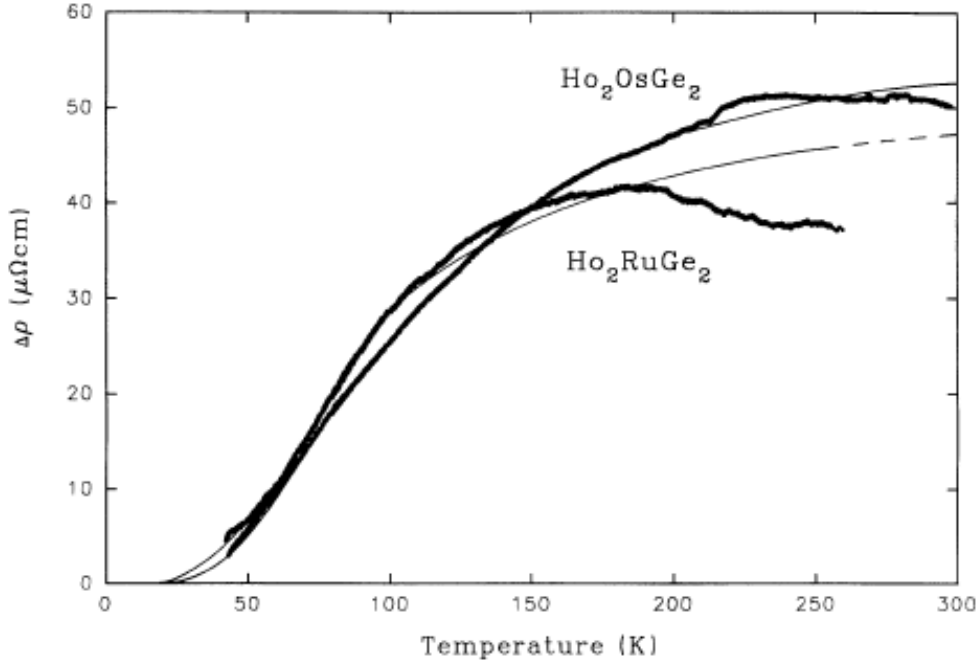
T_N

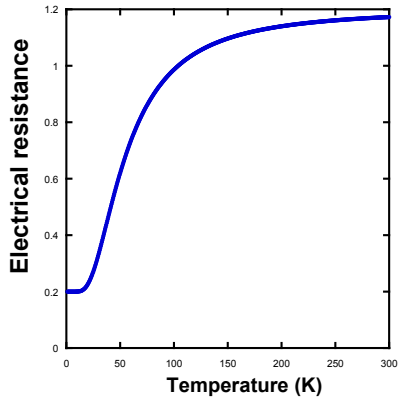
DeGennes-Friedel





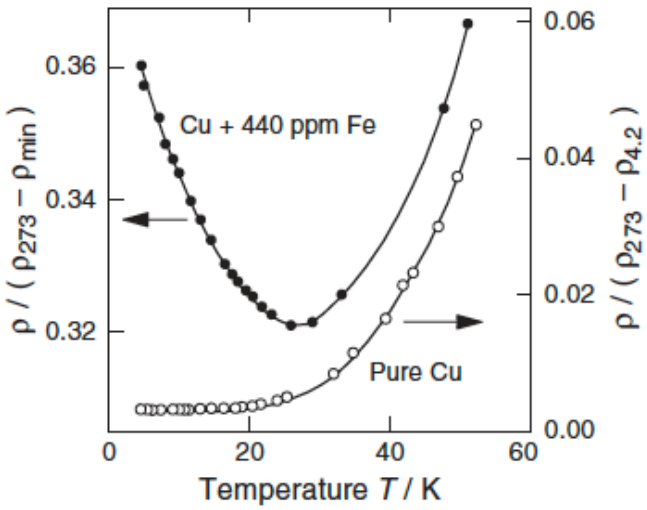
Crystal Field



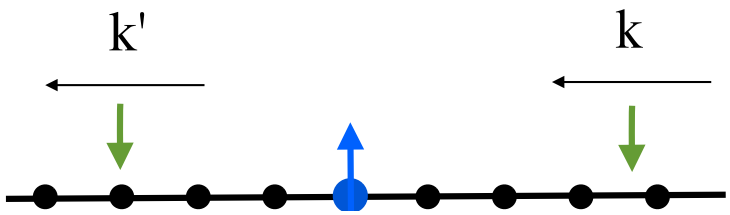


Magnetic scattering : Kondo Effect

Mysterious minimum in resistivity 1950's
Kondo 1964



First order scattering gives impurity scattering *Ro*

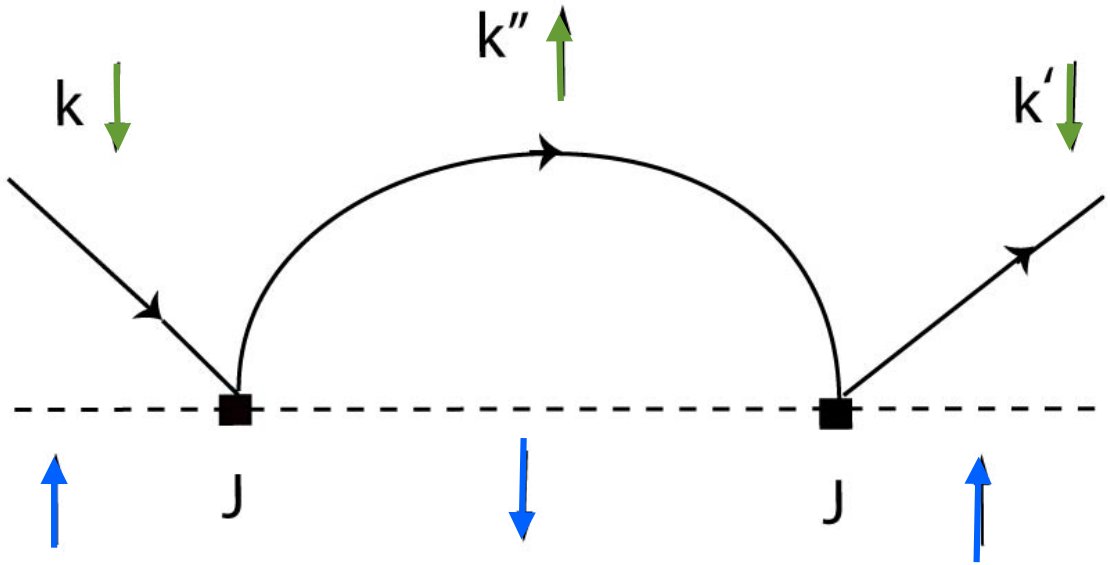


$$\sum_{k''} J(k\downarrow,\uparrow\rightarrow k''\uparrow,\downarrow).J(k''\uparrow,\downarrow\rightarrow k'\downarrow,\uparrow)\frac{1-f_{k''}}{\epsilon_k-\epsilon_{k''}}$$

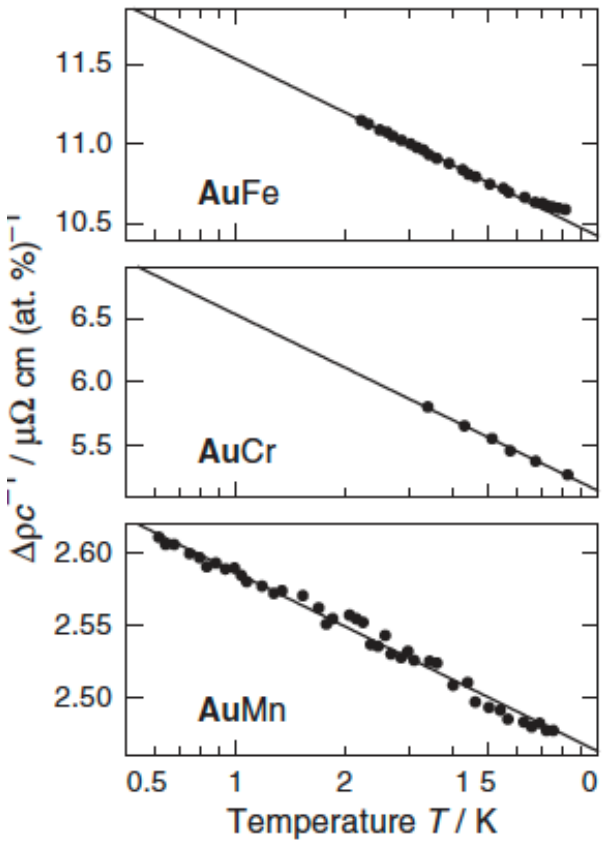
$$J^2\rho\int\frac{1-f_{k''}}{\epsilon_k-\epsilon_{k''}}d\epsilon_{k''}=J^2\rho\int_{\epsilon_F}^{D}\frac{1}{\epsilon_k-\epsilon_{k''}}d\epsilon_{k''}$$

$$J^2\rho\log\left(\left|\frac{\epsilon_k-\epsilon_F}{\epsilon_k-D}\right|\right)$$

$$\boxed{R(T)=R_0\left[1+2J\rho\log\left(\left|\frac{k_BT}{D-\epsilon_F}\right|\right)\right]}$$

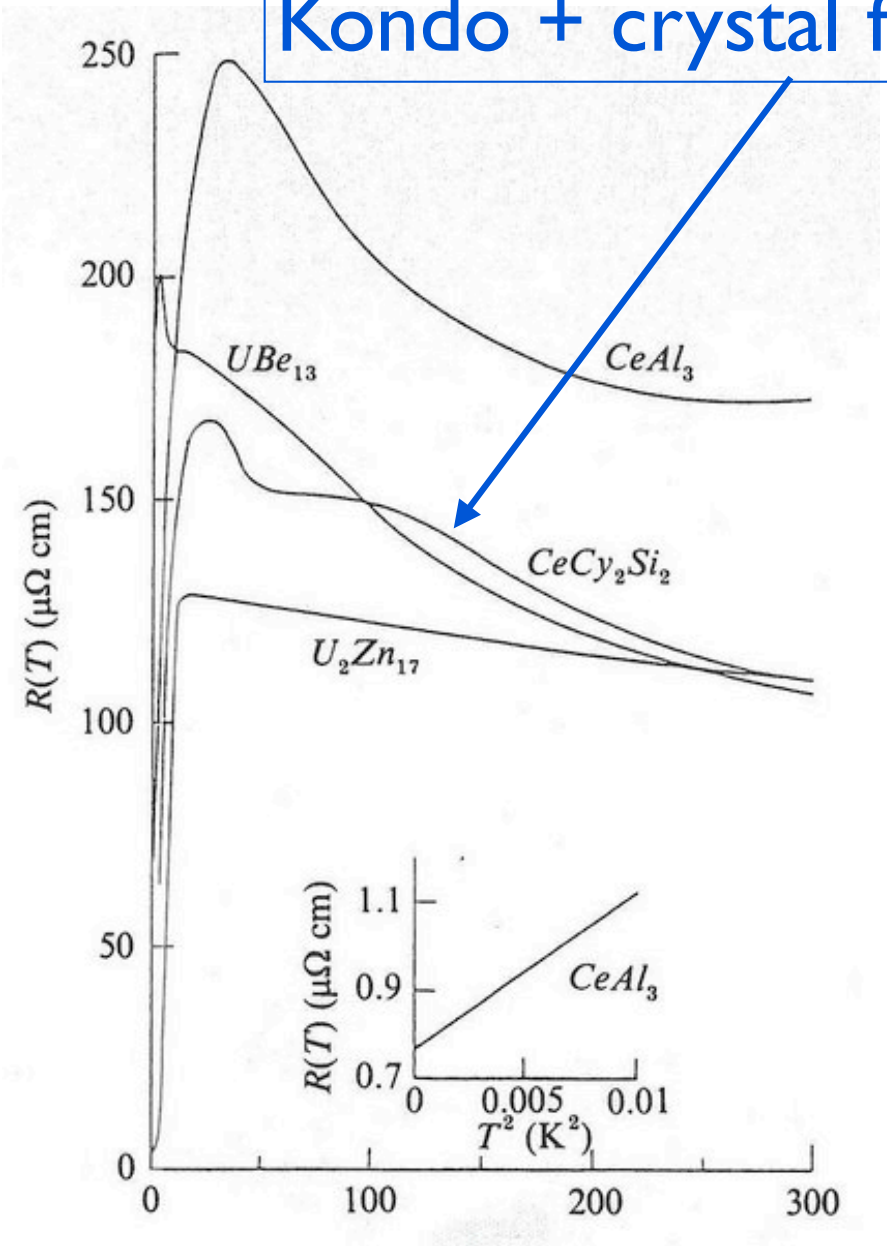


Magnetic scattering : Kondo Effect



$$R(T) = R_0 \left[1 + 2J\rho \log \left(\left| \frac{k_B T}{D - \epsilon_F} \right| \right) \right]$$

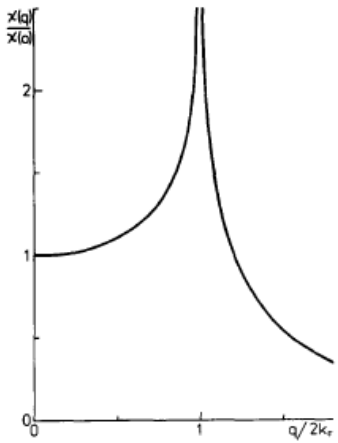
Kondo + crystal field

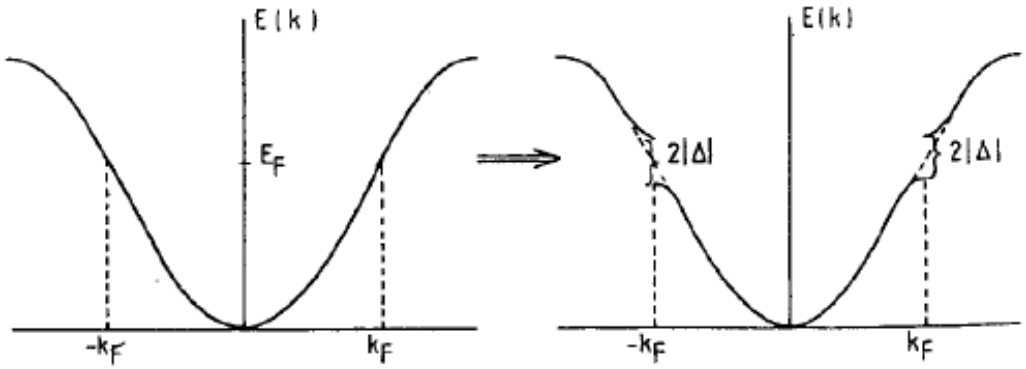


Kondo Lattice

Itinerant antiferromagnetism
Spin density waves
Charge density waves

$$\chi_Q = \sum_k \frac{(f_k - f_{k+Q})}{(\varepsilon_k - \varepsilon_{k+Q})}$$





Nesting wavevector

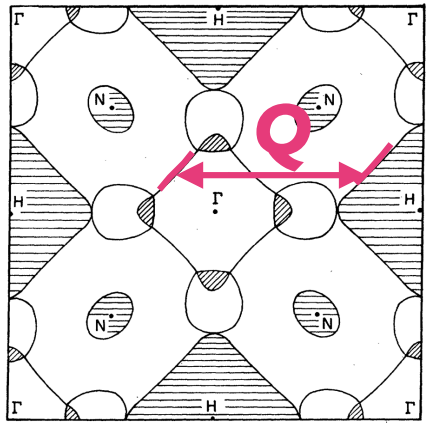


FIG. 4. Fermi-surface cross section: (001) plane.

portions of
Fermi surface
gapped &
lost for conduction

$$n \rightarrow n - n_{gapped}$$

SDW in Chromium

Itinerant antiferromagnetism
Spin density waves
Charge density waves

$$\rho \propto \frac{m^*}{n\tau}$$

Nordheim $\tau \propto (1-\Delta(T)^2)^{-1}$

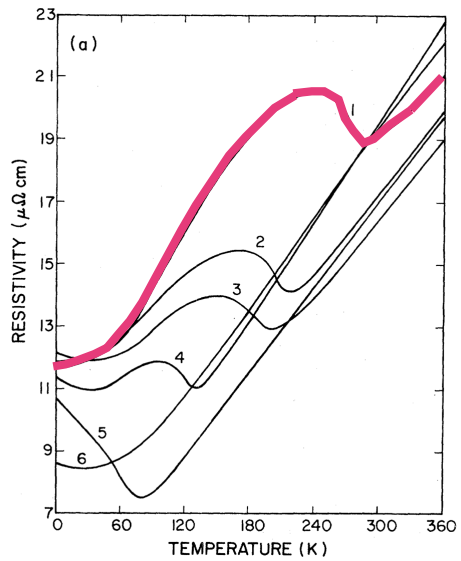
$$n_{gapped}(T) \propto \Delta(T)$$

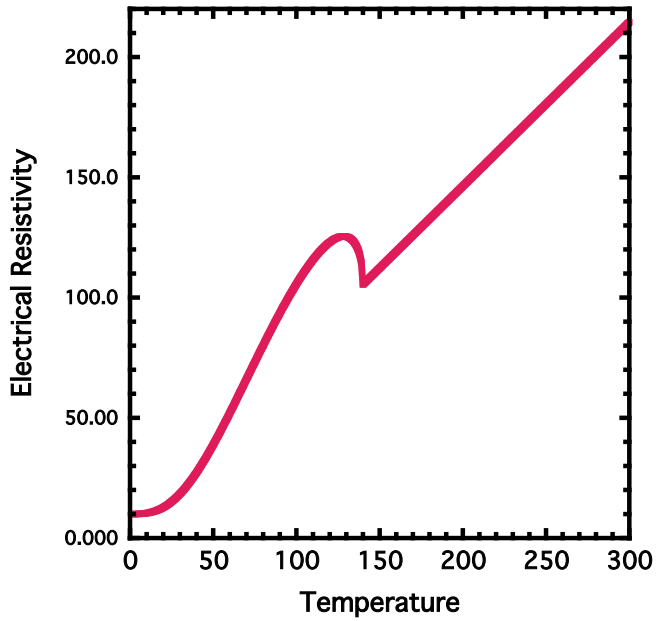
$$n = n - n_{gapped}(T)$$

$$\Delta(T) = \Delta_0 \sqrt{1-\left(\frac{T}{T_c}\right)^2}$$

$$\boxed{\rho \propto \frac{m^*}{n\tau} \propto \frac{m^* \cdot \left(1-\Delta^2(T)\right)}{\left[n - n_{gapped} \cdot \Delta(T)\right]} + \rho_{ph} T}$$

SDW in Chromium

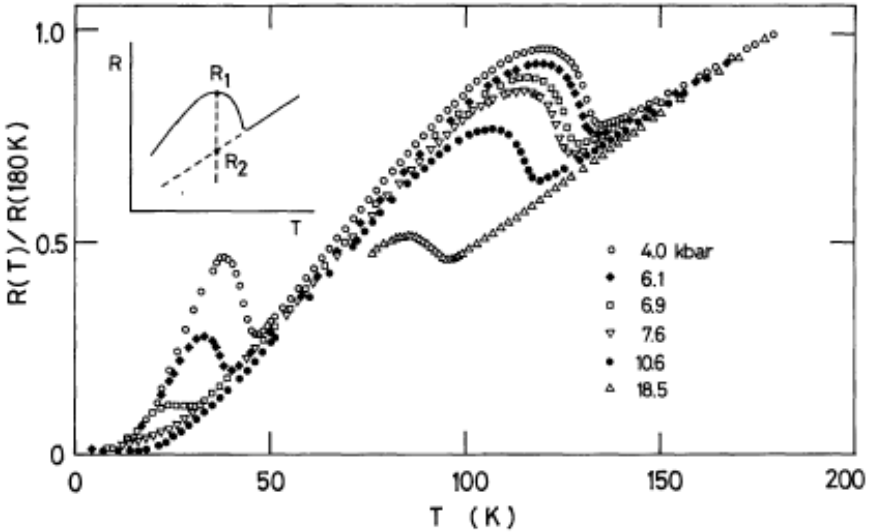




Examples with two DW

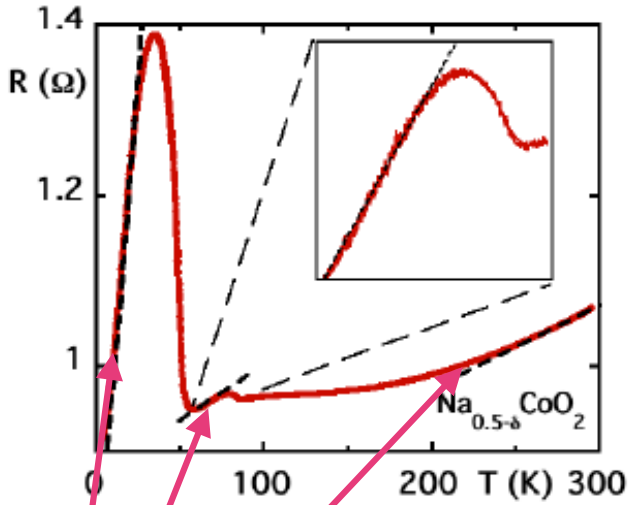
CDW

$$1D - NbSe_3$$



SDW

$$2D - Na_{0.5}CoO_2$$



$$R = \frac{12\pi^3\hbar}{e^2\int_{FS}\Lambda_k dS_F} \approx \frac{12\pi^3\hbar}{e^2\Lambda}\frac{1}{\int_{FS} dS_F}$$

From the ratio of the slopes we can estimate the percentage of the FS that disappears at each transition

Semiconductors

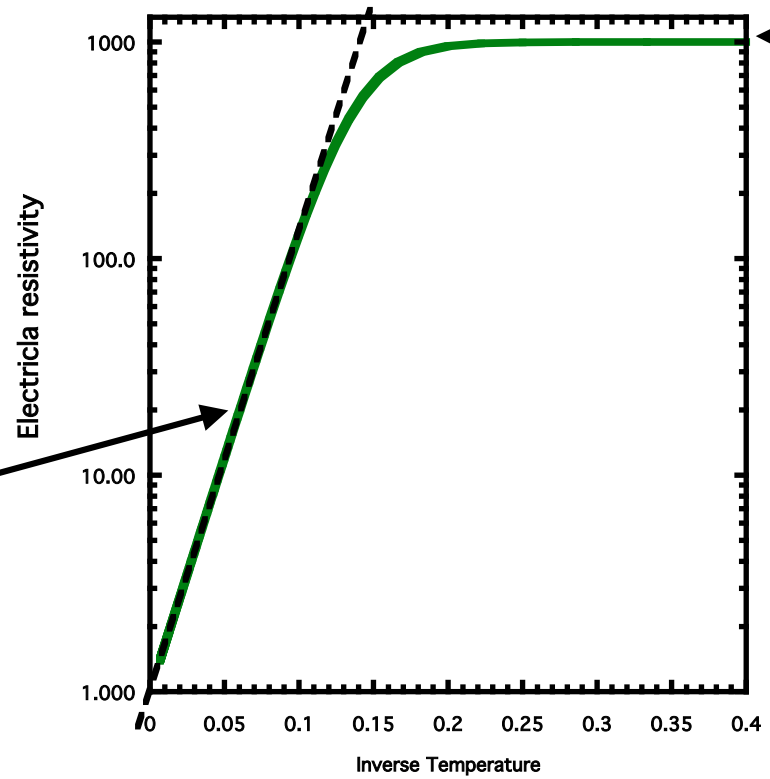
$$n \propto e^{-\frac{\Delta}{k_B T}}$$

The exponential carrier population controls the temperature resistivity dependance in intrinsic semiconductors

$$\therefore \quad \rho \propto e^{\frac{\Delta}{k_B T}}$$

But impurities give extrinsic carriers

$$\rho \propto \frac{m^*}{n\tau} \propto \frac{1}{\left(n_{\text{int}} e^{-\frac{\Delta}{k_B T}} + n_{ext} \right)}$$

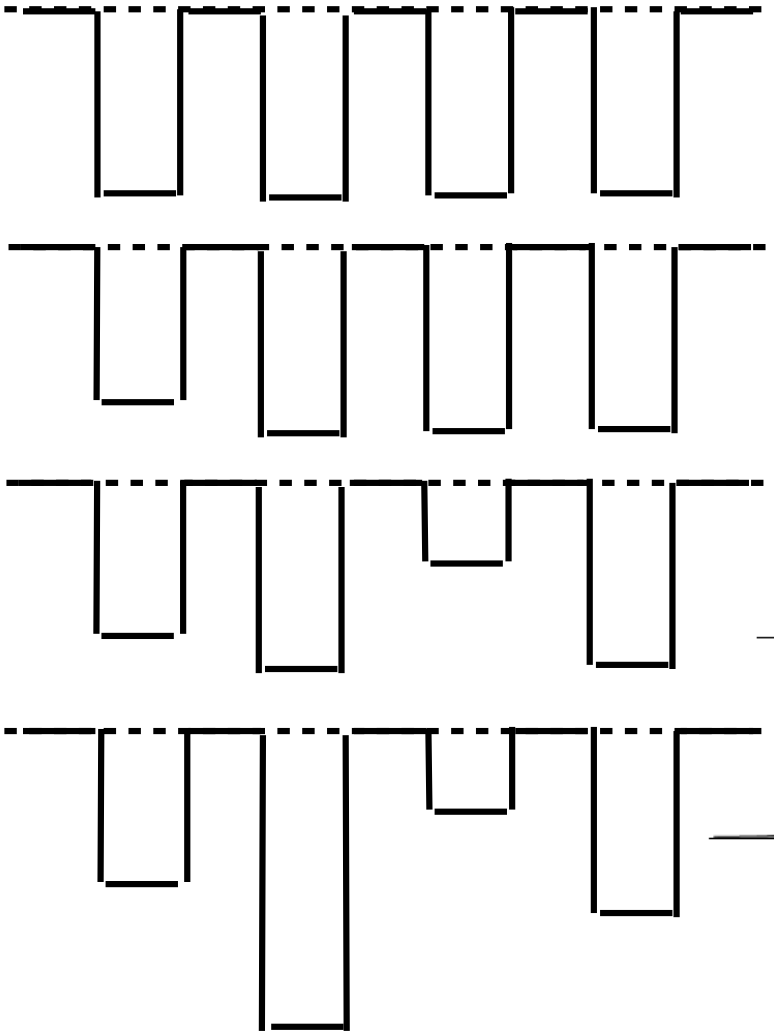


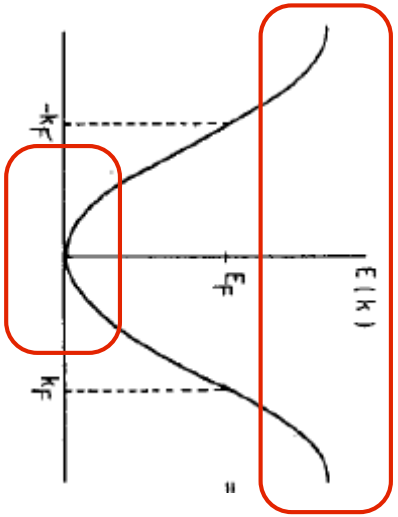
extrinsic impurity band conduction

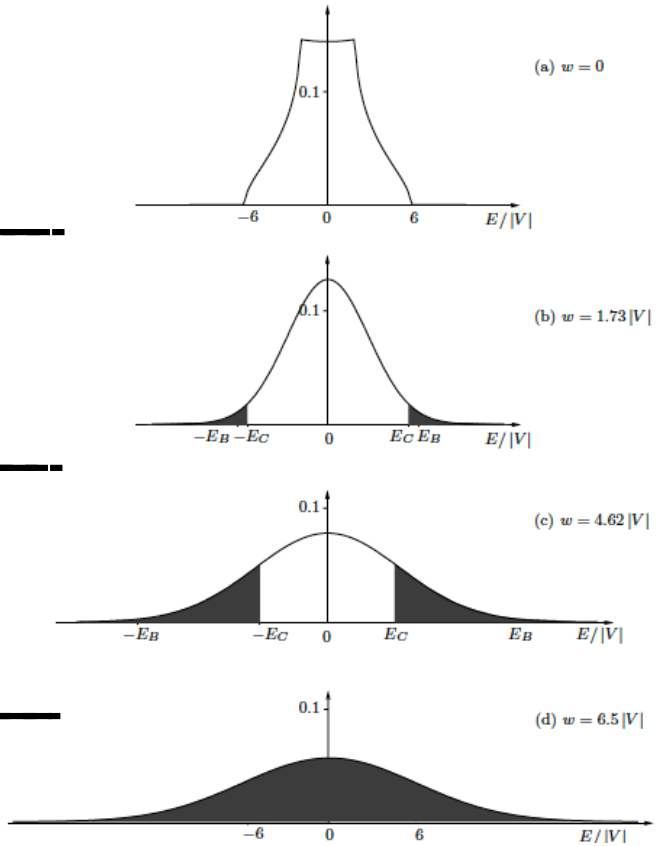
semiconducting gap Δ

Localization

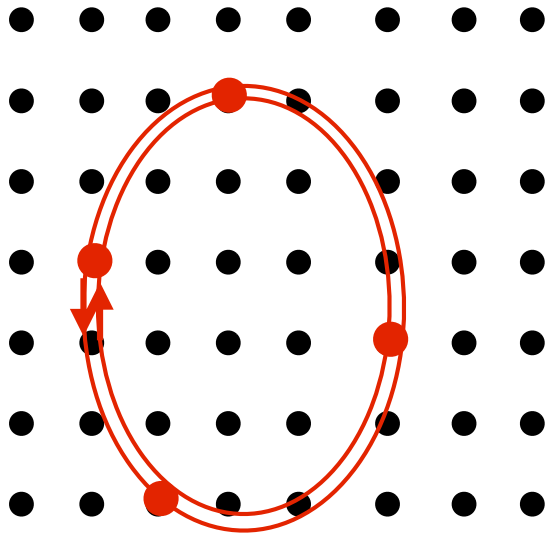
Defects cause localized states where effective masses are higher







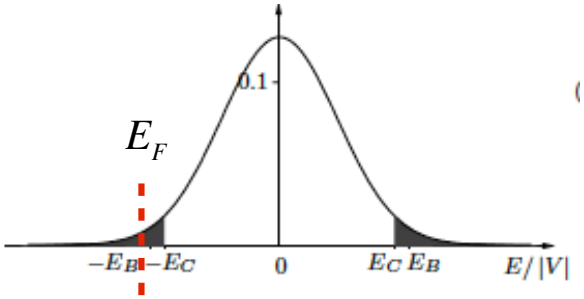
Interference through elastic scattering also causes localization



Localization

1. Conduction by thermal activation above mobility edge *Ec*

$$\sigma_1 \propto e^{-\frac{(E_c - E_F)}{k_B T}}$$

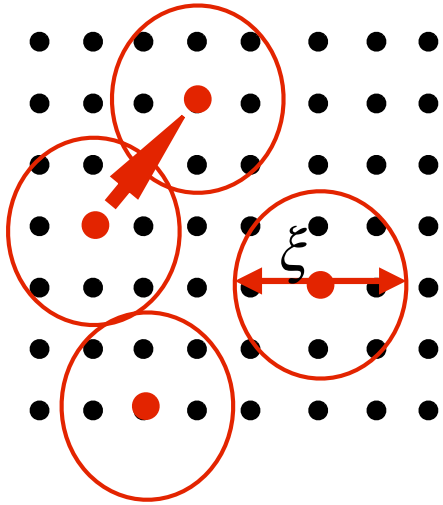


2. Activation to a neighbouring localized state

$$\sigma_2 \propto e^{-\frac{\Delta_\xi}{k_B T}}$$

average energy separation

$$\Delta_\xi \sim \left[n(0) \xi^d \right]^{-1}$$



3. Variable range hopping i.e. between sites of similar energy

$$\Delta_L \sim \left[n(0) L^d \right]^{-1} \sim \Delta_\xi \left(\frac{\xi}{L} \right)^d \qquad (L \gg \xi)$$

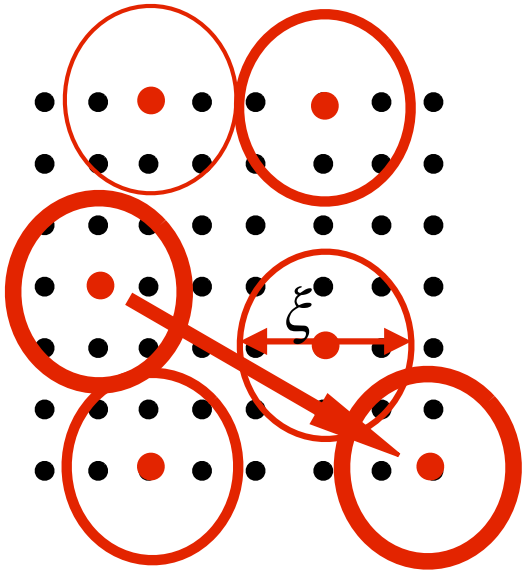
dependence of hopping with distance $\sim e^{2L/\xi}$

Total dependence $$e^{-\frac{2L}{\xi} - \frac{\Delta_L}{k_B T}} \sim e^{-\frac{2L}{\xi} - \Delta_\xi \left(\frac{\xi}{L} \right)^d}$$

optimizing

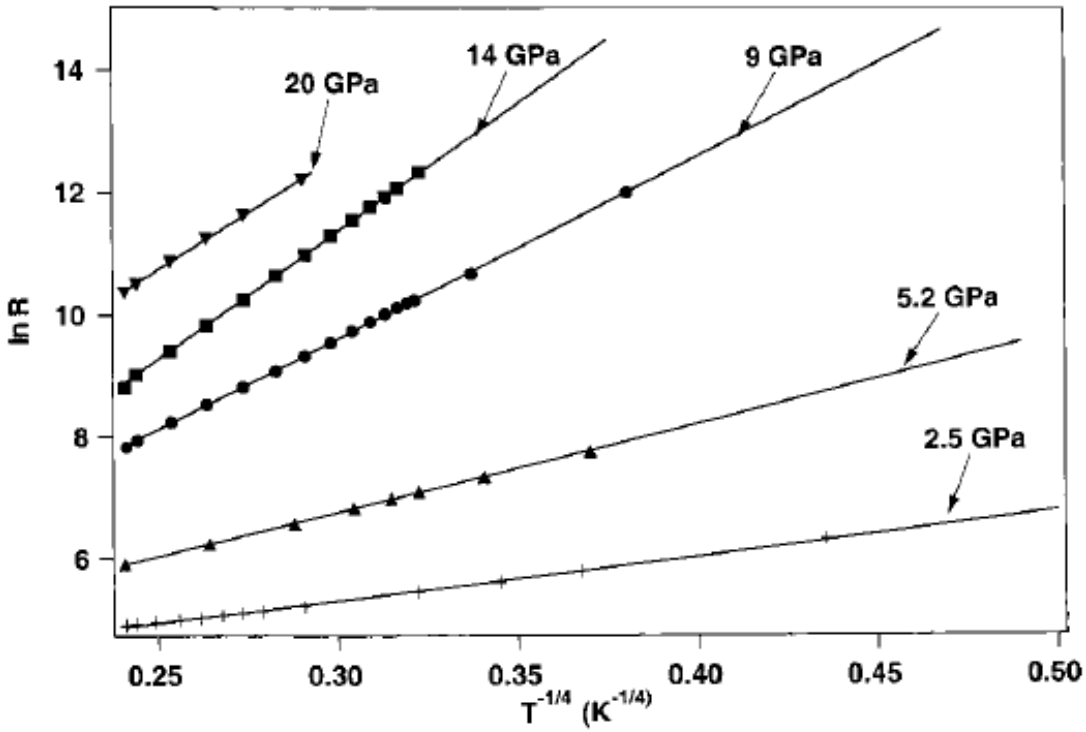
$$\sigma_3 \propto e^{-C \left(\frac{T_0}{T} \right)^{1/(d+1)}}$$

where $k_B T_0 \sim \Delta_\xi$

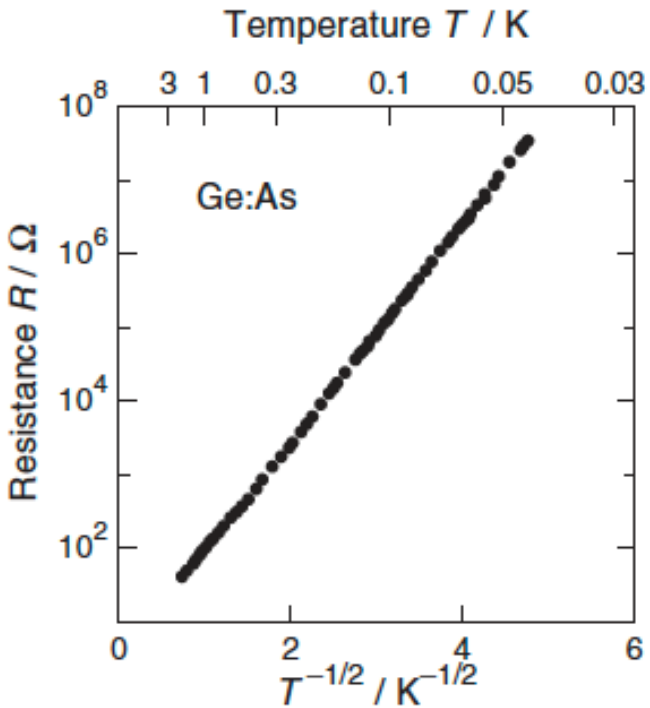


Localization

$$2D-(Bi,Pb)_2Ba_3Co_2O_y$$



$$\frac{1}{4} \rightarrow \frac{1}{3+1} \rightarrow 3D$$

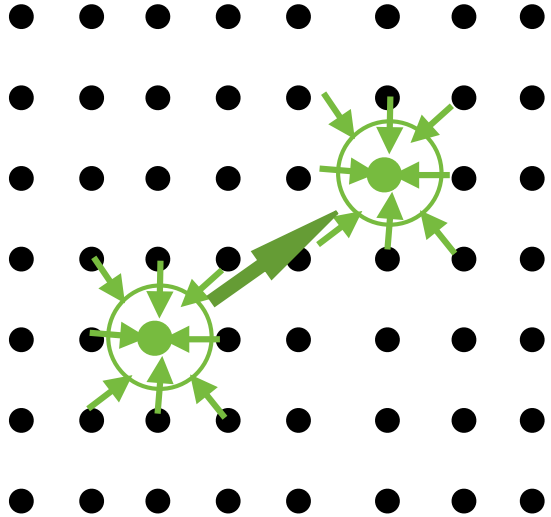


$$\frac{1}{2} \; not \; \frac{1}{1+1} \; but \; "Coulomb" \; Gap!$$

In bulk materials only
3D behaviour

“Heavy Carriers” : Polarons

The carrier is dressed by a lattice deformation must jump with it



Thermal activated hopping $\langle w \rangle \sim e^{-\hbar\omega^* / k_B T}$

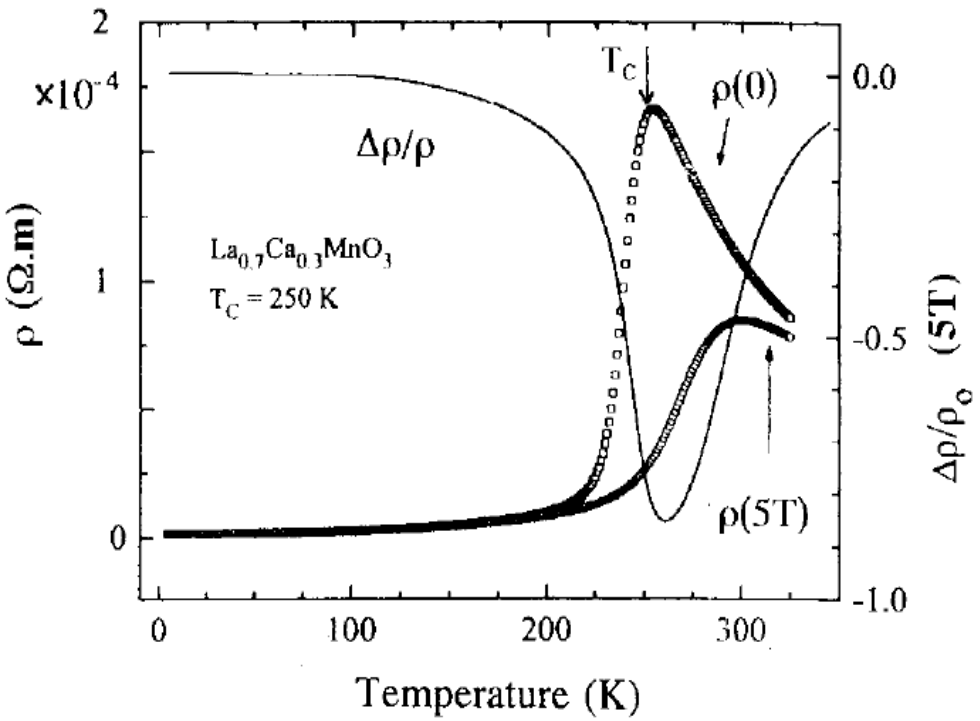
the mobility is $\mu = \dfrac{2ea^2}{k_B T}\langle w \rangle$ (*Einstein diffusion relation*)

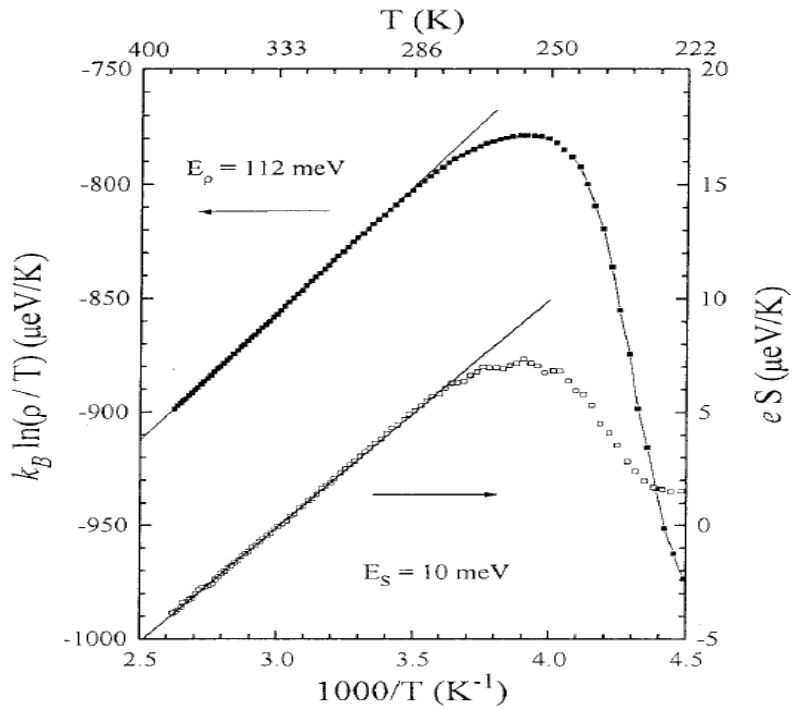
hence

$$\boxed{\rho \sim \frac{1}{\mu} \sim T \cdot e^{\hbar\omega^* / k_B T}}$$

“Heavy Carriers” : Polarons

Manganites

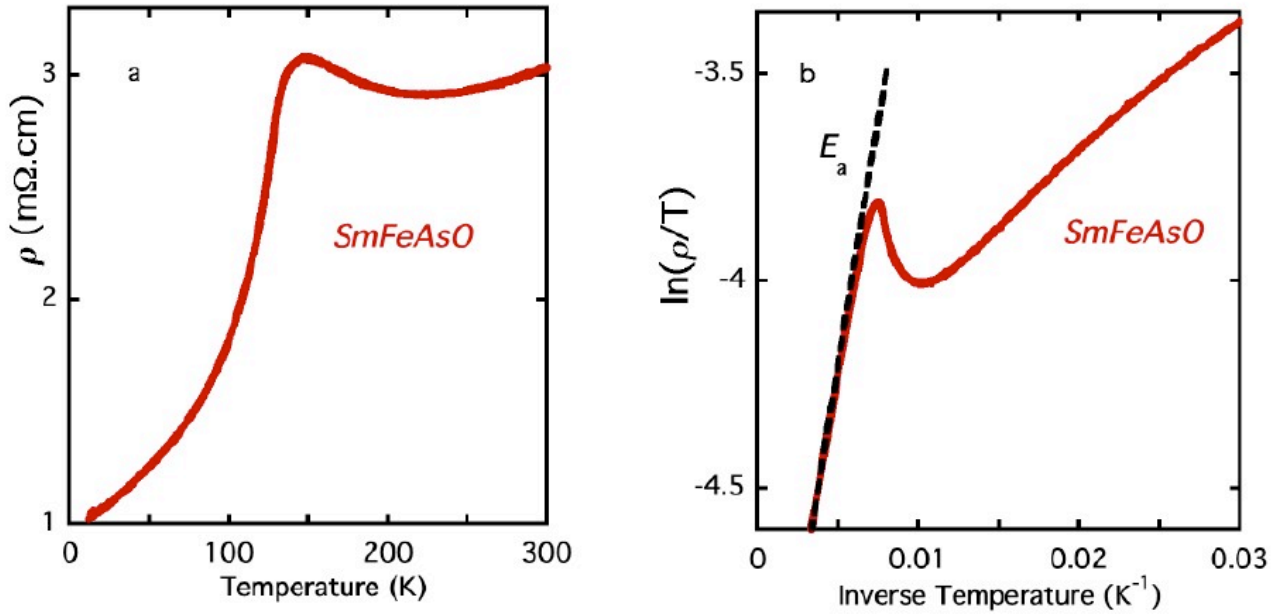




“Heavy Carriers” : Polarons

Ln-1111

Also “polaron-like” behaviour in Iron superconductors



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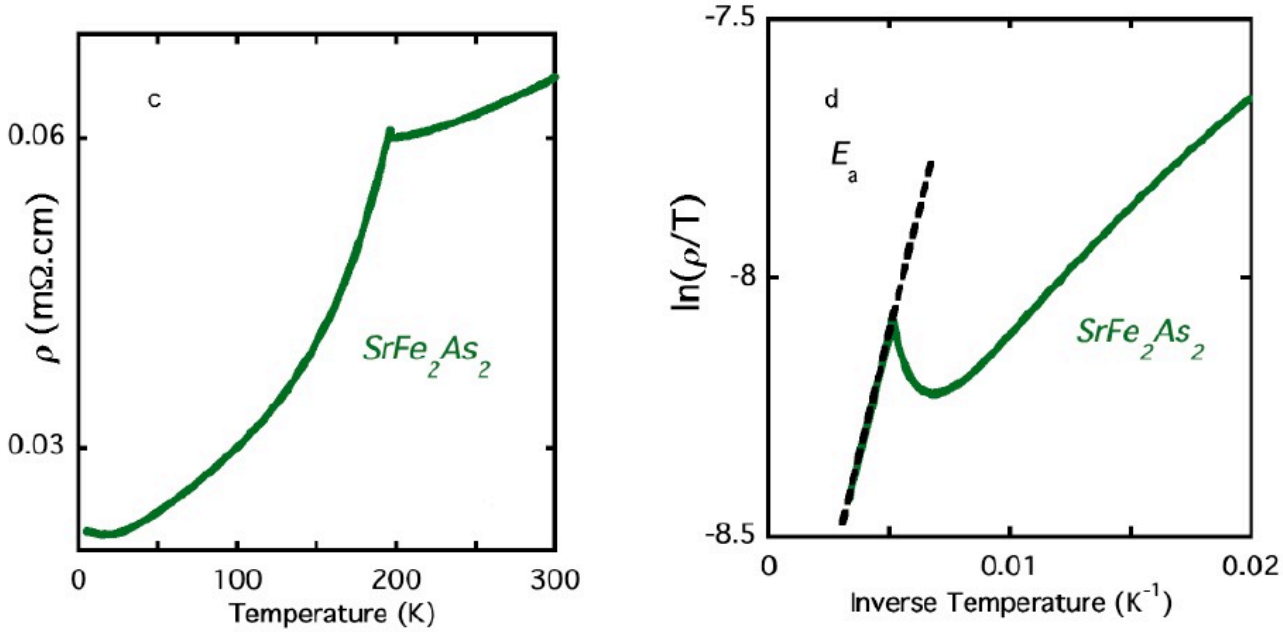
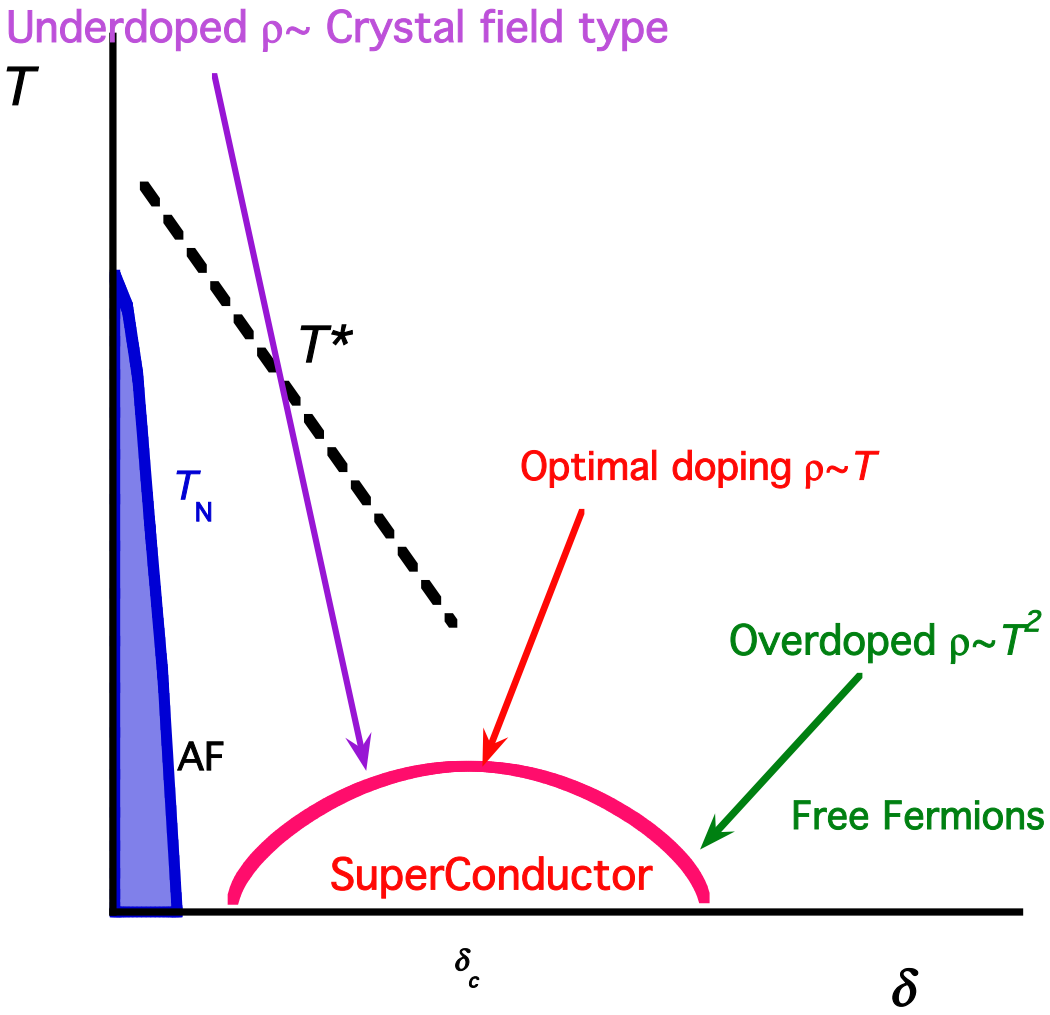


Cuprates


Underdoped ρ~ Crystal field type
T
T*
T_N
AF
Optimal doping ρ~T
Overdoped ρ~T^2
Free Fermions
SuperConductor
δ_c
δ

Conclusions

The behavior of the the electrical resistivity can yield many hints into the understanding of the physics of materials