SQUIDs and SQUID-microscopy

Klaus Hasselbach
outline

• Basic principles of SQUIDs
• Applications of SQUIDs
• SQUID microscopy
Basic principles of SQUIDs

• Flux quantization in superconducting Ring
• DC and AC Josephson effect

Realization of SQUIDs
Flux quantization in superconducting Ring

\[ \psi = \rho_s^{1/2} \exp(i\varphi(r)) = \psi = \rho_s^{1/2} \exp(i\tilde{\rho} r / \hbar) \]

\[ \tilde{p} = \tilde{p}_{\text{kin}} + \tilde{p}_{\text{pot}} = 2m\tilde{v}_s - 2e|\tilde{A} | \]

\[ \tilde{j}_s = -2e|\rho_s \tilde{v}_s | \]

\[ \tilde{p} = -(m / e|\rho_s | - 2e|\tilde{A} | \]

Rigid phase:
total phase change must be single valued

\[ \Delta \varphi = \oint (\tilde{p} / \hbar) \cdot d\tilde{l} = \frac{m}{e|\rho_s \hbar} \oint \tilde{j}_s \cdot d\tilde{l} - \frac{2e}{\hbar} \oint \tilde{A} \cdot d\tilde{l} = 2\pi n \]

\[ \varphi' = (m / 2\rho_s e^2) \oint \tilde{j}_s \cdot d\tilde{l} + \phi = n(2\pi \hbar / (2e)) = n\phi_0 \]
Flux quantization in superconducting Ring

\[ \phi' = \left( \frac{m}{2 \rho_s e^2} \right) \oint \mathbf{j_s} \cdot d\mathbf{l} + \phi = n(2\pi)\hbar / (2|e|) = n\phi_0 \]

\[ = 0 \text{ in bulk sc as } J_s = 0 \text{ in bulk} \]

\[ \phi' = \phi = n\phi_0 \]

\[ \Phi_0 = \frac{\hbar}{2e} = 2.07 \cdot 10^{-15} Tm^2 \]

Flux(oid) quantization
Deaver and Fairbanks (PRL 7, 43 (1961)) Doll and Naebauer (PRL 7, 51 (1961))
Experiment proof of flux quantization

Torsional oscillator
Doll and Näbauer (PRL 7, 51 (1961))
Torsional oscillator
Deaver and Fairbanks (PRL 7, 43 (1961))
Magnetometry of tin coated copper wire
Josephson tunneling- transfer of Cooper pairs rather than single electrons

\[ I_s = I_0 \sin \varphi \]

- \( I_s \) - Cooper pair current across junction

\[ V = \frac{\hbar}{2e} \frac{d\varphi}{dt} \]

- \( V \) - voltage
- \( \varphi \) - difference in pair phases across junction

B.D. Josephson (PRL 1, 251(1962))
Josephson Junctions

Tunnel barrier

SNS Junction

Micro Bridge junction

M. Tinkham
DC-SQUID

\( I_1, I_2 \) – critical current of junctions
\( L_1, L_2 \) - inductances of arms of SQUID loops

\[
I_B = I_1 \sin(\varphi_1) + I_2 \sin(\varphi_2)
\]

\[
2\pi n = \varphi_2 - \varphi_1 + \frac{2\pi}{\Phi_0} (\Phi_a + L_2 I_2 - L_1 I_1)
\]

\( n = 0 \) Maximize \( I_B \) for each value of \( \Phi_a \)

\[\Phi_a/\Phi_0\]
Design considerations

SQUID size (≈ inductance L) not too small
$I_c$ sufficient big compared to thermal noise

JR Kirtley
Magnetic field scales

Magnetic Fields

- 1 tesla: Conventional MRI
- $10^{-2}$: Earth’s field
- $10^{-4}$: Urban noise
- $10^{-8}$: Car at 50 m
- $10^{-10}$: Human heart
- $10^{-12}$: Fetal heart
- $10^{-14}$: Human brain response
- 1 femtotesla: SQUID magnetometer
- $10^{-16}$: SQUID magnetometer

J. Clarke
Operation of SQUID

Fig. 1. DC SQUID. (a) Schematic. (b) $I-V$ characteristic. (c) $V$ versus $\Phi/\Phi_0$ at constant bias current $I_b$. 

R. Kleiner
Noise in DC SQUID

- Nyquist noise of shunt resistor
- Spectral density of flux noise
  \[ S_\phi(f) \approx 16k_B TL^2 / R \]
- \( L=200 \) pH, \( R=6 \) Ohm \( T=4.2 \) K
  \[ S_\phi^{1/2}(f) \approx 1.2 \cdot 10^{-6} \phi_0 / \sqrt{Hz} \]
- Noise energy
  \[ \varepsilon(f) = S_\phi(f) / 2L \approx 10^{-32} JHz^{-1} \approx 100\hbar \]

C. Tesche, J Clarke 1977
Flux noise in the SQUID

\[ S_\phi(\theta) (\Phi_0^2 \text{Hz}^{-1}) \]

- White noise: \( 2 \times 10^{-6} \Phi_0 \text{Hz}^{-1/2} \)

J. Clarke
Applications of SQUIDs

• Magnetometer
  Geophysics
  Medical research
• Non destructive testing
• Current amplifier

How is the SQUID coupled to the world?
The SQUID is part of a feed-back loop, maintaining it at given Flux
-> allows to measure important fields
Transformer gradiometer...

![Diagram of Transformer Gradiometer](image)

Fig. 5. Superconducting, wire-wound flux transformers. (a) Magnetometer. (b) First-derivative, axial gradiometer. (c) Second-derivative, axial gradiometer.

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Noise thermometry

\[ S_\phi(f, T) = \frac{4k_b TM^2}{R(1 + (f/f_c)^2)} \]

\[ f_c = \frac{R}{2\pi L} \]

Pd resistor in the circuit of 1 mOhm (green)
Measure the fluctuating currents in a piece of copper (thermal magnetic flux noise) -> better thermalization of electrons.

\[ S_\phi(f,T) = \frac{4k_b T M^2}{R(1+(f/f_c)^{2a})^b} \]
Noise thermometry
Nano-SQUID

FIG. 1. Electron micrograph of the experimental device. On the left is the ring etched in GaAs 2DEG (labeled 1) (the dashed line has been added because of the poor contrast) with the two gates, (2) and (3). On the right is the calibration coil (4). On the top is the first level of the SQUID fabrication (5) with the two microbridge junctions on the right. The picture has been taken before the second level of the SQUID fabrication.

Mailly PRL 1993
Wernsdorfer PRL 1996

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SQUID Microscopy

Sensitivity and spatial resolution depend on size of pickup area and spacing to sample

JR Kirtley

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Variable sample temperature scanning SQUID susceptometer

- Sample mount
- SQUID
- Cu cantilever
- Sapphire disk
- Cu thermal anchors
- Cold plate

Field coil

SQUID pickup loop
3μm diameter
Weizmann SQUID on Tip
Grenoble NanoSQUID microscope

- Former PhD students:
  - C. Veauvy, V.O. Dolocan, D. Hykel
- Present PhD
  - Z.S. Wang
- PostDoc
  - D. Hazra

- Technical support:
- Collaborations:
  - K. Schuster IRAM, J.R. Kirtley,.....
SQUID

Flux penetrating the SQUID loop modulates the critical current
Hysteretic Nano-SQUID

K. Hasselbach et al.
*Physica C 332(2000)140–147*
Critical current measurement

Field range ± 100 G

Magnetic sensitivity

\[ 1.2 \cdot 10^4 \Phi_0 / \sqrt{Hz} \]

\[ 2 \cdot 10^{-3} G / \sqrt{Hz} \]
SQUID tip first generation

Aluminium

0.6 µm / 1.1 µm diameter

To minimize the distance between SQUID and sample we cut the wafer
SQUID tip second generation

Deep Si etching allows for better edge SQUID alignment

T. Crozes (Neel),
A. Barbier, K. Schuster (IRAM)
Squid Force Microscopy

Quartz tuning fork -> Distance Control

Review of Scientific Instruments C. Veauvy et al. 73 3825 2002
AFM Principle

TF is a force sensor

Its resonance frequency increases with decreasing tip sample distance
AFM Electronic

DSP contains
Two PI in series
10 kHz bandwidth

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Microscope

- attocube motors
- z-piezo stack
- excitation piezo
- tuning fork
- SQUID chip
- slider
- spring
- z-motor

11 cm

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Vortices in a perforated Al film

Flux quantization at each hole

Vortex transitions:
Fusion and localized SC

T=0.4K  T=1.18 K  T=1.2 K


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Magnetic anisotropy in the superconducting state of $\text{Sr}_2\text{RuO}_4$ state

Crossing vortex lattices: in-plane vortex pin crossing vortices in $\text{Sr}_2\text{RuO}_4$

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Penetration depth and vortex stray field

E. H. Brandt et al. PRB, 61, 6370

\[
h_z = \frac{\Phi_0}{(2\pi \lambda_{ab})^2} \int d^2k e^{i\vec{k} \cdot \vec{r}} \frac{e^{k(d/2-z)}}{\alpha [\alpha + k \cdot \coth(\alpha d/2)]}
\]

\[
\vec{r} = \{x, y\}, \, \vec{k} = \{k_x, k_y\}, \, k = \sqrt{k_x^2 + k_y^2}, \, \text{and} \, \alpha = \sqrt{k^2 + \lambda_{ab}^{-2}}.
\]


\[
(r^2 + z^2)^{1/2} \gg \lambda \quad h_z = \frac{\Phi_0}{2\pi} \frac{z + \lambda_{eff}}{[r^2 + (z + \lambda_{eff})^2]^{3/2}}
\]

\[
\lambda_{eff} = \lambda \coth(d/2\lambda)
\]

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04/01/12
Absolute value of penetration depth

$z = 0.45\mu m$

$\lambda = 101\text{nm}$

$\lambda = 103\text{nm}$

$T_c = 1.6\text{K}$

ZS Wang 2011

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Scientific projects

• Vortex state and penetration depth (doping) in Pnictide SC
• Domain structure in superconducting Ferromagnets
• Imaging noise sources in Qbit circuits
• .......

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Outlook Imaging

Smaller SQUID

better aligned

500 nm SQUID loop

1 \mu m alignment of etch

1 \mu m SQUID loop

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Outlook Imaging

At least 10 times higher sensitivity at 4.2 K

\[ 0.2 \cdot 10^{-7} \Phi_0 / \sqrt{Hz} \]

L. Hao et al. NPL PTB
APL 92 192507 (2008)
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  D. Mailly, C. Chapelier, A. Benoit PRL 70 2020 1993
• Nucleation of Magnetization Reversal in Individual Nanosized Nickel Wires

Martin Huber, Nick Koshnick, et al., RSI 79,053704 (2008)
Koshnick et al., APL 93, 243101 (2008)

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