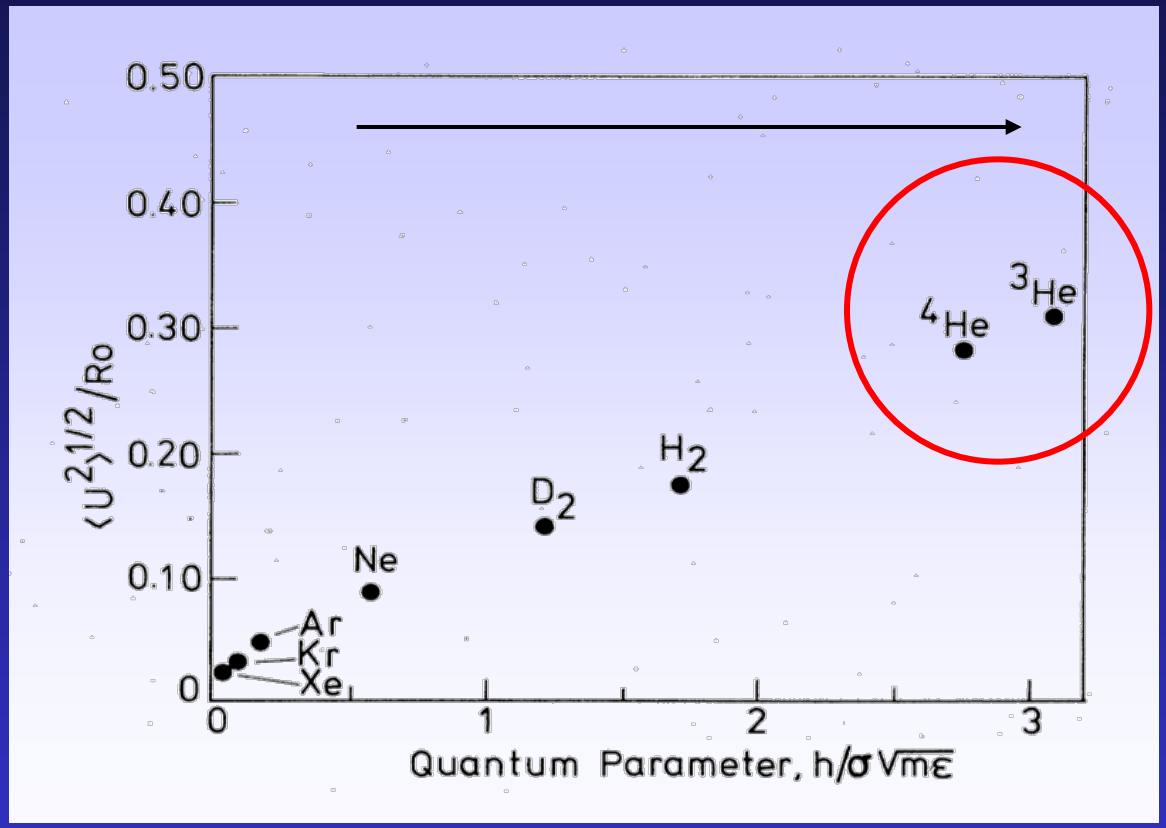
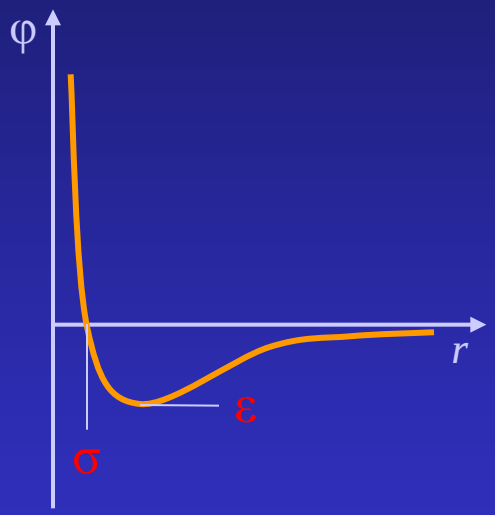
Quantum Fluids and Solids

Christian Enss (4He) & Henri Godfrin (3He)

What Do We Consider as Quantum Matter ?



Lennard Jones Potential



De Boer quantum parameter: $$\Lambda^\star = \frac{\Lambda_B}{\sigma} = \frac{h}{\sigma\sqrt{2m\varepsilon}}$$

De Broglie wavelength: $$\Lambda_B = \frac{h}{p}$$

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Overview

Introduction
- Phase Diagrams

He-II Experimental Observations
- Superfluidity
- Viscosity
- Second sound

Two Fluid Model
- Hydrodynamics
- Thermodynamical effect
- Sound propagation

Bose-Einstein Condensate
- Condensate fraction

Macroscopic wave function
- Quantized vortices

Normal Fluid Component
- Excitation spectrum
- Critical Velocity

Quantum Crystals

Fermions: ^{3}He
- Phase diagram
- Melting curve

Normal Fluid ^{3}He
- Fermi liquids
- Heat capacity
- Magnetic susceptibility

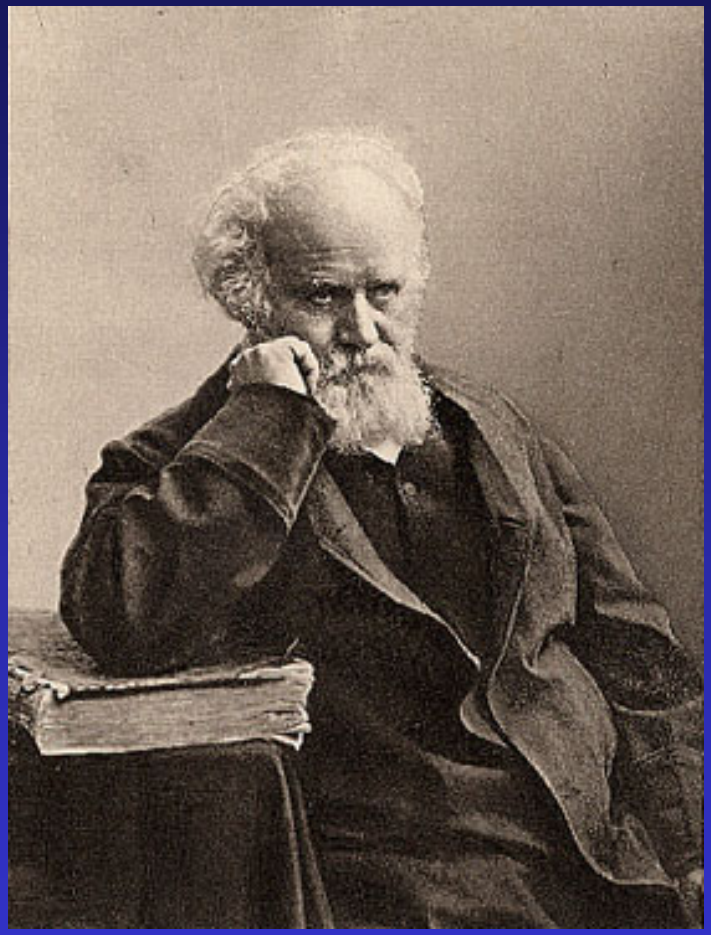
Superfluid ^{3}He
- Cooper pairs
- A and B phases
- Vortices

Dirac multi-spin exchange
- U2D2 phase
- CNAF phase
- Susceptibility and heat capacity

2D Systems
- Heisenberg ferromagnet
- Spin liquid

Conclusions

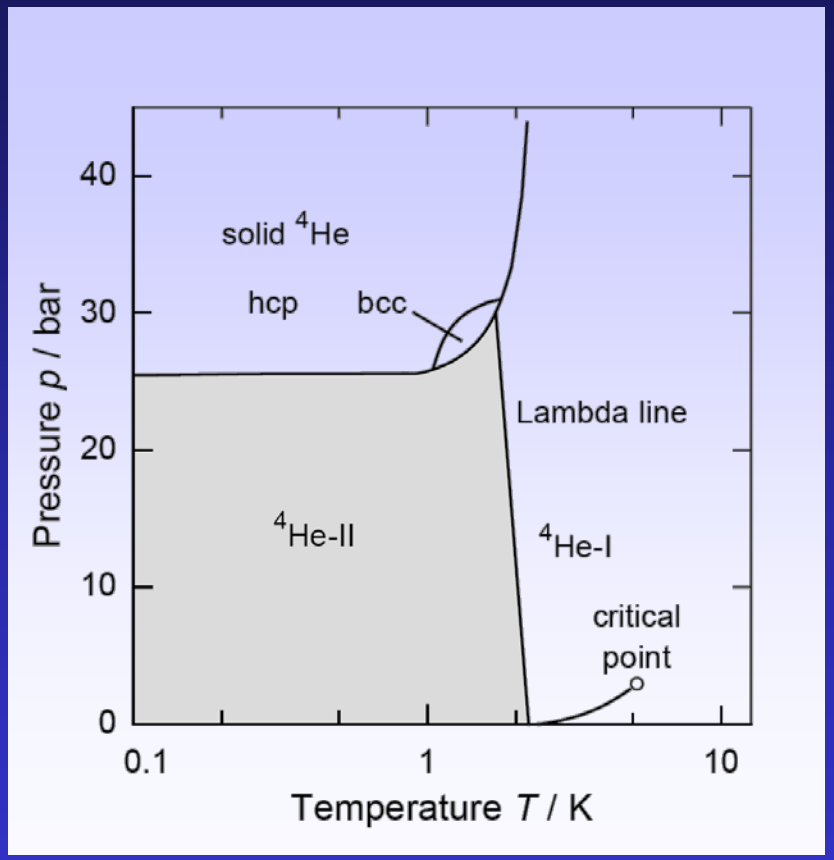
Discovery of Helium

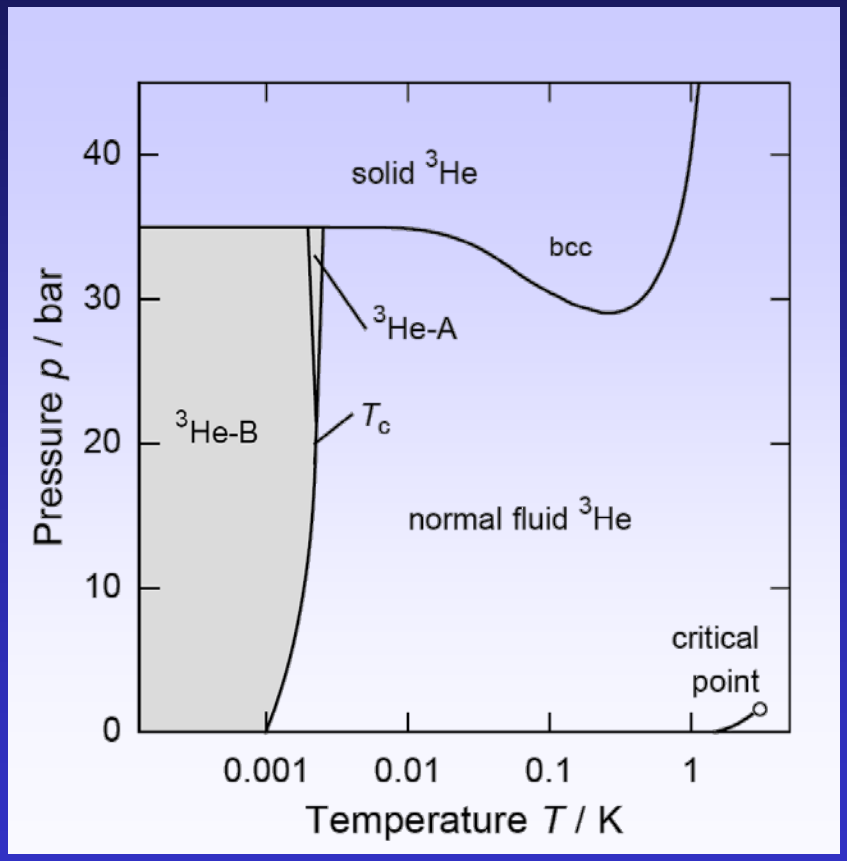


Pierre Jules César Janssen

Sir William Ramsay

Phase Diagrams of ^{3}He and ^{4}He

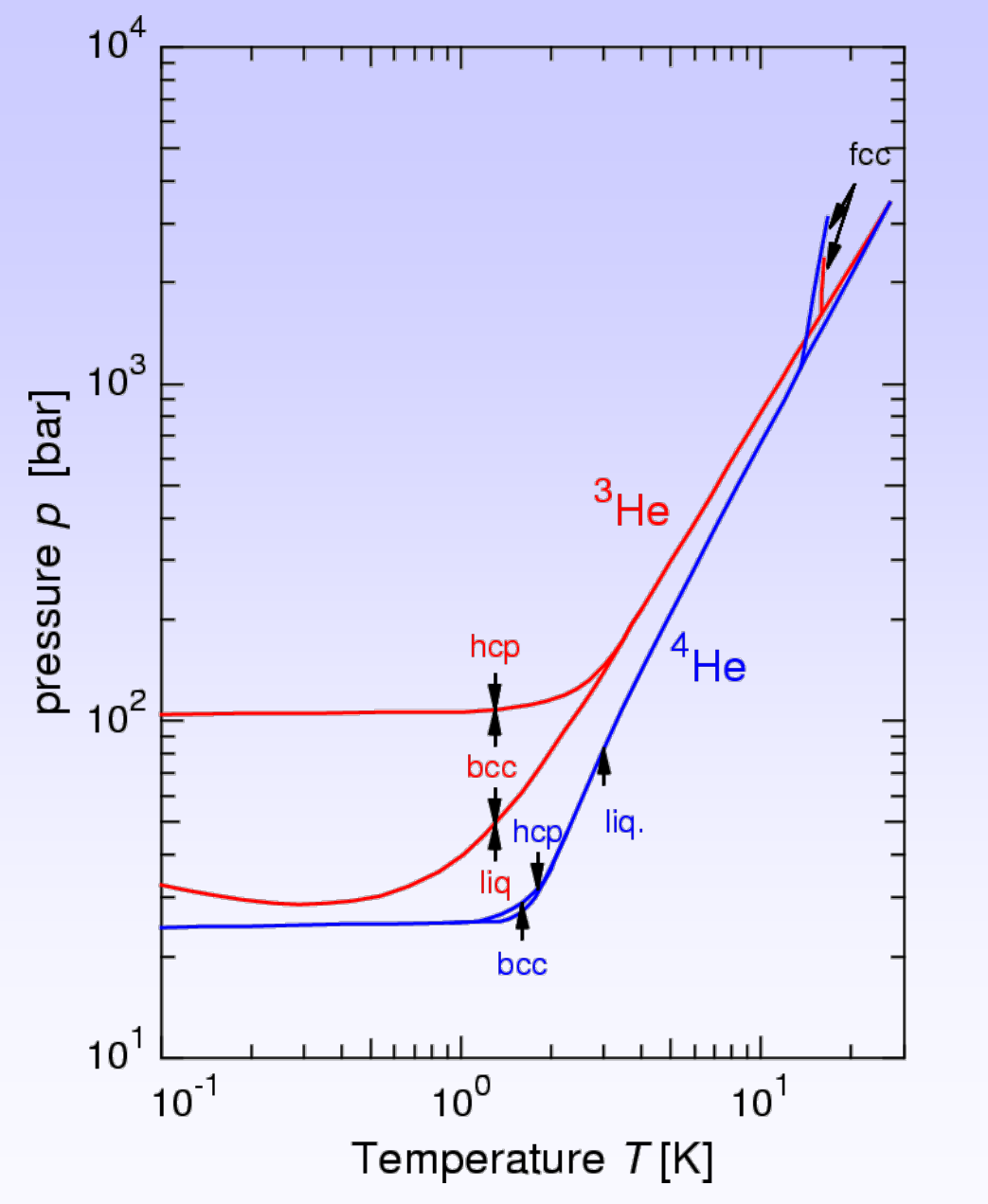




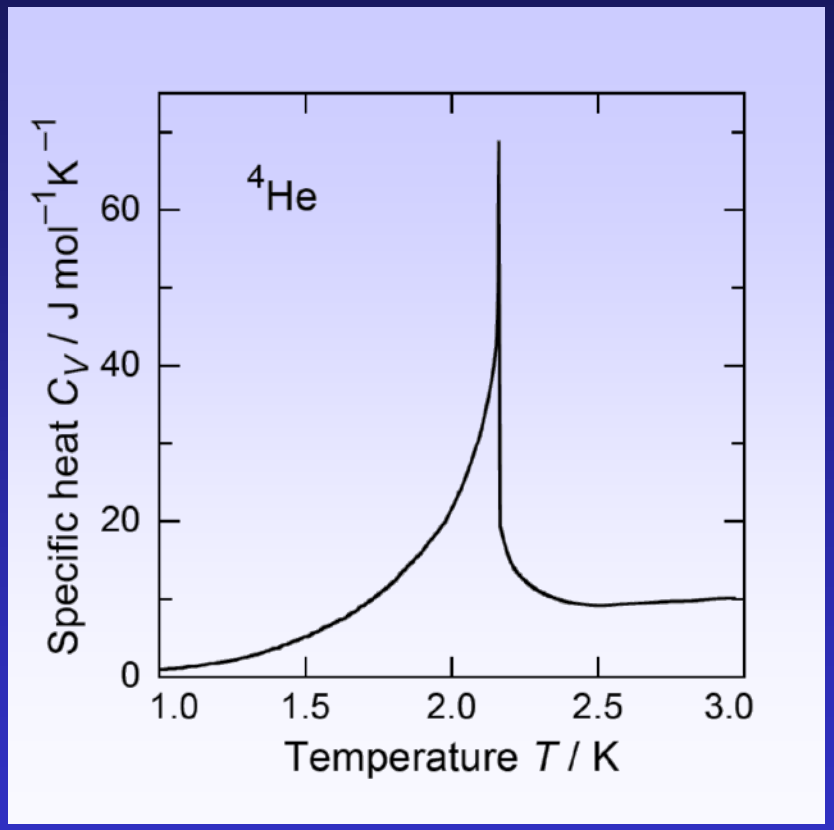
Solid Phases

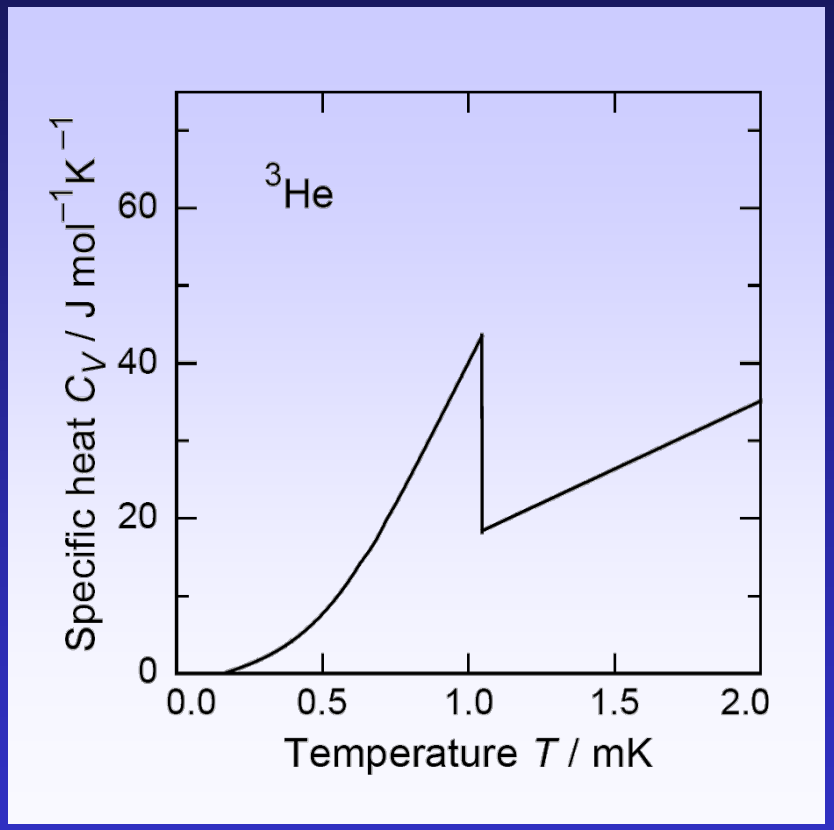
bcc phase much larger for ^{3}He
➡ less dense because of higher zero point energy

zero point energy unimportant at high temperatures



Specific Heat at the Superfluid Transition





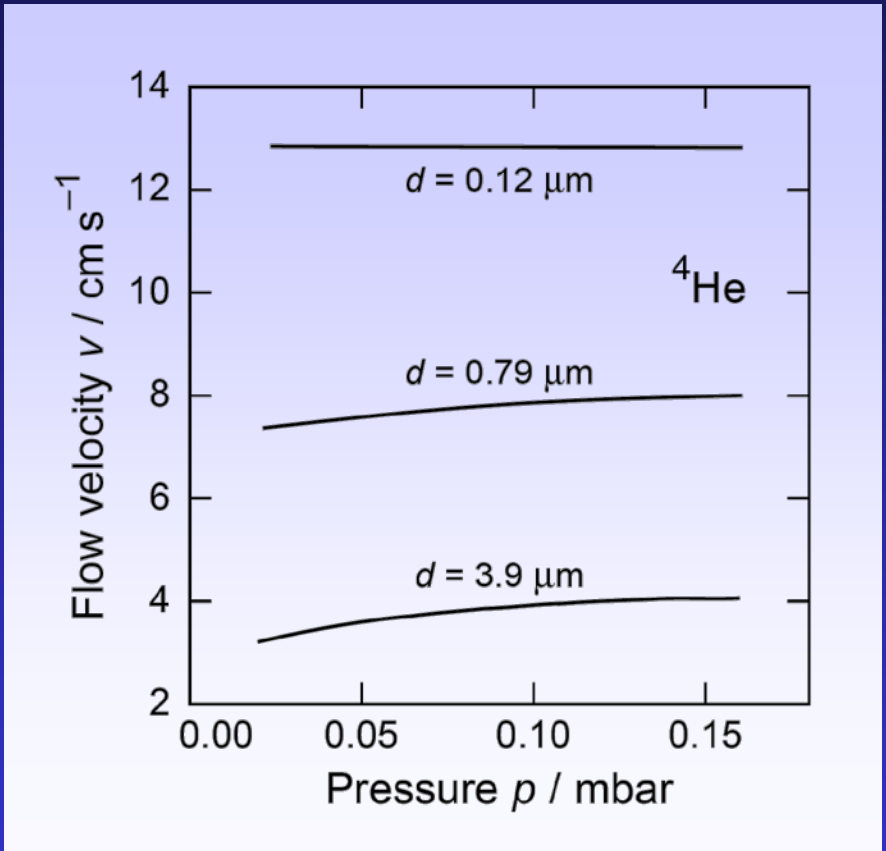
Superfluidity: Flow Through Thin Capillaries

Hagen-Poiseuille law

$$\dot{V} = \frac{\pi r^4}{8} \frac{1}{\eta} \frac{\Delta p}{L}$$

Flow velocity

$$v = \dot{V}/(\pi r^2)$$



Experimental result:

flow velocity is independent of pressure
and increases with decreasing diameter

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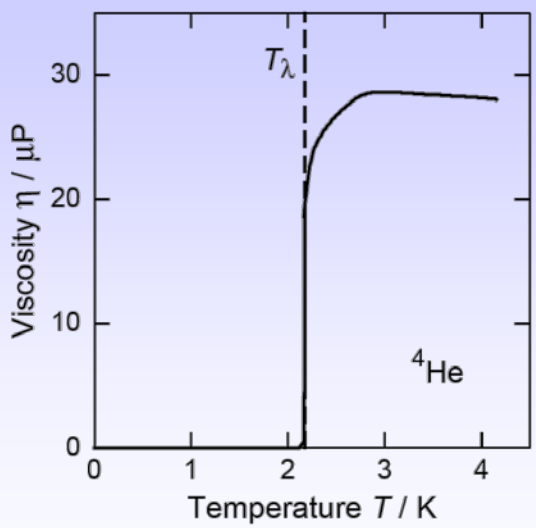
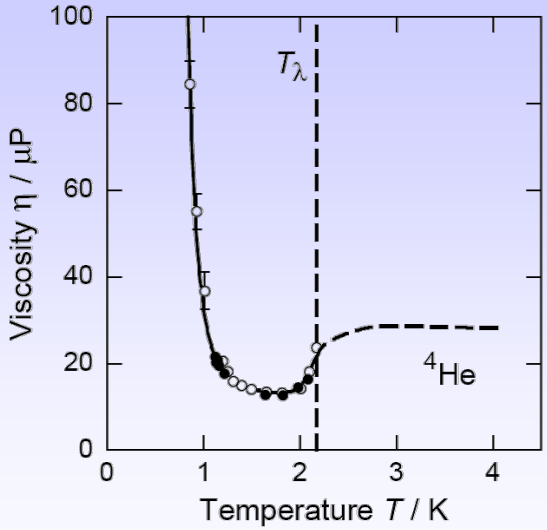
Temperature Dependence of Viscosity

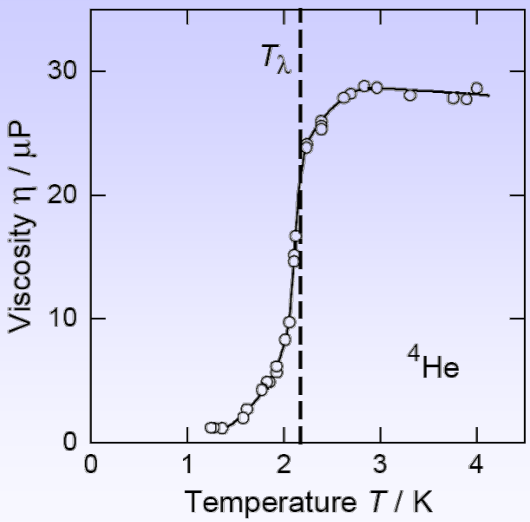
flow through capillary



rotary viscosimeter

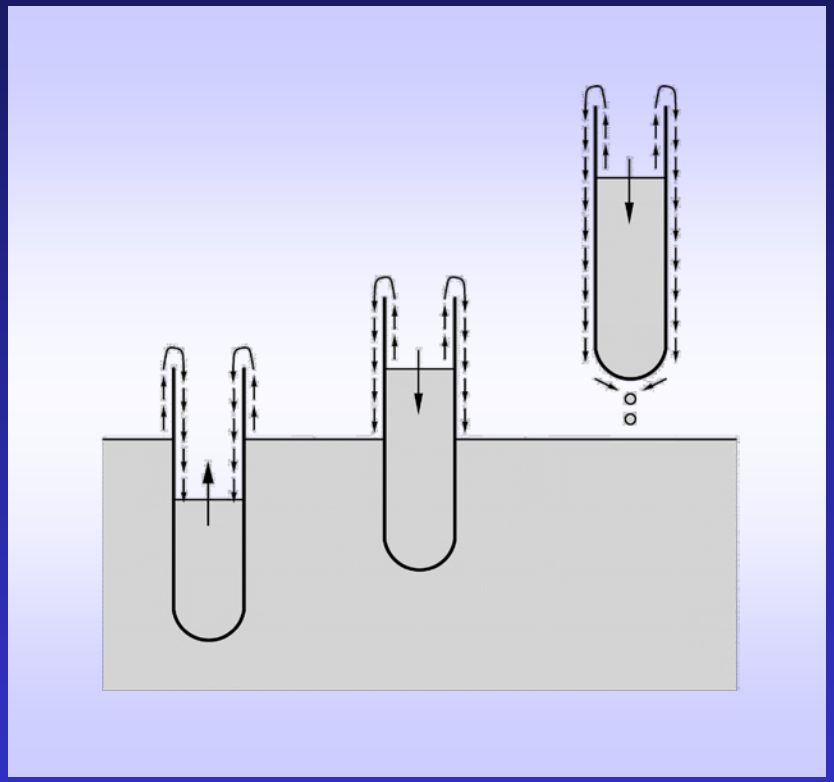


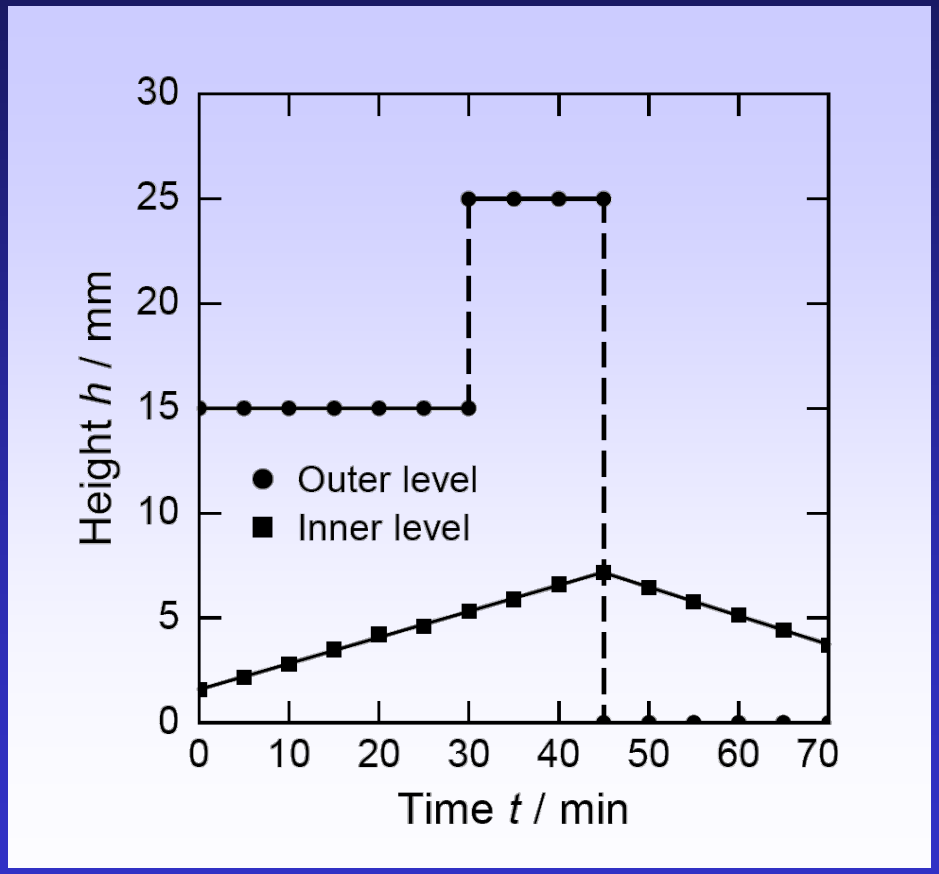
oscillating disc



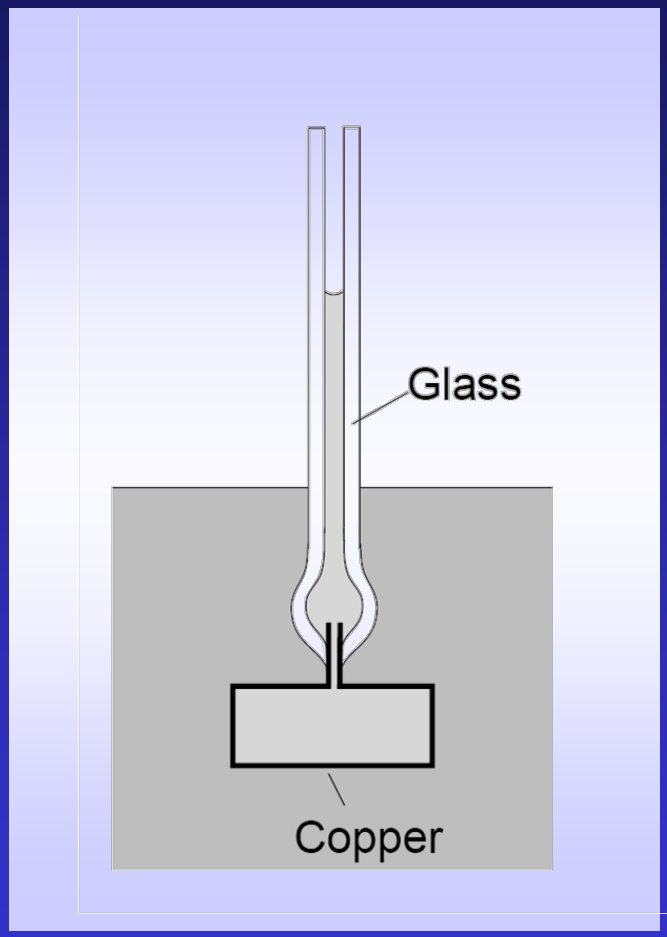
Three different results and all experiments are correct! What is wrong?

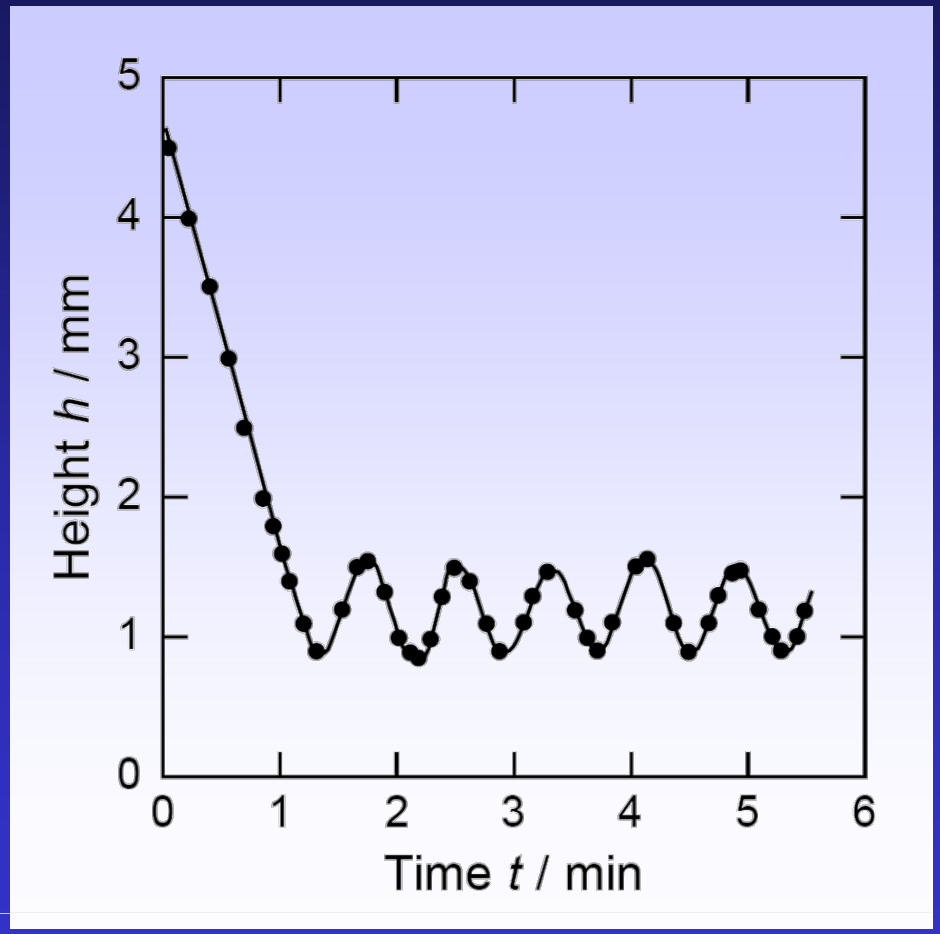
Beaker Experiments



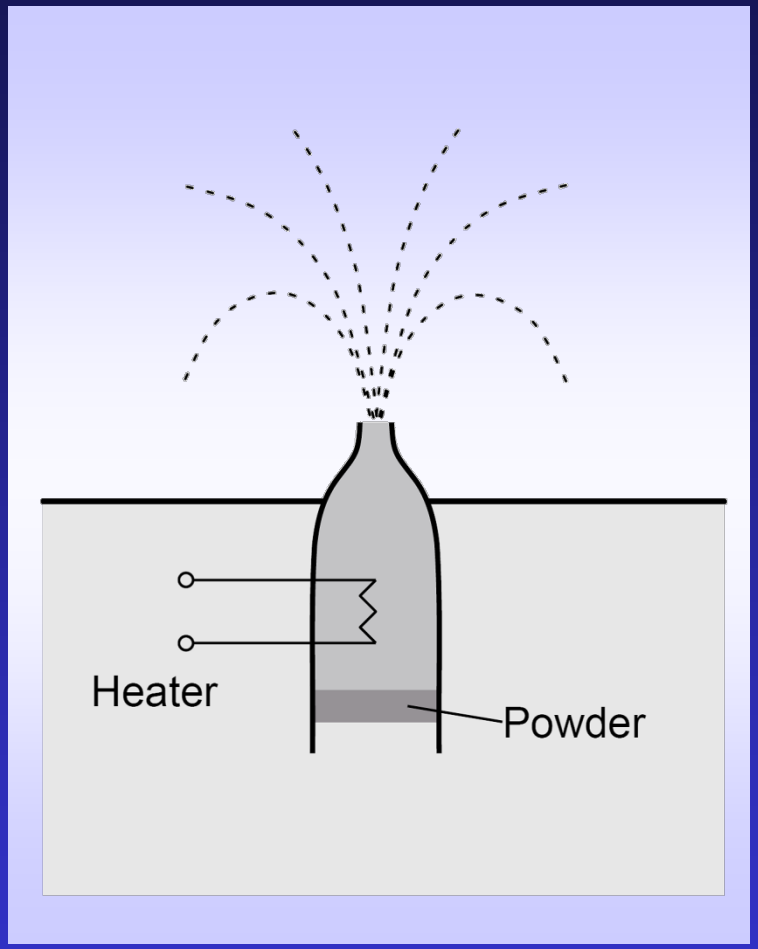


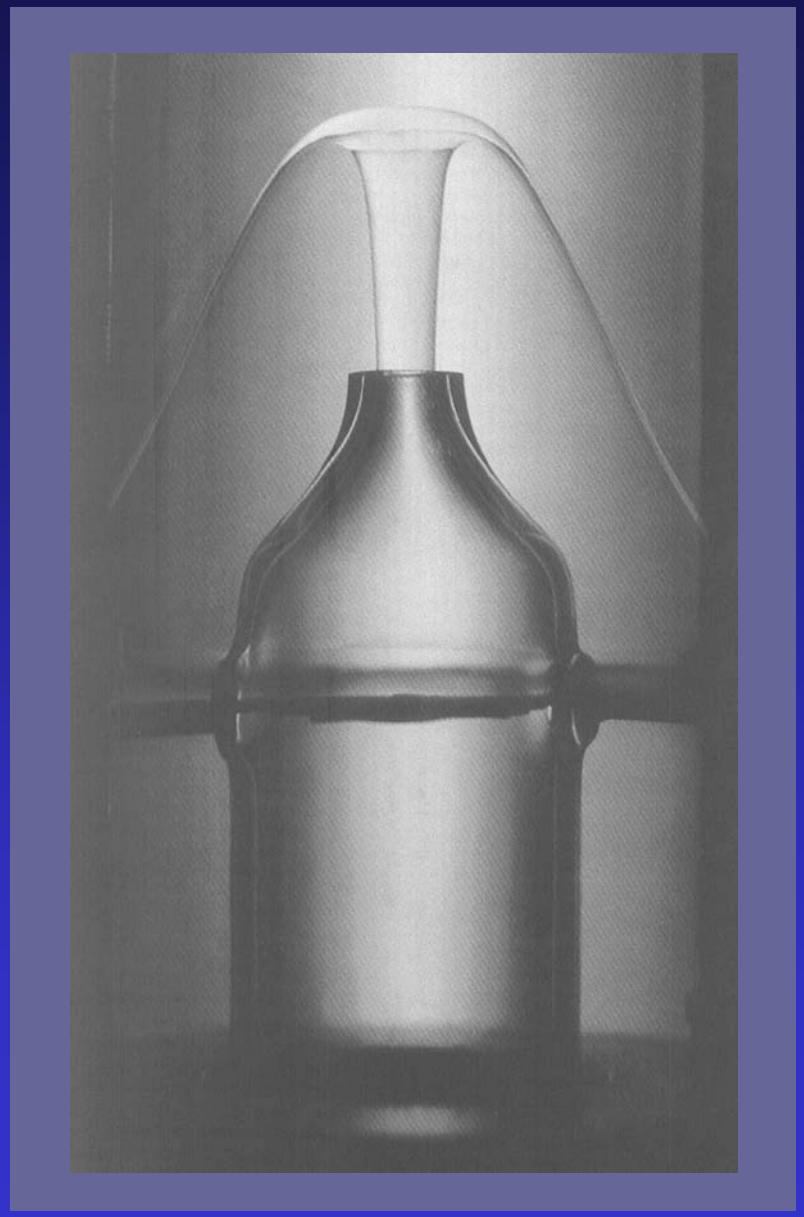
Beaker Experiments





Fountain Effect





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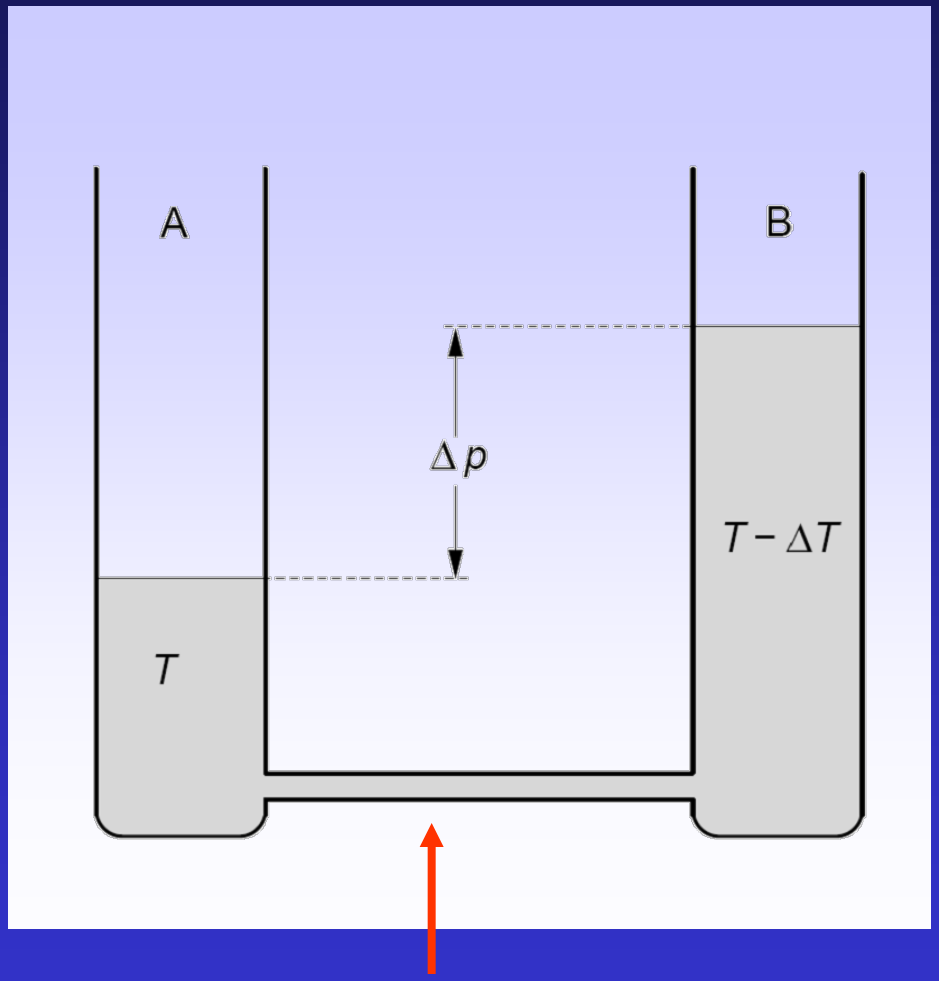
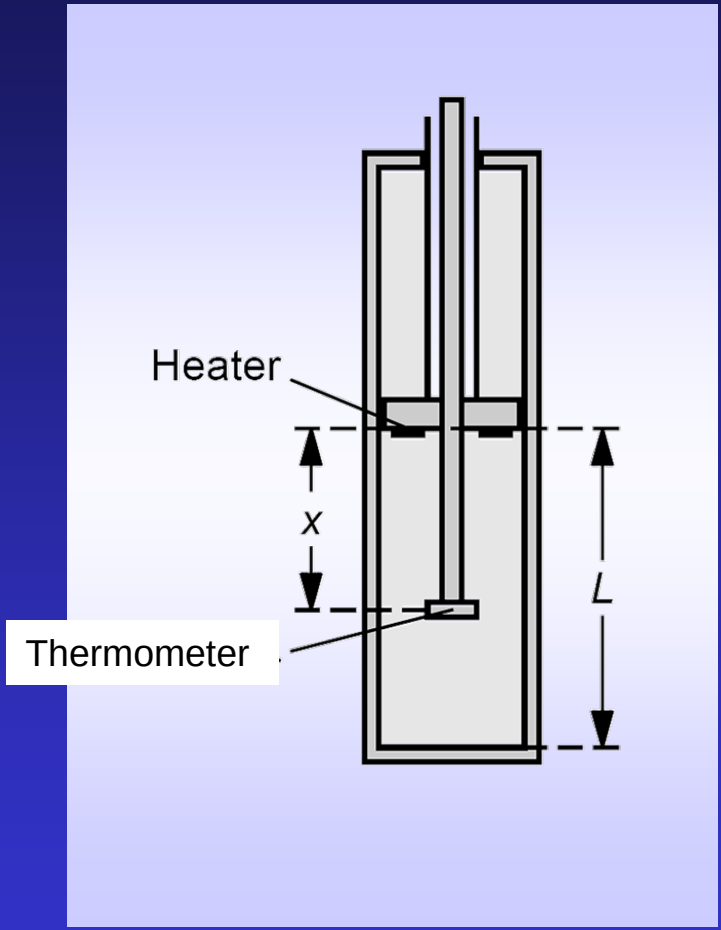
Thermomechanical Effect

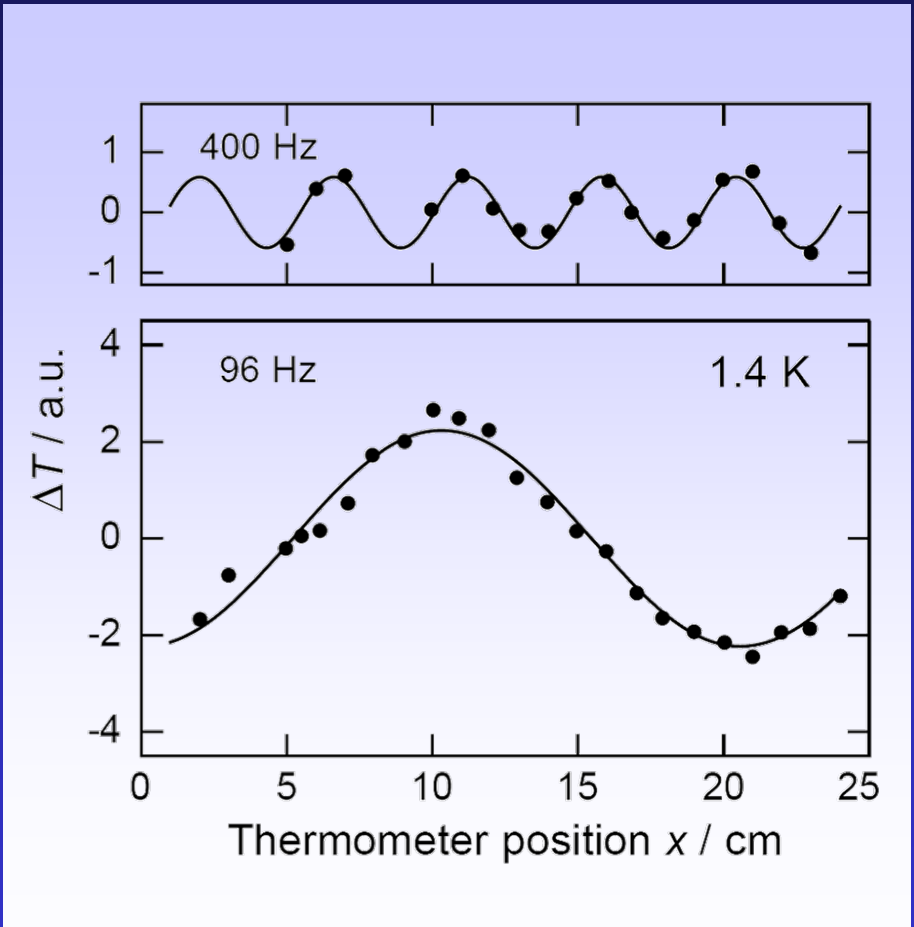


direction of mass transport
is opposite to heat transport

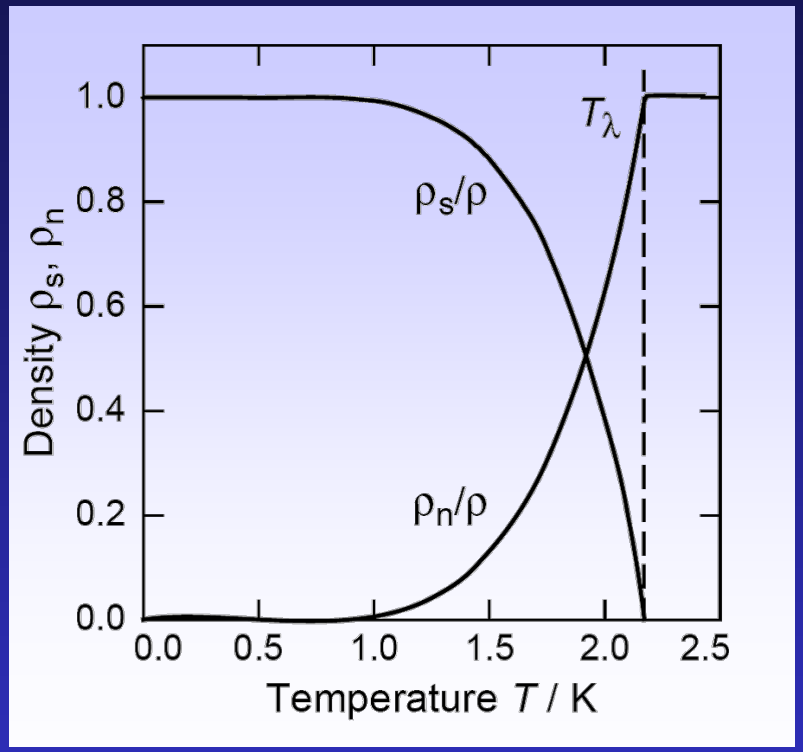
very thin capillary (superleak)

Second Sound: Temperature waves





Two Fluid Model



basic idea

$$\varrho = \varrho_{\mathrm{n}} + \varrho_{\mathrm{s}}$$

$T = T_\lambda$

$$\varrho_{\mathrm{s}} = 0 \qquad \varrho_{\mathrm{n}} = \varrho$$

$T = 0$

$$\varrho_{\mathrm{s}} = \varrho \qquad \varrho_{\mathrm{n}} = 0$$

	density	viscosity	entropy
normal fluid component	ϱ_{n}	$\eta_{\mathrm{n}} = \eta$	$S_{\mathrm{n}} = S$
suprafluid component	ϱ_{s}	$\eta_{\mathrm{s}} = 0$	$S_{\mathrm{s}} = 0$

Two Fluid Hydrodynamics

mass flow

$$\boldsymbol{j} = \varrho_{\mathrm{n}} \boldsymbol{v}_{\mathrm{n}} + \varrho_{\mathrm{s}} \boldsymbol{v}_{\mathrm{s}}$$

mass conservation

$$\frac{\partial \varrho}{\partial t} = -\mathrm{div}\, \boldsymbol{j}$$

Euler equation

$$\frac{\partial \boldsymbol{j}}{\partial t} + \underbrace{\varrho \boldsymbol{v} \cdot \mathrm{grad}\, \boldsymbol{v}}_{\approx 0} = -\mathrm{grad}\, p$$

entropy conservation

$$\frac{\partial (\varrho S)}{\partial t} = -\mathrm{div}(\varrho S \boldsymbol{v}_{\mathrm{n}})$$

equation of motion
superfluid compontent

$$\frac{\partial \boldsymbol{v}_{\mathrm{s}}}{\partial t} = S\, \mathrm{grad}\, T - \frac{1}{\varrho}\, \mathrm{grad}\, p$$

Viscosity Experiments

Flow Through Capillaries:

Superfluid component is flowing at critical velocity

→ no viscosity below Lambda point

Rotary Viscosimeter:

Torque: $M_{\mathrm{r}} = \pi \eta \omega d_{\mathrm{r}}^2 d_{\mathrm{s}}^2 / (d_{\mathrm{s}}^2 - d_{\mathrm{r}}^2)$

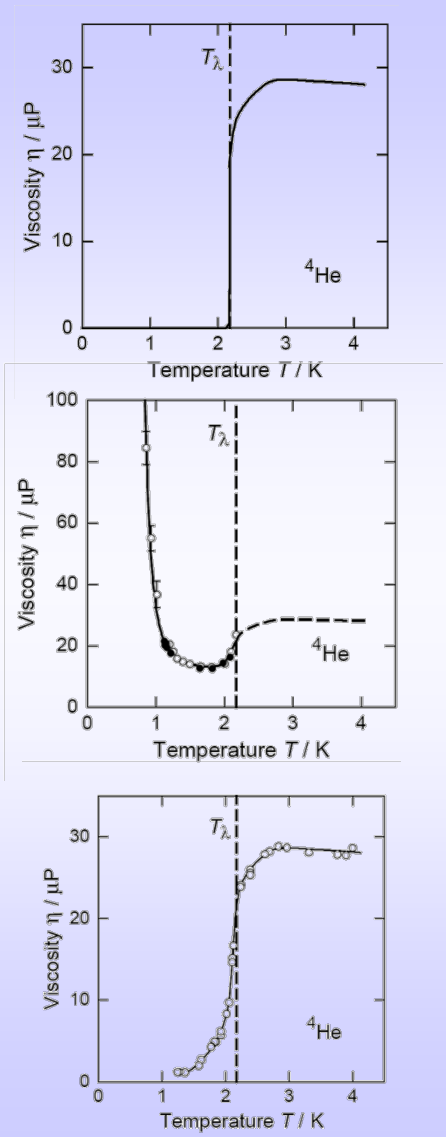
Normal fluid component is observed below Lambda point

→ viscocity depends on mean free path of excitations

Oscillating Disc Viscosimeter:

Torque: $M_{\mathrm{d}} = \pi \sqrt{\varrho \eta}\, \omega^{3/2} r^4\, \Theta(\omega)$

Product of normal fluid density and normal fluid viscocity is observed



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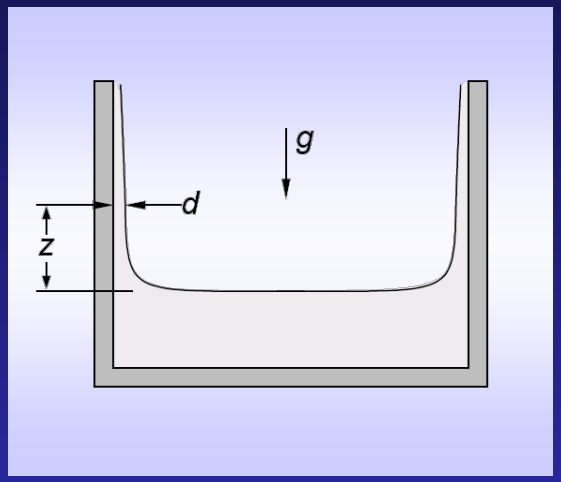
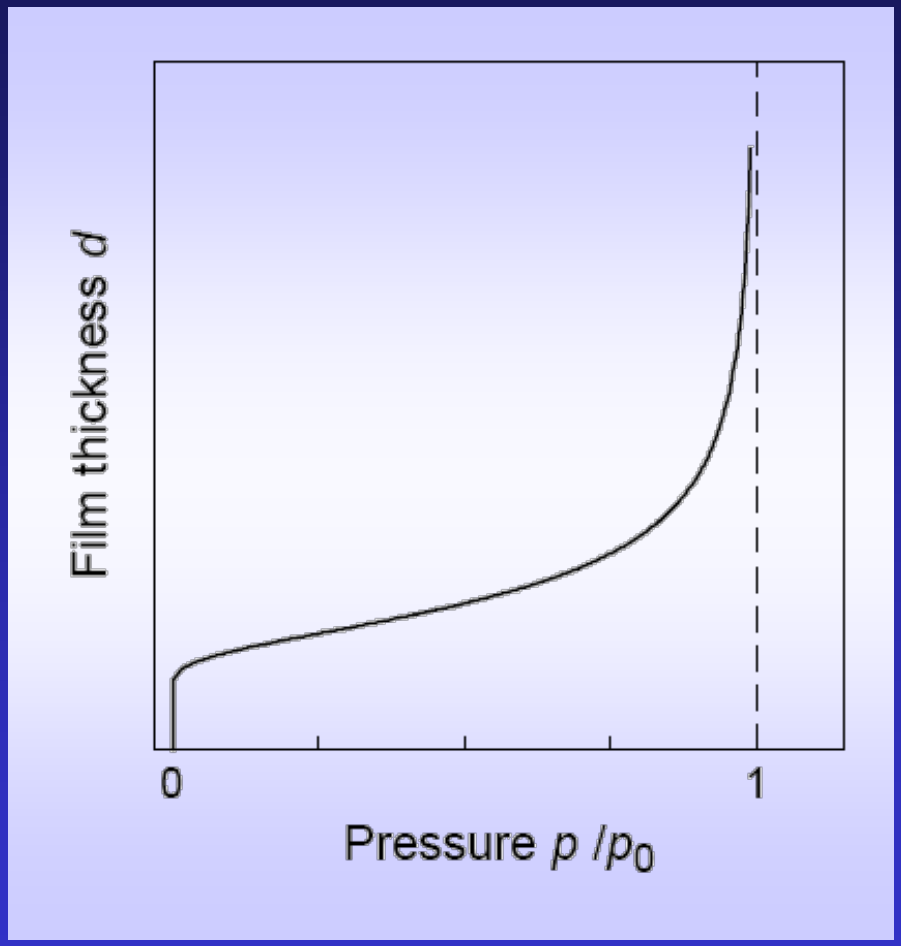
Beaker flow: Helium films





Film thickness at saturated vapor:

$$d = \sqrt[3]{\frac{\alpha}{gz}}$$

For $z = 10\,\mathrm{cm}$

$\Rightarrow$ $200\,\text{Å}$

Film flows at the critical velocity

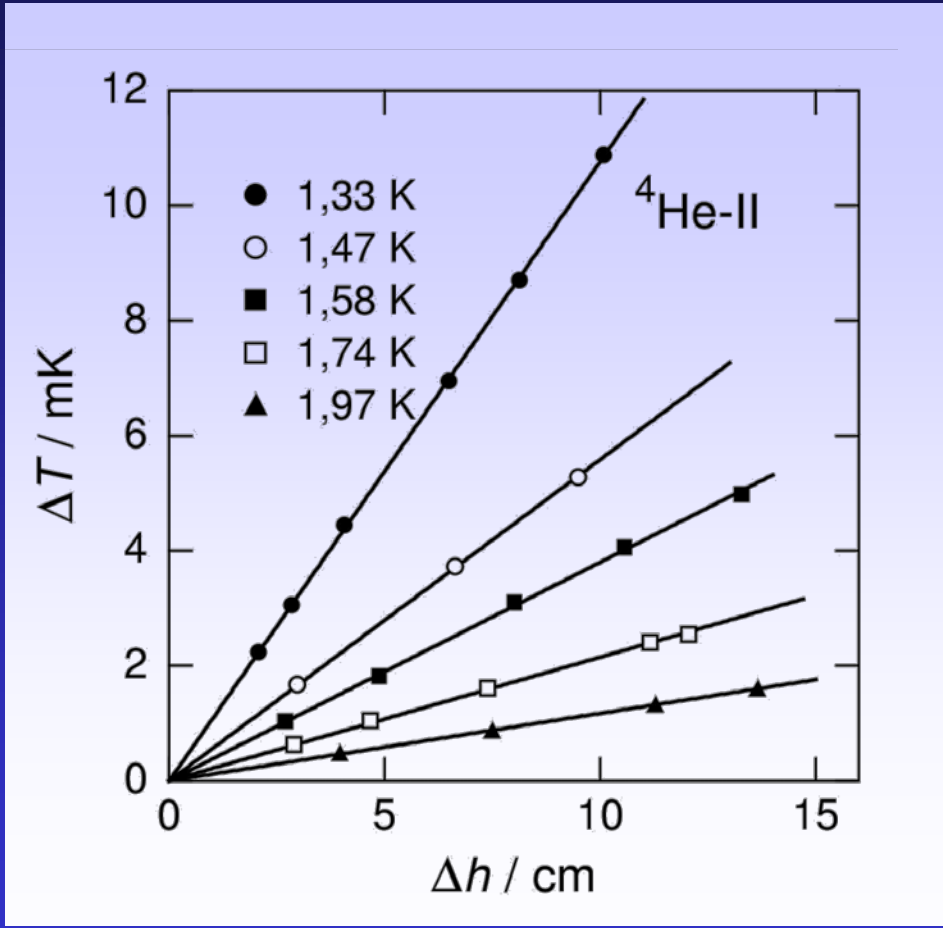
Thermo-mechanical Effect

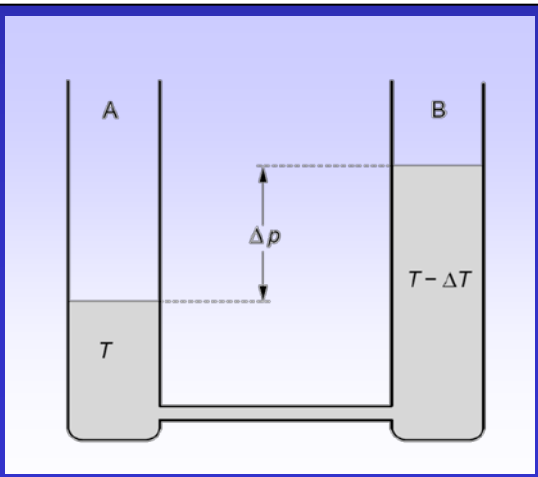
In equilibrium:

$$\frac{\partial \boldsymbol{v}_{\mathrm{s}}}{\partial t} = S \,\mathrm{grad}\, T - \frac{1}{\varrho}\,\mathrm{grad}\, p = 0$$

(H.) London equation:

$$\frac{\Delta p}{\Delta T} = \varrho S$$





Sound Propagation

From two-fluid hydrodynamics:

$$\frac{\partial^2 \varrho}{\partial t^2} = \nabla^2 p \qquad \frac{\partial^2 S}{\partial t^2} = \frac{\varrho_{\mathrm{s}}\, S^2}{\varrho_{\mathrm{n}}} \nabla^2 T$$

Can be transformed into:

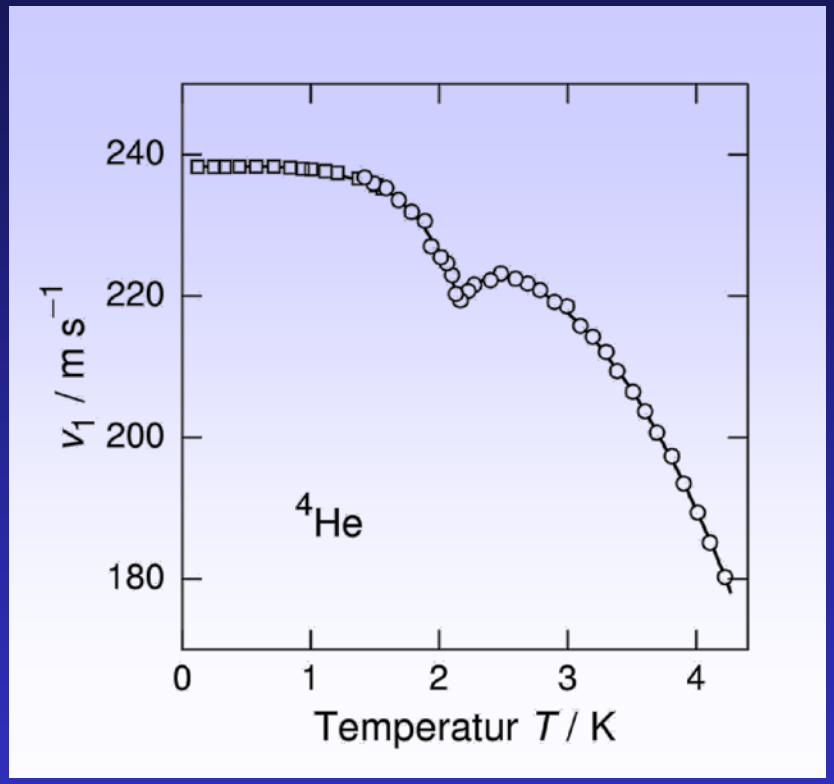
$$\left[\left(\frac{v}{v_1}\right)^2 - 1\right]\left[\left(\frac{v}{v_2}\right)^2 - 1\right] = \frac{C_p - C_V}{C_p}$$

two weakly coupled modes

with

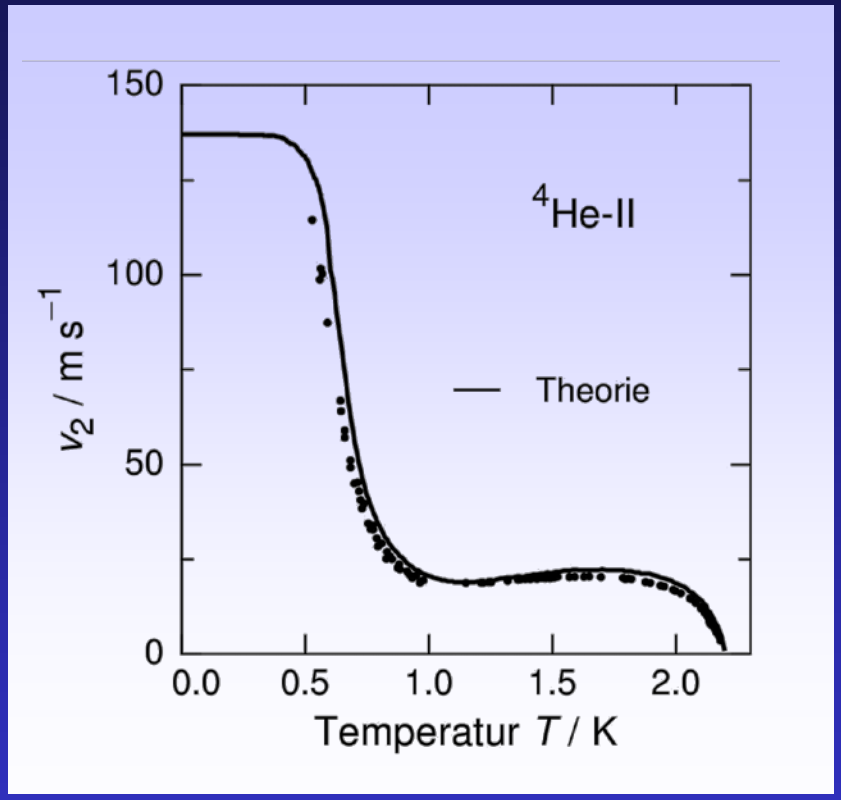
$$v_1^2 = \left(\frac{\partial p}{\partial \varrho}\right)_S \quad \text{and} \quad v_2^2 = \frac{\varrho_{\mathrm{s}}}{\varrho_{\mathrm{n}}} S^2 \left(\frac{\partial T}{\partial S}\right)_\varrho$$

First Sound



$$\boldsymbol{v}_{\mathrm{n}} = \boldsymbol{v}_{\mathrm{s}}$$

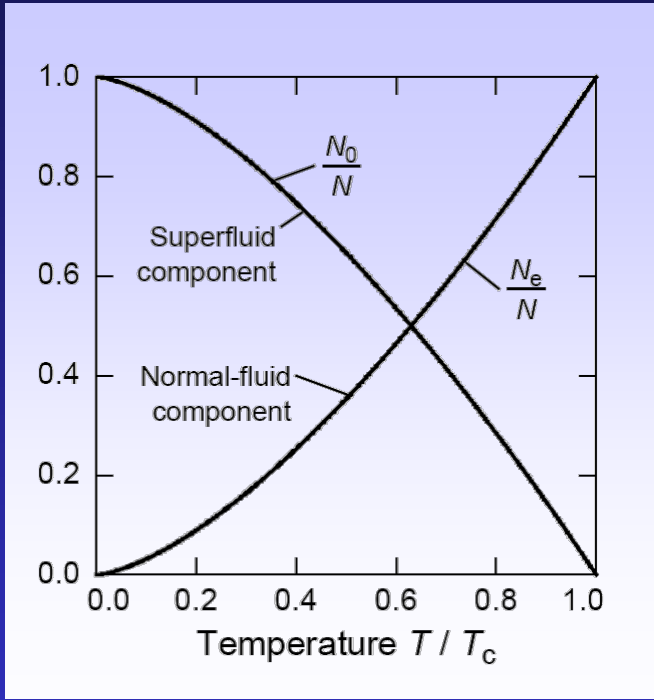
Second Sound



$$\varrho_{\mathrm{n}}\boldsymbol{v}_{\mathrm{n}} + \varrho_{\mathrm{s}}\boldsymbol{v}_{\mathrm{s}} = 0$$

$$T \to 0 \quad v_2 \to v_1/\sqrt{3} \simeq 137\,\mathrm{m/s}$$

Bose-Einstein Condensation



$$N = N_0 + \frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty \frac{\sqrt{E}}{e^{(E-\mu)/k_B T} - 1}\,dE$$

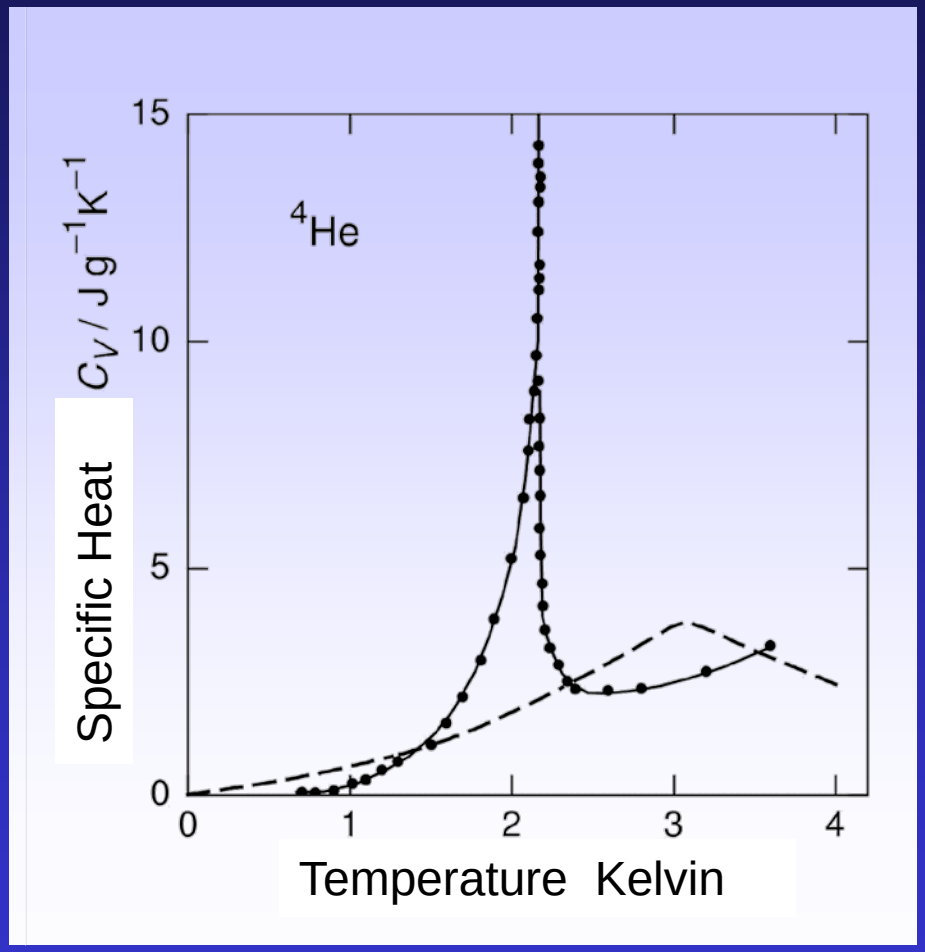
$$N \approx N_0 + 2.6\,\frac{V}{V_Q}$$

$$N_e(T_c) = N \text{ and } N_0(T_c) = 0$$

$$\frac{N_e}{N} = \left(\frac{T}{T_c}\right)^{3/2}$$

$$T_c = \frac{2\pi\hbar^2}{k_B m}\left(\frac{N}{2.6V}\right)^{2/3}$$

Specific Heat



- - - ideal Bose-Gas

● liquid Helium

Macroscopic Quantum State

Macroscopic wave function

$$\psi(\boldsymbol{r}) = \psi_0 \, \mathrm{e}^{\mathrm{i}\varphi(\boldsymbol{r})}$$

$$\psi^\star \psi = |\psi_0|^2 = \frac{\varrho_\mathrm{s}}{m_4}$$

$$-\mathrm{i}\hbar\nabla\psi = \boldsymbol{p}\,\psi \qquad \boldsymbol{p} = \hbar\nabla\varphi(\boldsymbol{r}) = m_4 \boldsymbol{v}_\mathrm{s}$$

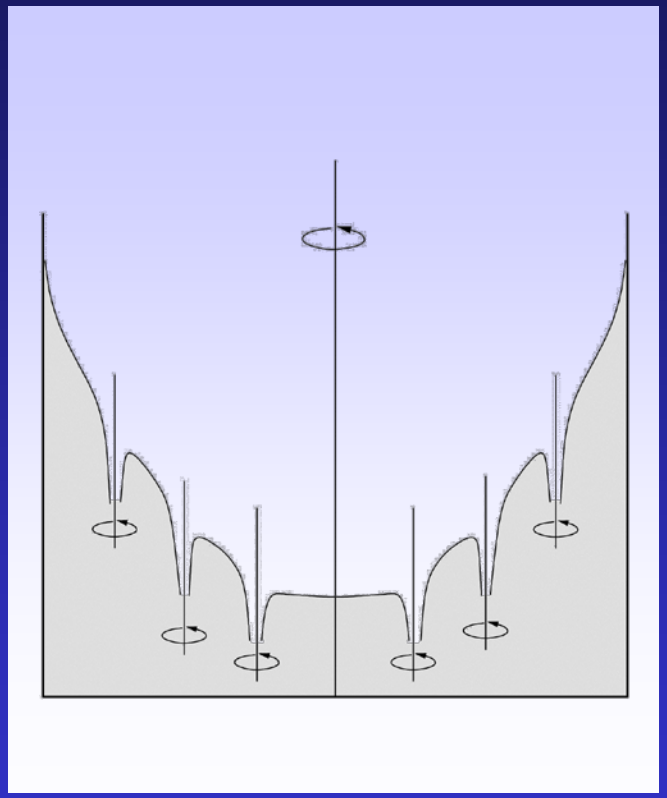
$$\boldsymbol{v}_\mathrm{s} = \frac{\hbar}{m_4}\nabla\varphi(\boldsymbol{r})$$

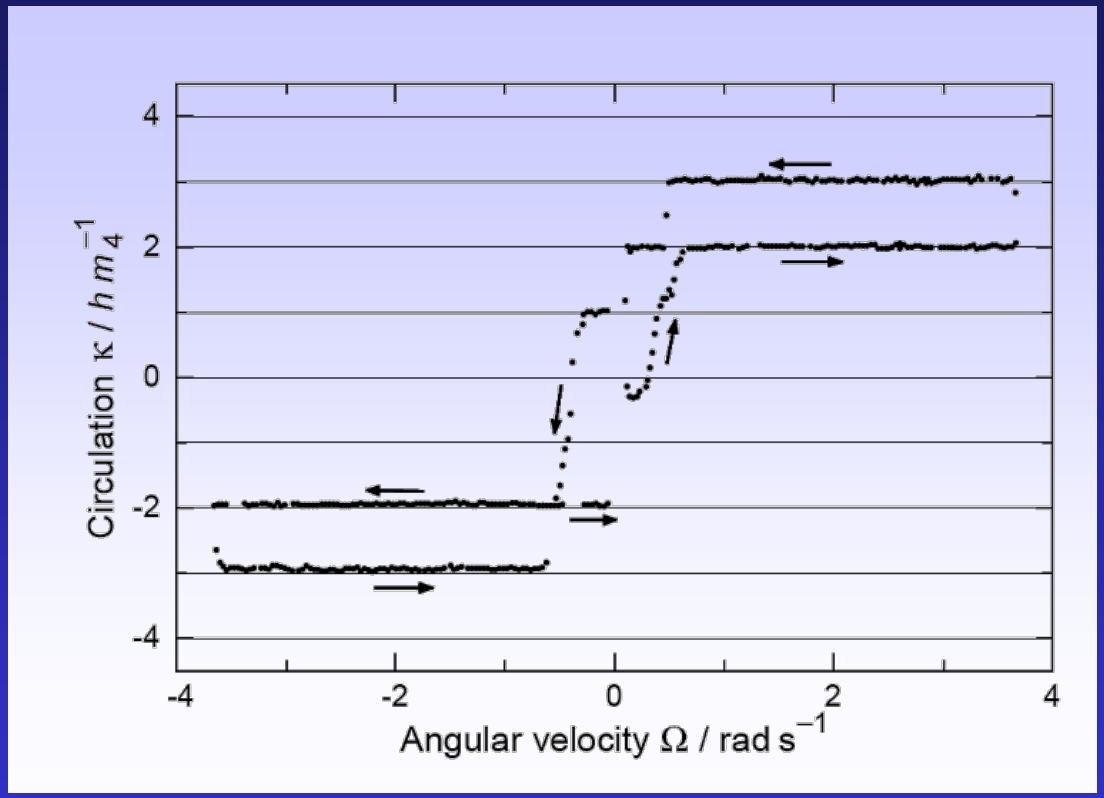
$$\kappa = \oint_L \boldsymbol{v}_\mathrm{s} \cdot \mathrm{d}\boldsymbol{l} \qquad \Delta\varphi = 2\pi\, n$$

Quantized circulation

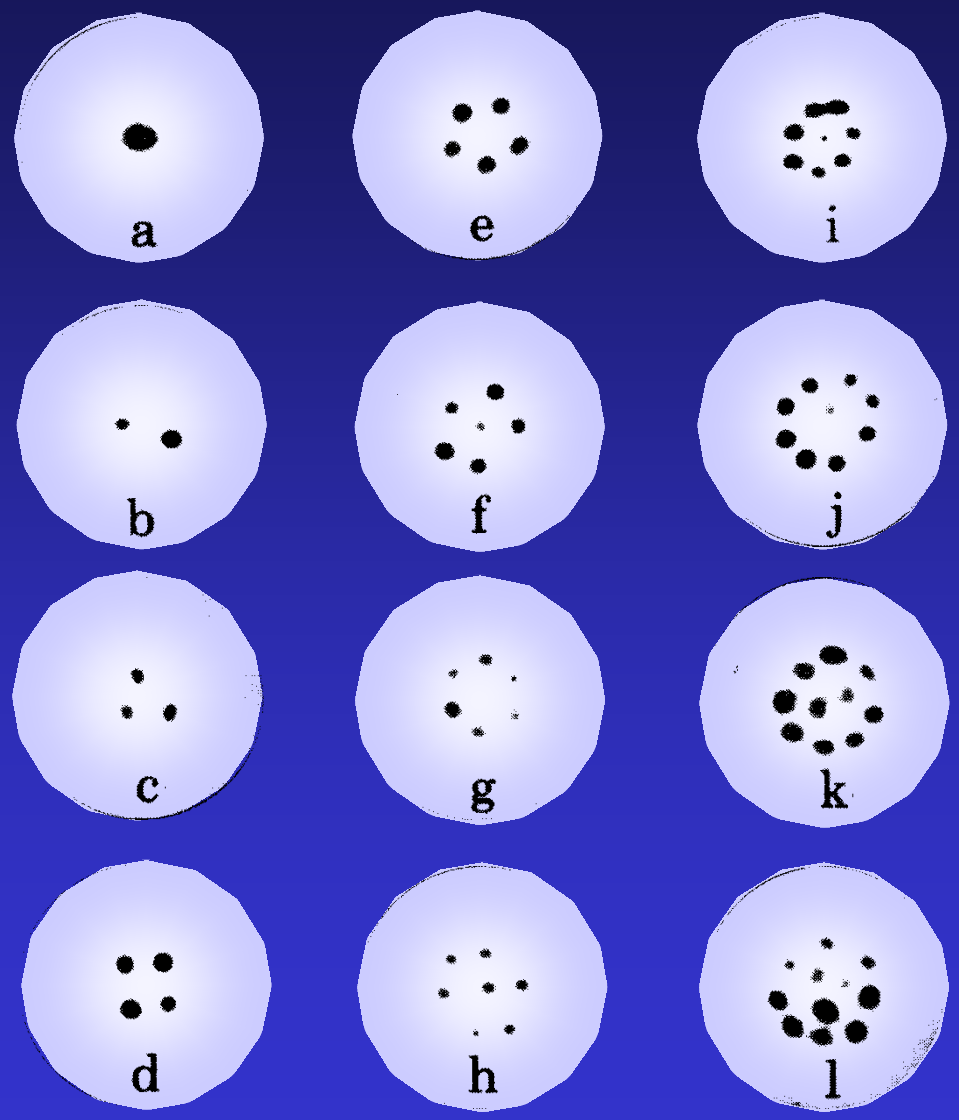
$$\boxed{\kappa = \frac{h}{m_4}\, n}$$

Quantization of Circulation





Vortices with Quantized Circulation



T = 0,1 K

d = 2 mm

l = 25 mm

(a) ⟶ (l)

3 ⟶ 8 Rev./min

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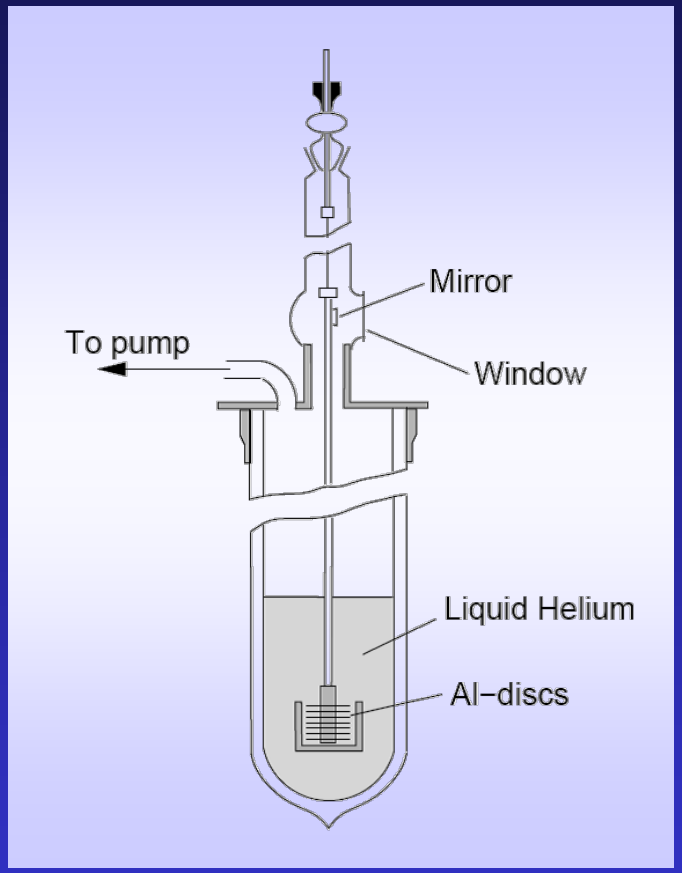
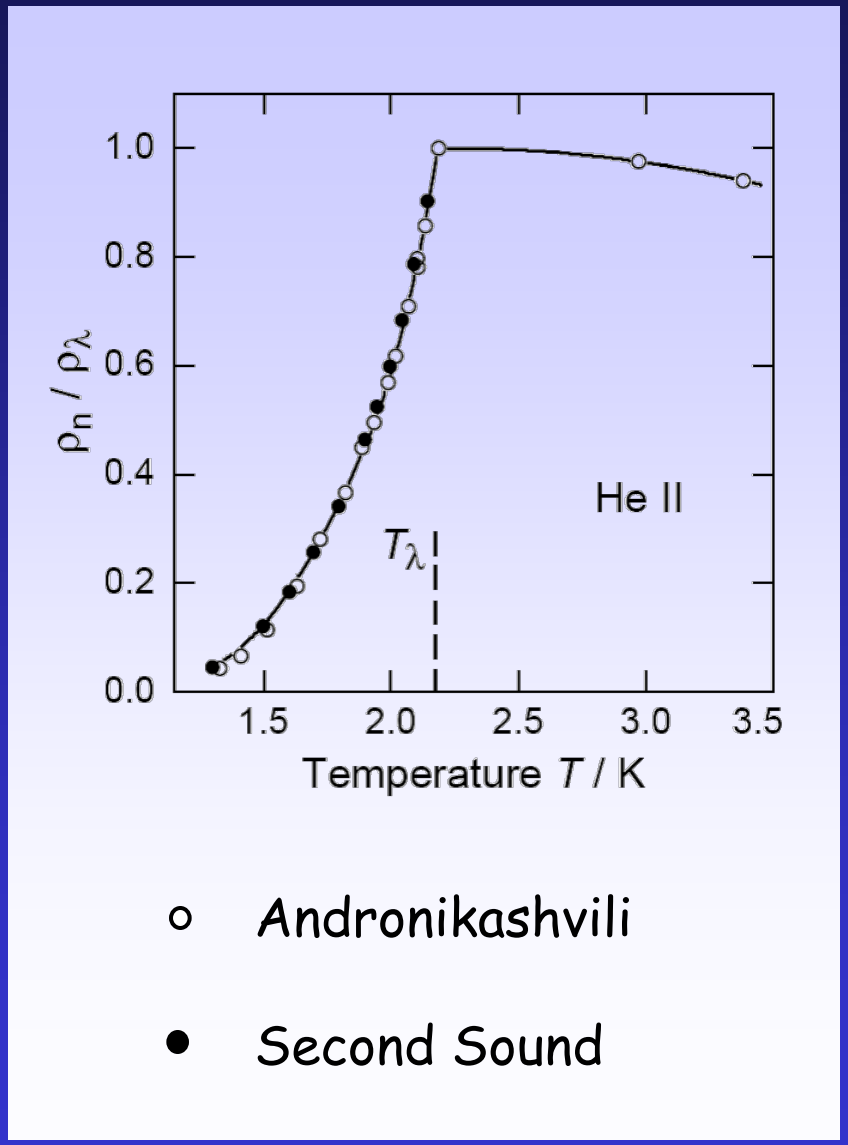
Determination of ρ_n

Andronikashvili 1948





50 Aluminium discs

Thickness 13 μm

Diameter 3,5 cm

Distance 210 μm

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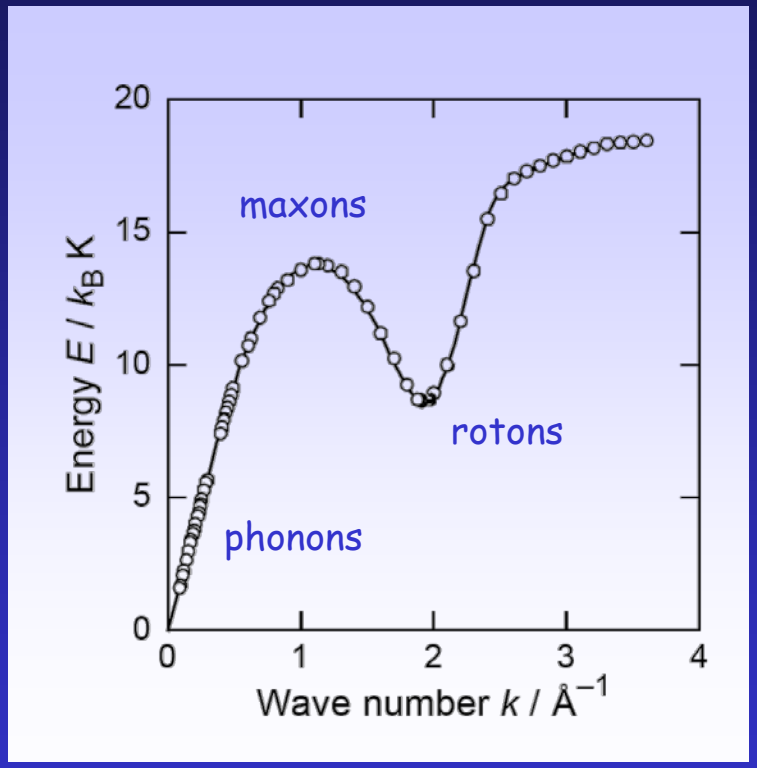
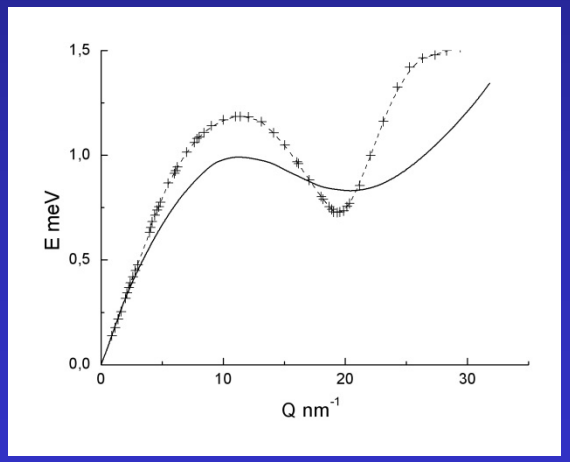
Excitation Spectrum of superfluid 4He



Well-defined collective excitations

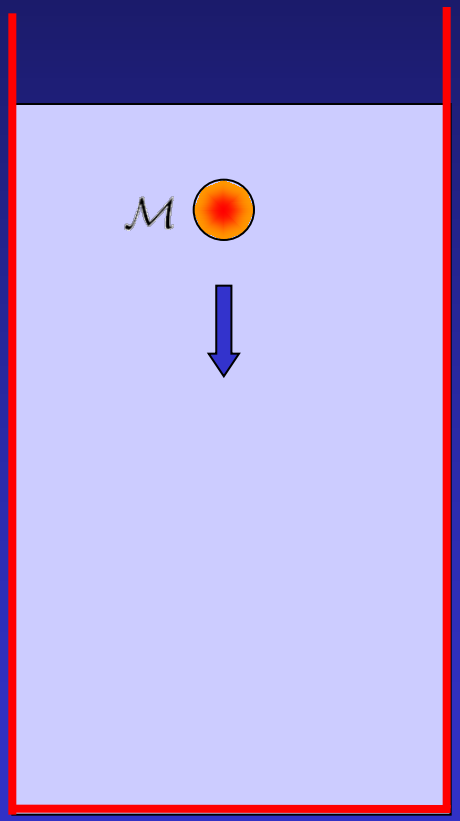
Phonons and Rotons

No single particle excitations



Theory: Landau

Concept of a Critical Velocity



$$T = 0$$

one excitation with: energy $\mathcal{E}$ and momentum $\boldsymbol{p}$

energy conservation

$$\frac{1}{2}\mathcal{M}v^2 = \frac{1}{2}\mathcal{M}v'^2 + \mathcal{E}$$

momentum conservation

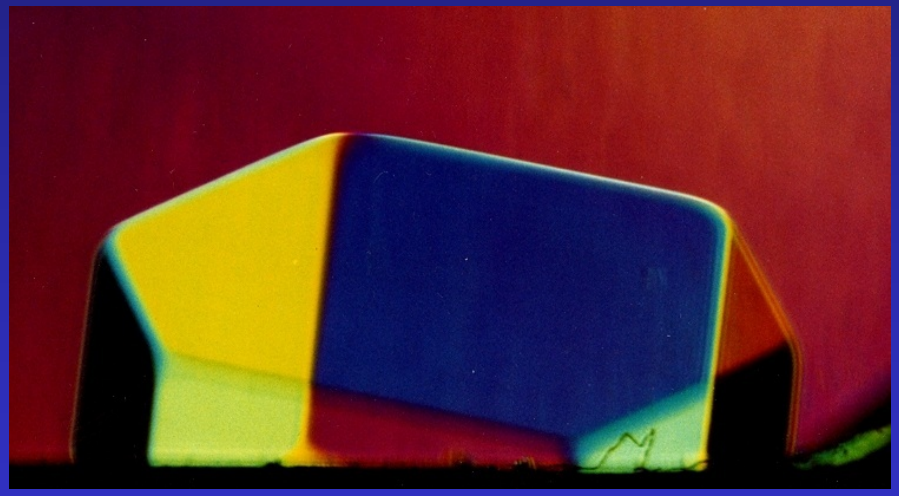
$$\mathcal{M}\boldsymbol{v} - \boldsymbol{p} = \mathcal{M}\boldsymbol{v}'$$

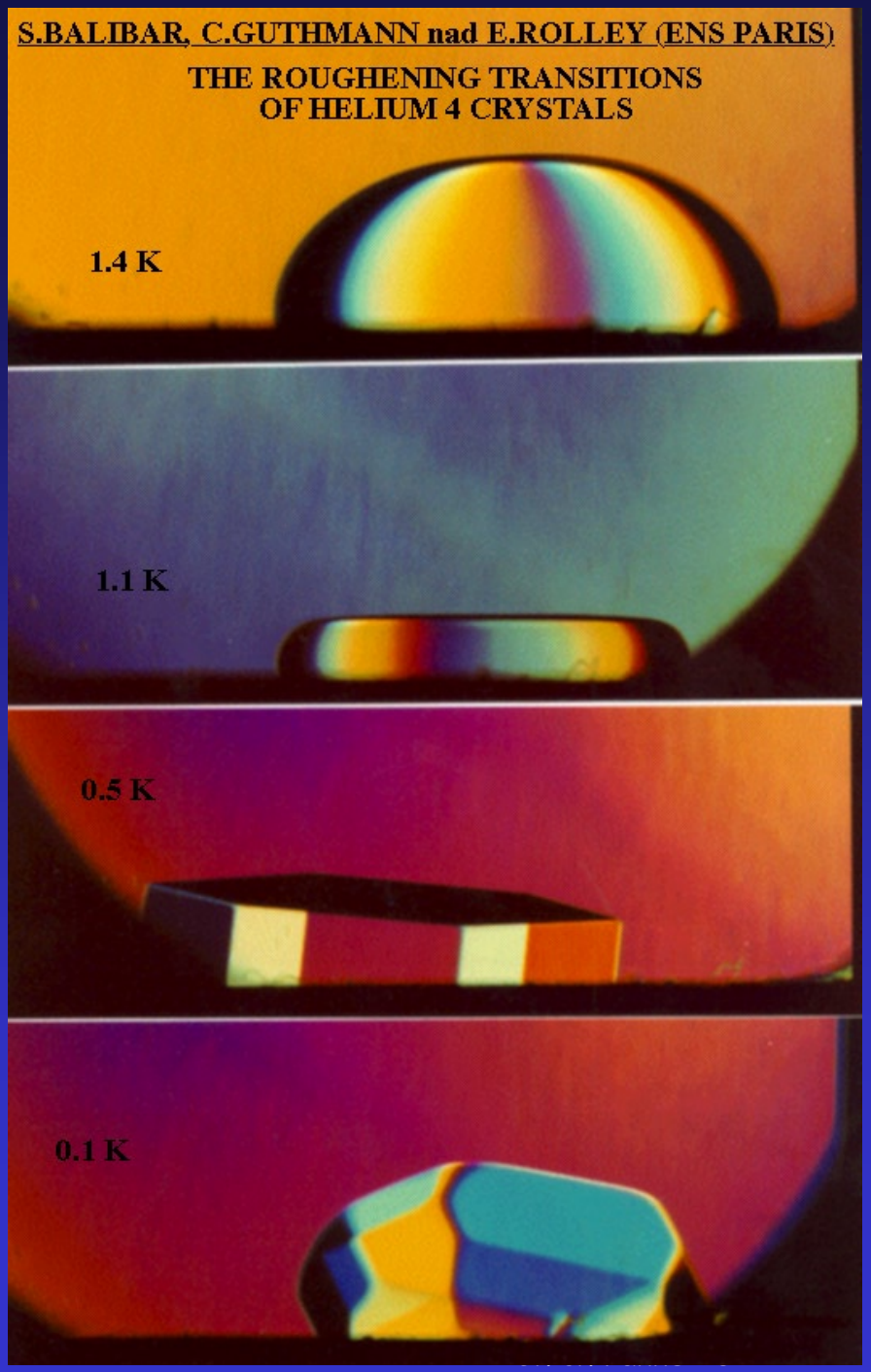
$$\boldsymbol{v}\cdot\boldsymbol{p} - \frac{1}{2\mathcal{M}}p^2 = \mathcal{E}$$

Critical velocity

$$\boxed{v_c = \frac{\mathcal{E}}{p}}$$

Solid ^{4}He

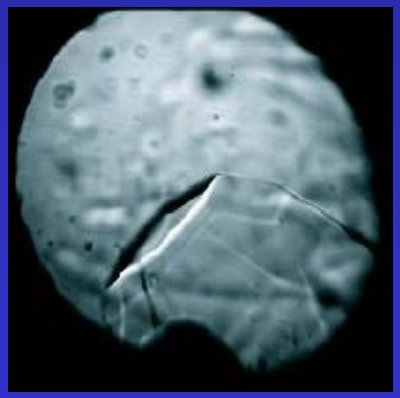


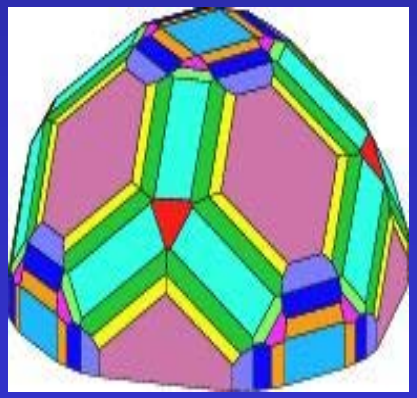

S.BALIBAR, C.GUTHMANN nad E.ROLLEY (ENS PARIS)
THE ROUGHENING TRANSITIONS
OF HELIUM 4 CRYSTALS
1.4 K
1.1 K
0.5 K
0.1 K

Crystallization waves in helium-4.

Photograph taken by S. Balibar, C. Guthmann and E. Rolley in Paris.

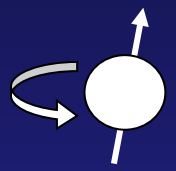
Growing helium-3 crystal at 2.2 mK





Eleven different types of facets observed on the surface of helium-3 crystals.
from Wagner *et al.*, Phys Rev. Lett. **76**, 263 (1996).

Nuclear magnetism of ^{3}He



^{3}He atom : nuclear spin $\frac{1}{2}$ **Fermion!**

Nuclear magnetic moment : $\mu = -2.1274\ \mu_n$
$\mu/k_B = 7.78255\ 10^{-4}$ Kelvin/Tesla
$\gamma/2\pi = 32.435$ MHz/Tesla

Spin $\frac{1}{2}$, large magnetic moment, good nucleus for NMR!!!

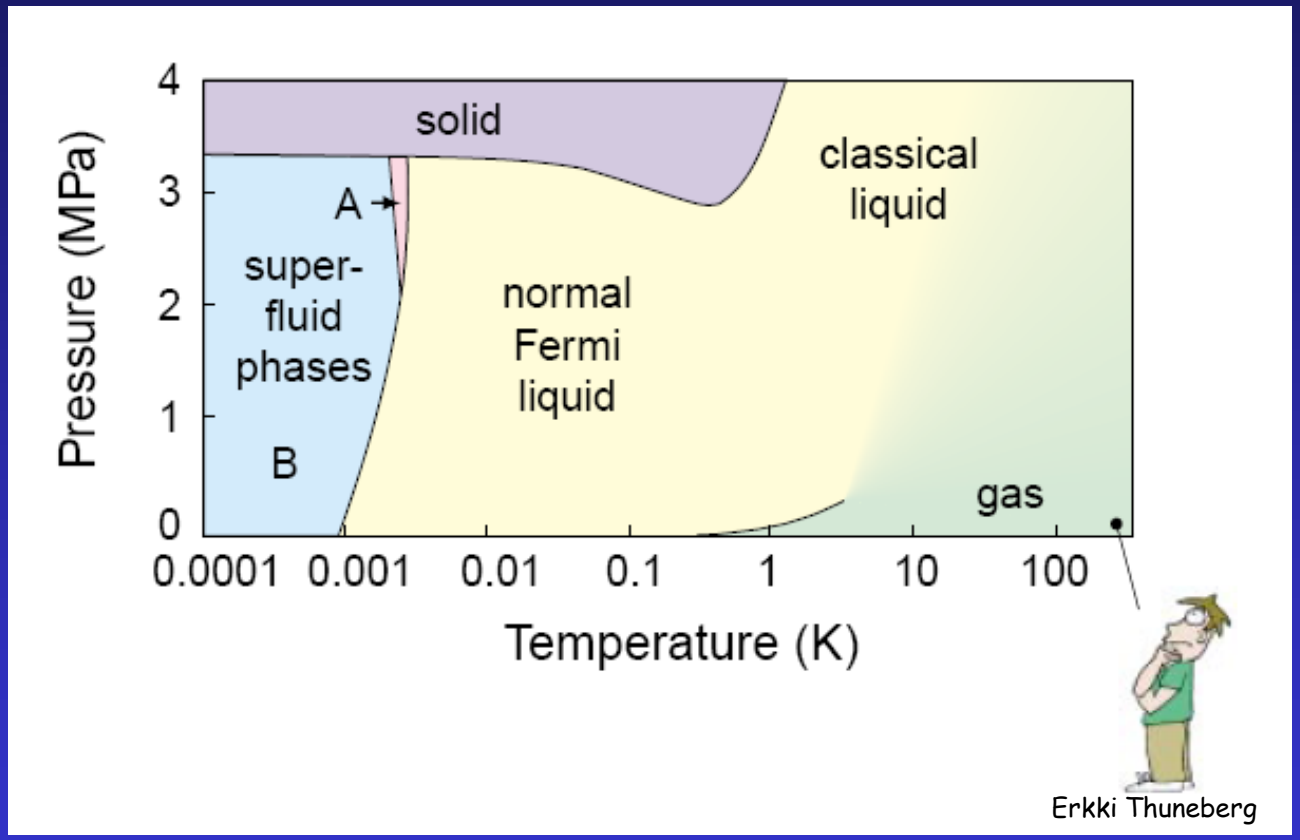
In the solid phases the atoms are <u>quasi-localized</u>

Zero point energy is comparable to the potential well depth, about 10 K (structural energies!).

Large tunneling of atoms (frequency of order MHz)
Quantum exchange interactions : $J \sim 1$ mK.

Small dipole-dipole interactions $E_D \sim \mu^2/a^3 \sim \mu K$

Phase Diagram of ^{3}He



Melting curve

T > 1 K : classical phase diagram

T < 1 K : the entropy of the solid is larger than that of the liquid

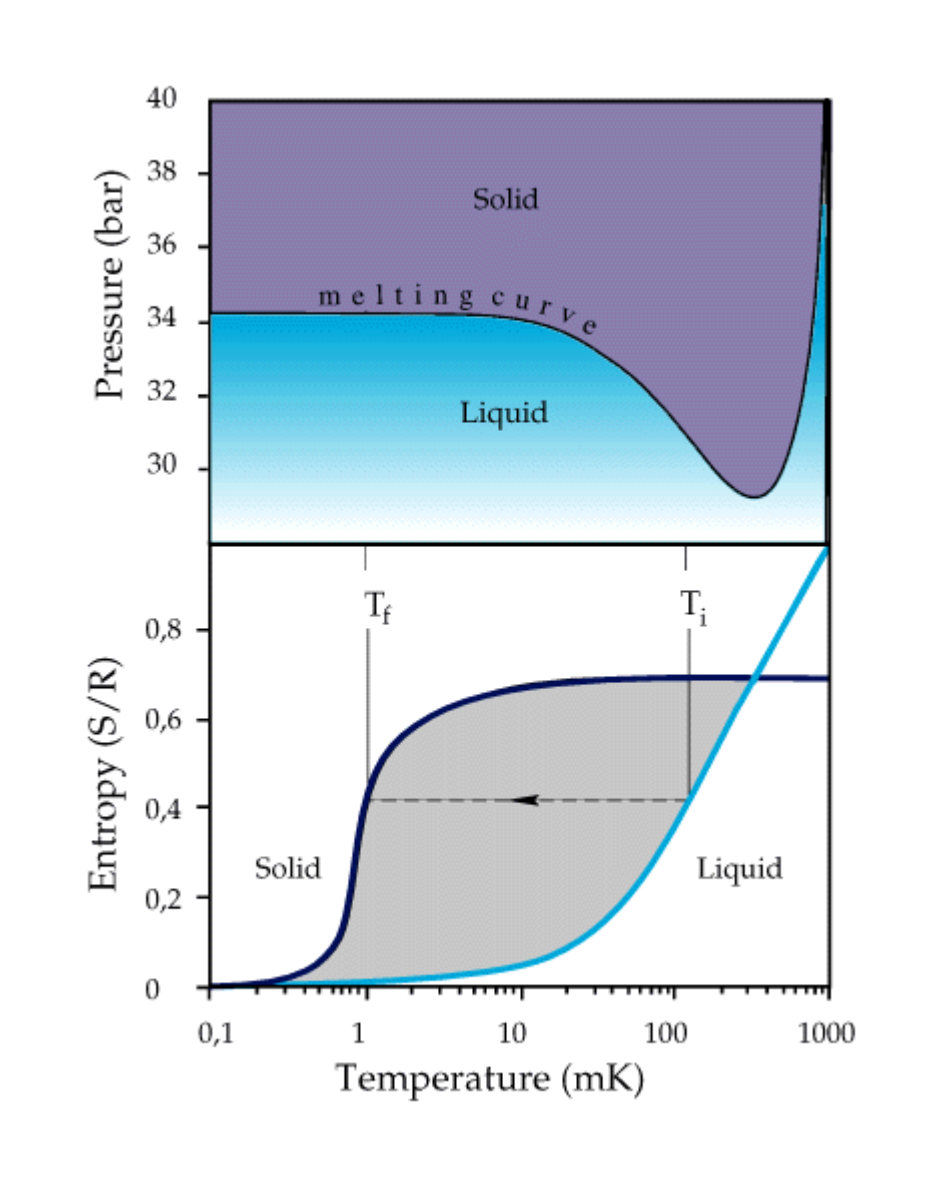
Disordered solid (spin entropy

Liquid is ordered in k-space (FermiLiquid)

→Minimum of the melting curve

(Clausius-ClapeyronEquation)

Solid nuclear order at T ~ 1 mK



Landau theory of Fermi liquids

$C = \gamma T \quad (T \ll T_F)$

$\chi = c/T_F^{**}$

$\eta \propto T^{-2}$

$\kappa \propto T^{-1}$

« Fermi Gas » with renomalised parameters

Theory is subtle!

See Pines and Nozières books

Effective mass m*

$C/C_{id} = m^*/m$ 2,8 to 5,8
increases with pressure

Reinforced magnetism 9,2 to 23,7
increases with pressure

$$\frac{C_v}{C_v^{id}} = \frac{m^*}{m} = 1 + \frac{1}{3} F_1^s$$

$$\frac{\chi(0)}{\chi^0(0)} = \frac{m^*}{m} \cdot \frac{1}{1 + F_0^a}$$

Heat Capacity

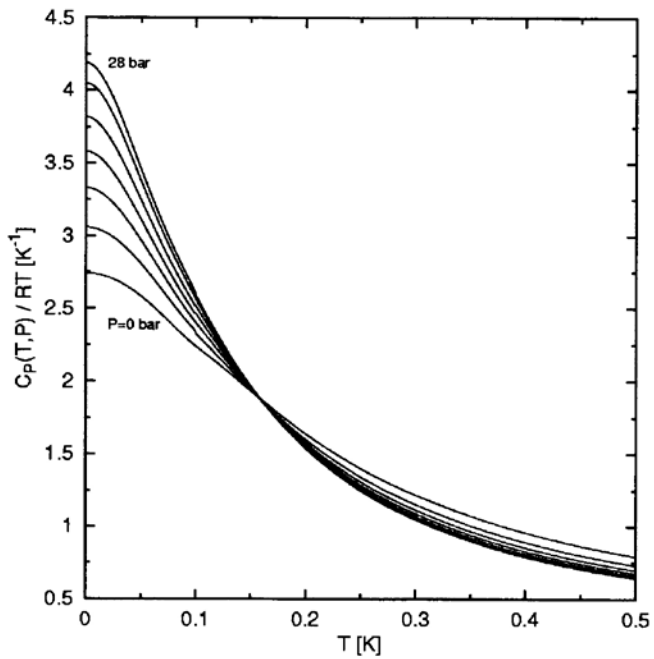


FIG. 5. Specific heat at constant pressure $C_P(T,P)$ divided by temperature vs T at pressures $P=0$, 5, 10, 15, 20, 25, 28 bar. The small kinks at the temperature $T_0=0.1$ K in this and other figures are artefacts caused by the different interpolation formulas (Ref. 16) for $C_V(T,V)$ used below and above T_0.

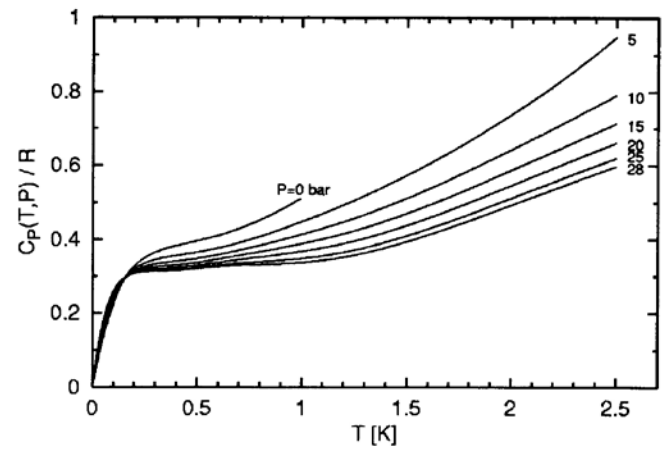


FIG. 6. High-temperature behavior of the specific heat at constant pressure $C_P(T,P)$.

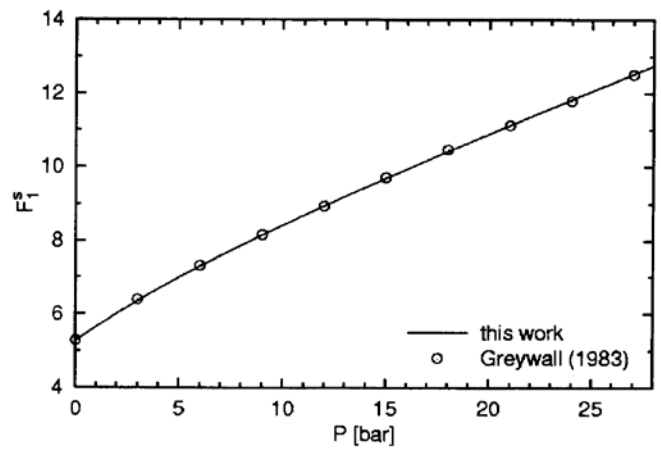
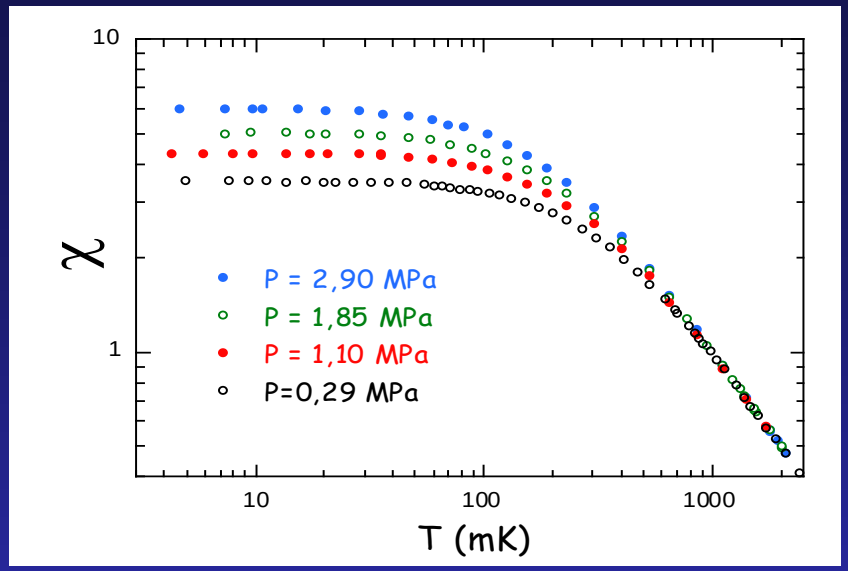
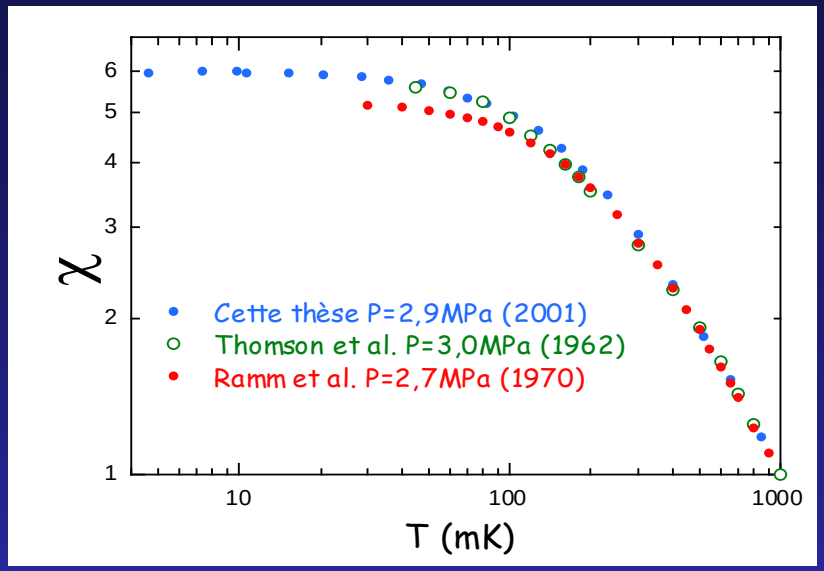


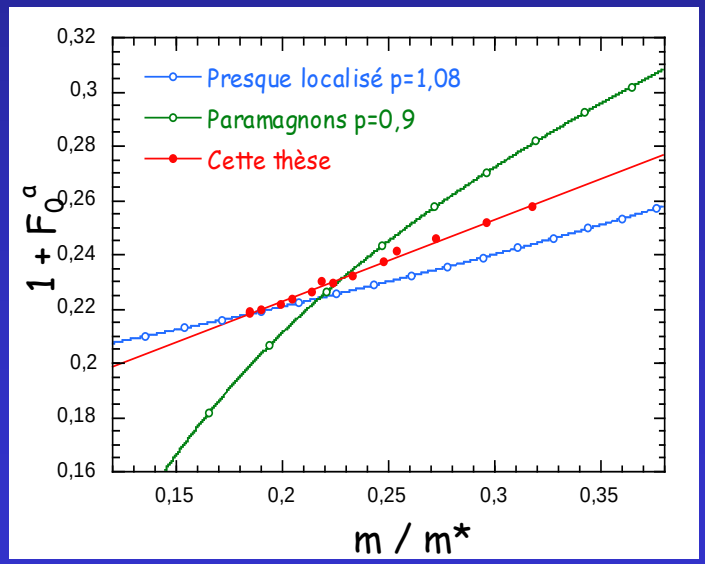
FIG. 9. Fermi-liquid parameter F_1^s vs P, compared to the data obtained by Greywall (Ref. 16).

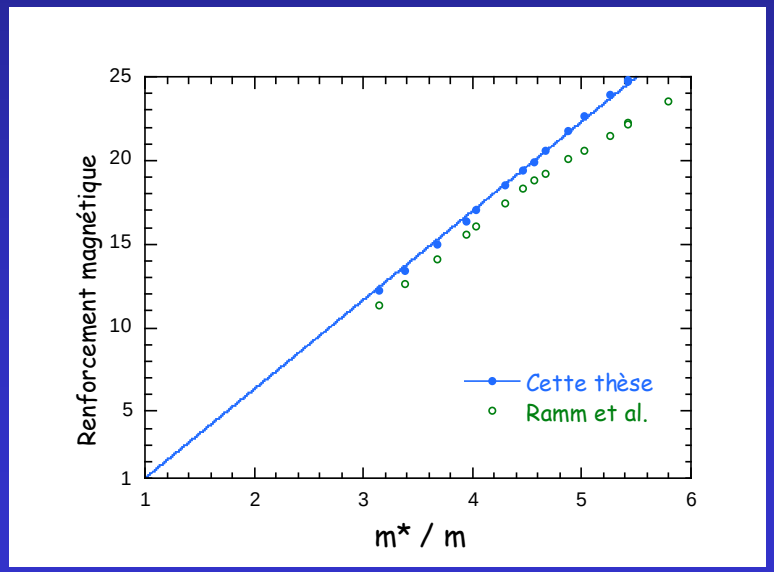
D.S. Greywall (1983)

Magnetic susceptibility





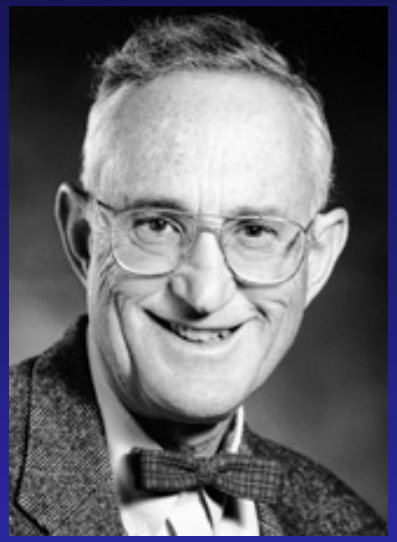




(PhD thesis S. Triqueneaux, 2001; V. Goudon, 2006)

The Nobel Prize in Physics 1996

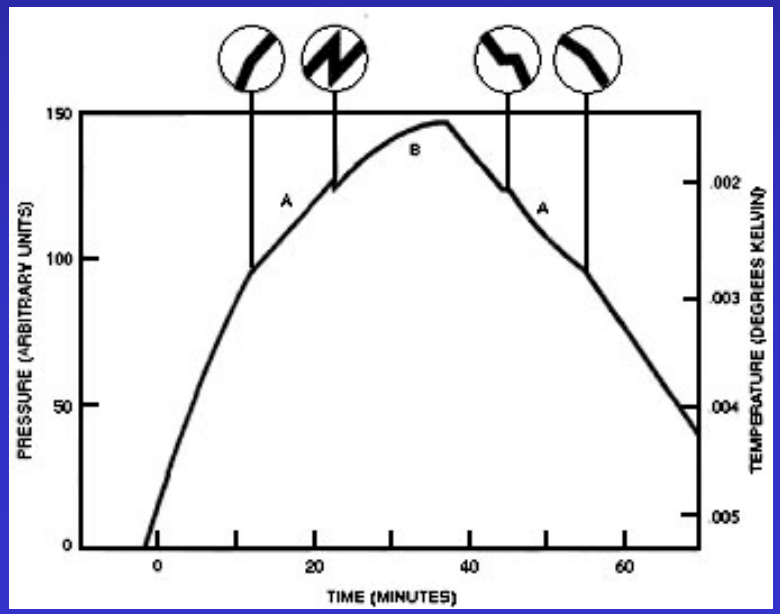
"for their discovery of superfluidity in helium-3"



David M. Lee

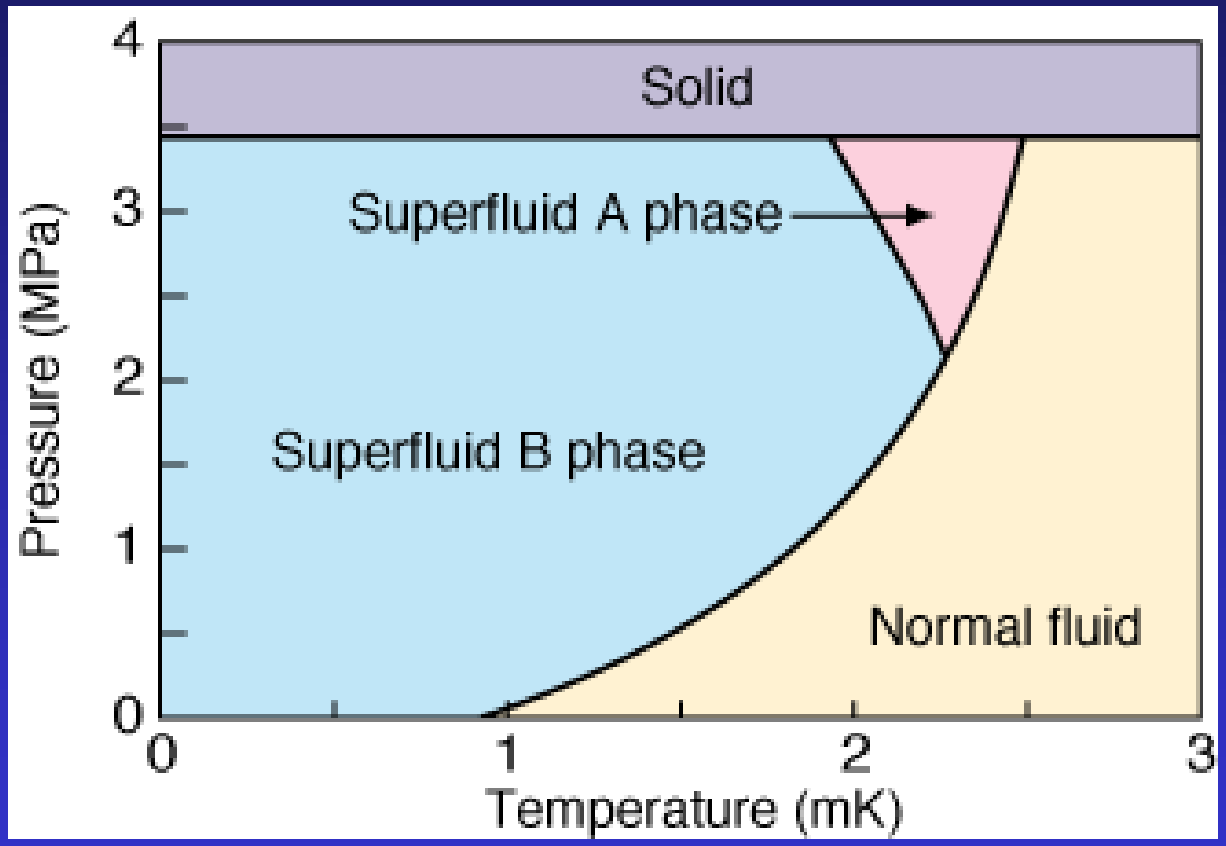
Douglas D. Osheroff

Robert C. Richardson



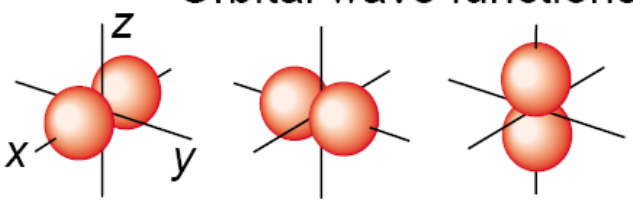
The figure shows the pressure inside a sample containing a mixture of liquid helium-3 and solid helium-3 ice. The sample is first subjected to increasing external pressure for about 40 minutes, whereafter the external pressure is reduced. Note the changes in the slope of the curve at A and B and the temperatures at which these occur. The graph resembles that published by D.D. Osheroff, R.C. Richardson and D.M. Lee in Physical Review Letters 28, 885 (1972) in which the new helium-3 phase transitions were first reported. .

Low temperature phase diagram



Cooper pairs in superfluid ^{3}He

Orbital wave functions (L=1)



Spin wave functions (S=1)

S_x=0: $(-\uparrow\uparrow + \downarrow\downarrow)$

S_y=0: $i(\uparrow\uparrow + \downarrow\downarrow)$

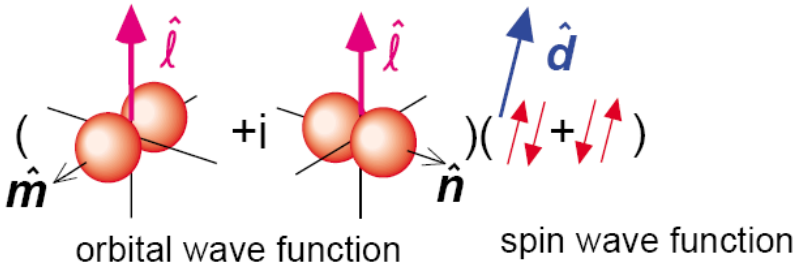
S_z=0: $(\uparrow\downarrow + \downarrow\uparrow)$

$$\begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix}$$

order parameter

The A phase

The order parameter $A_{\mu j} = \Delta \hat{d}_\mu (\hat{m}_j + i\hat{n}_j)$



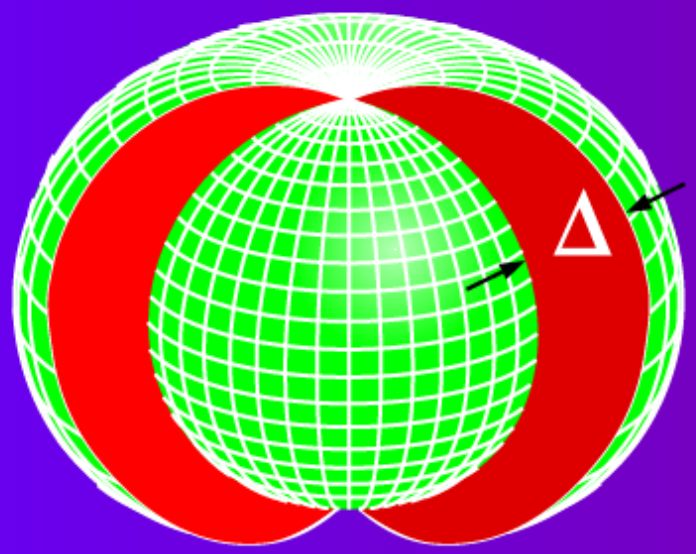
A phase factor $e^{i\chi}$ corresponds to rotation of $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ around $\hat{\mathbf{l}}$:

$$\begin{aligned} e^{i\chi}(\hat{\mathbf{m}} + i\hat{\mathbf{n}}) &= (\cos\chi + i\sin\chi)(\hat{\mathbf{m}} + i\hat{\mathbf{n}}) \\ &= (\hat{\mathbf{m}}\cos\chi - \hat{\mathbf{n}}\sin\chi) + i(\hat{\mathbf{m}}\sin\chi + \hat{\mathbf{n}}\cos\chi). \end{aligned}$$

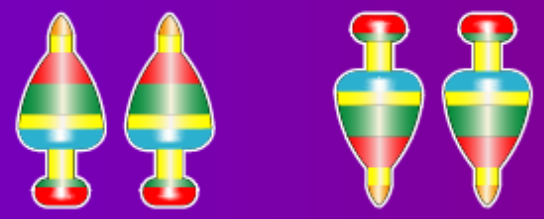
Superfluid velocity

$$\mathbf{v}_s = \frac{\hbar}{2m}\nabla\chi = \frac{\hbar}{2m}\sum_j \hat{m}_j \nabla \hat{n}_j.$$

A-phase: $A_{\mu i} = \Delta e^{i\varphi} d_{\mu}(m_i + i n_i)$



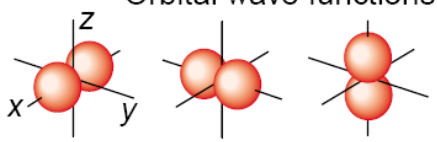
A-phase gap



A-phase has only spin ⇑⇑ pairs
and spin ⇓⇓ pairs

Superfluid B phase

Orbital wave functions (L=1)



Spin wave functions (S=1)

S_x=0: $(-\uparrow\uparrow + \downarrow\downarrow)$

S_y=0: $i(\uparrow\uparrow + \downarrow\downarrow)$

S_z=0: $(\uparrow\downarrow + \downarrow\uparrow)$

$$\begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} = A_0 e^{i\phi} \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix}$$

order parameter

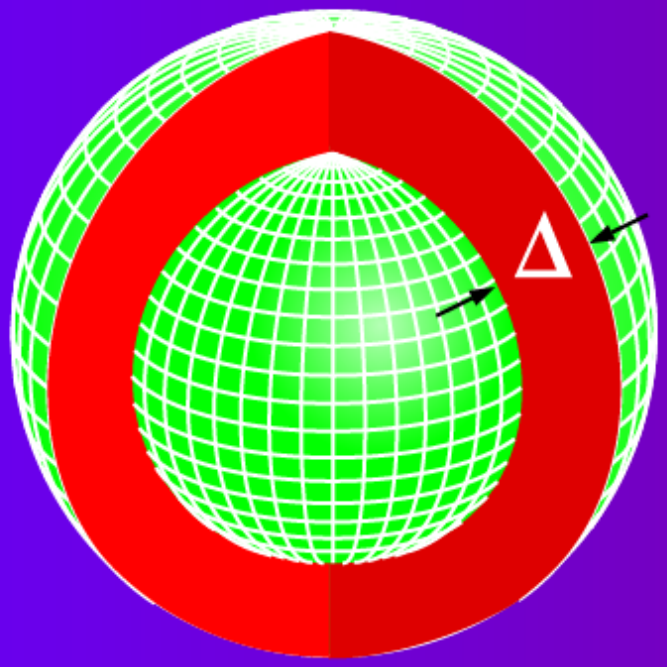
amplitude (real)

phase factor (complex)

rotation matrix (real)

$\begin{cases} \text{angle } \theta = 104^\circ \\ \text{axis } \mathbf{n} \end{cases}$

B-phase: $A_{\mu i} = |\Delta| e^{i\varphi} \mathbf{R}(\hat{\mathbf{n}}, \theta)_{\mu i}$ $\mu, i = 1,2,3$



B-phase gap

B-phase has all three components;
spin ⇑⇑ pairs, spin ⇓⇓ pairs
and spin ⇓⇑ pairs

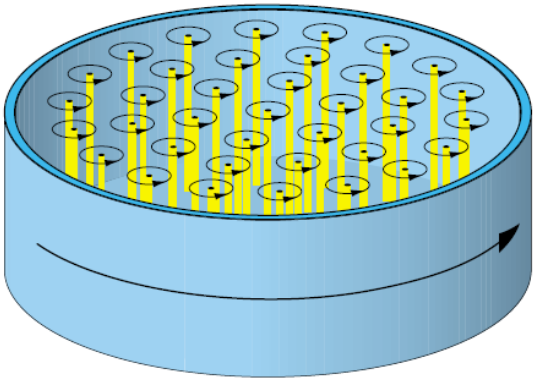
Rotation and vortices

Problem of a rotating superfluid

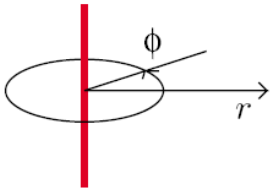
An uncharged superfluid cannot rotate homogeneously:

$$\mathrm{A}(\mathbf{r}) = \mathrm{A}_0 \exp[i\chi(\mathbf{r})]$$
$$\Rightarrow \mathbf{v} = \frac{\mathbf{p}}{2m} = \frac{-i\hbar\boldsymbol{\nabla}}{2m} = \frac{\hbar}{2m}\boldsymbol{\nabla}\chi$$
$$\Rightarrow \boldsymbol{\nabla} \times \mathbf{v} = 0.$$

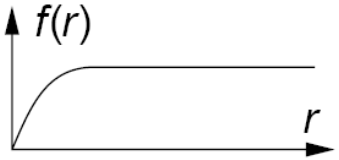
Rotation takes place via vortex lines



Simple model for a vortex



$$\mathrm{A}(r,\phi) = \mathrm{A}_0 e^{\mathrm{i}\phi} f(r)$$



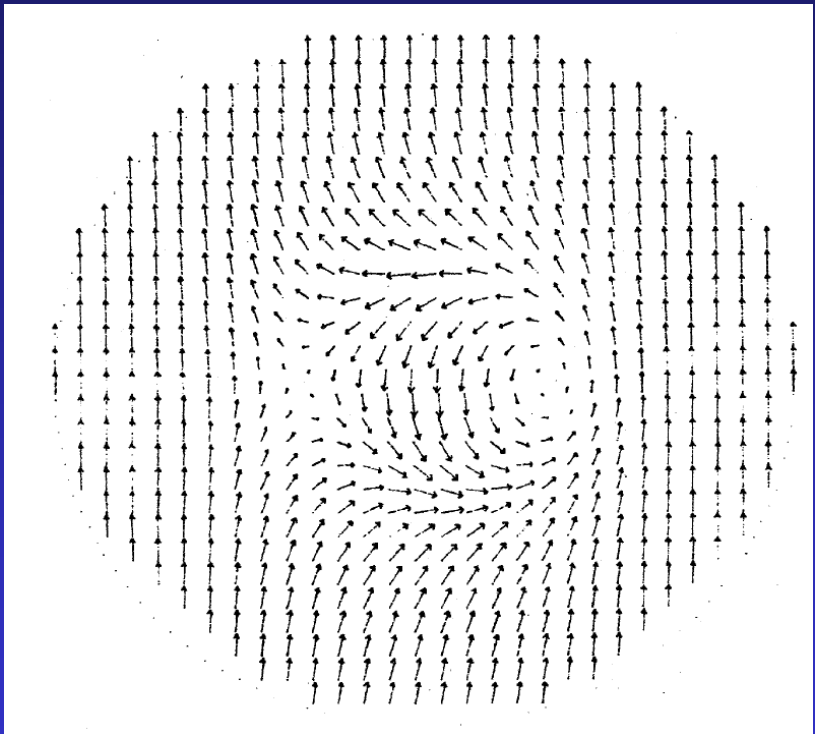
It follows that

$$\mathbf{v} = \frac{\hbar}{2m}\boldsymbol{\nabla}\phi = \frac{\hbar}{2m}\frac{\hat{\boldsymbol{\phi}}}{r}.$$

Circulation around a vortex line

$$\kappa = \oint d\mathbf{l}\cdot\mathbf{v} = \frac{\hbar}{2m}\oint d\mathbf{l}\cdot\boldsymbol{\nabla}\phi = \frac{h}{2m}.$$

Vortex in A phase



Vortex in B phase

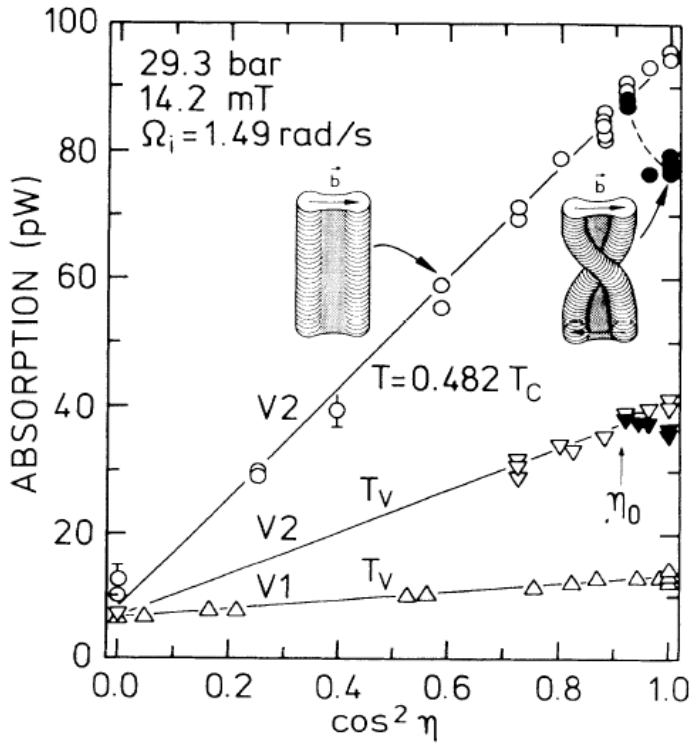


FIG. 2. HPD absorption of an isolated vortex cluster vs the orientation of the applied magnetic field: ○, $V2$ vortex in the untwisted state at $0.48T_c$ with $a_2/a_0 \simeq 10.5$ [cf. Eq. (1)] and, ●, in the twisted state; ▽, $V2$ vortex untwisted at T_V with $a_2/a_0 \simeq 4.87$ and, ▼, twisted; △, $V1$ vortex with $a_2/a_0 \simeq 1.02$.

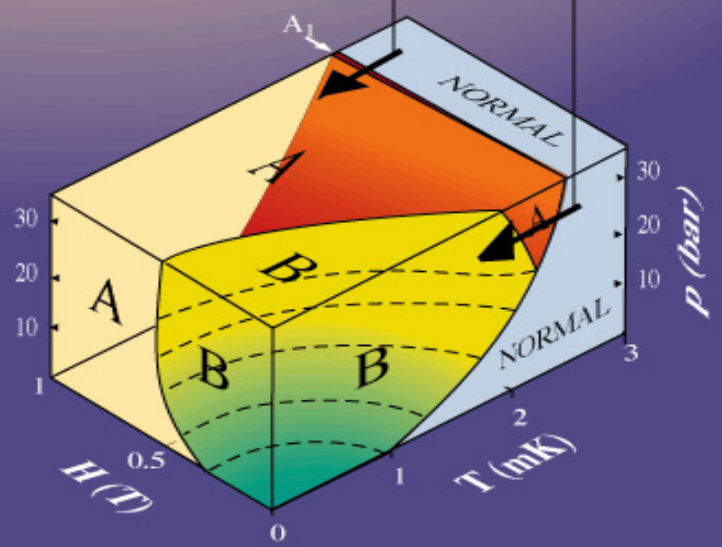
Big Bang

Symmetries broken

$SU(5) \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)$

$SO(3) \times U(1) \times U(1) \rightarrow U(1) \times U(1) \rightarrow U(1)$

$SO(3) \times SO(3) \times U(1) \rightarrow U(1) \times U(1) \rightarrow SO(3)$



Superfluid ^{3}He

$$\psi = \sqrt{\rho_s}\,[S]\,[L]\,e^{i\varphi}$$

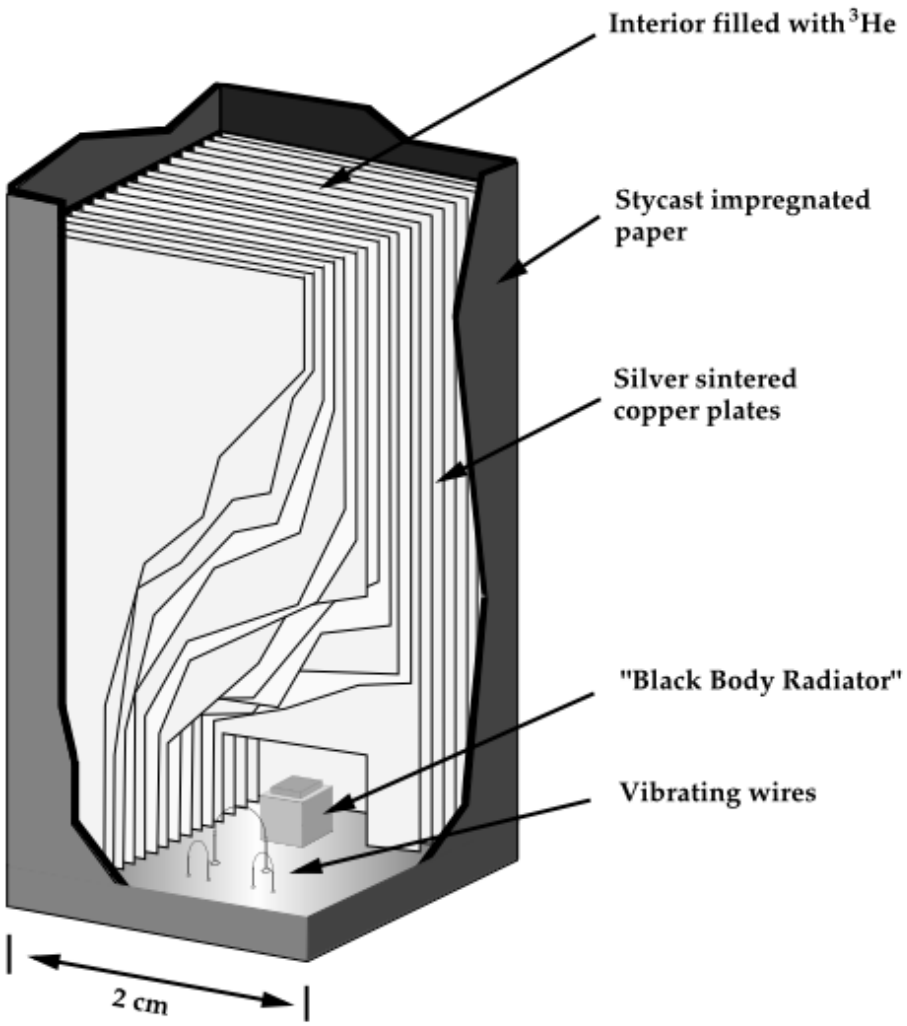
Spin part

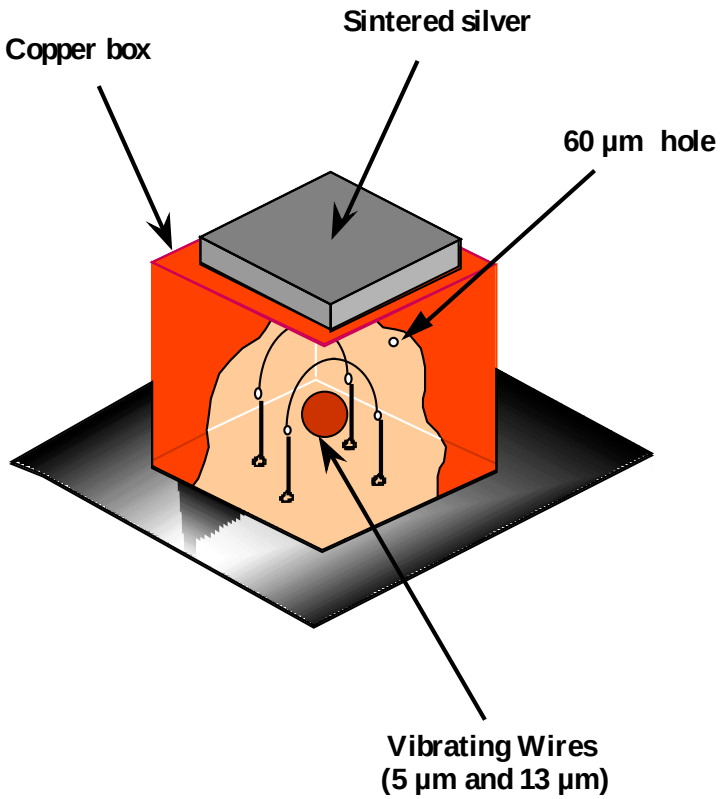
Orbital part

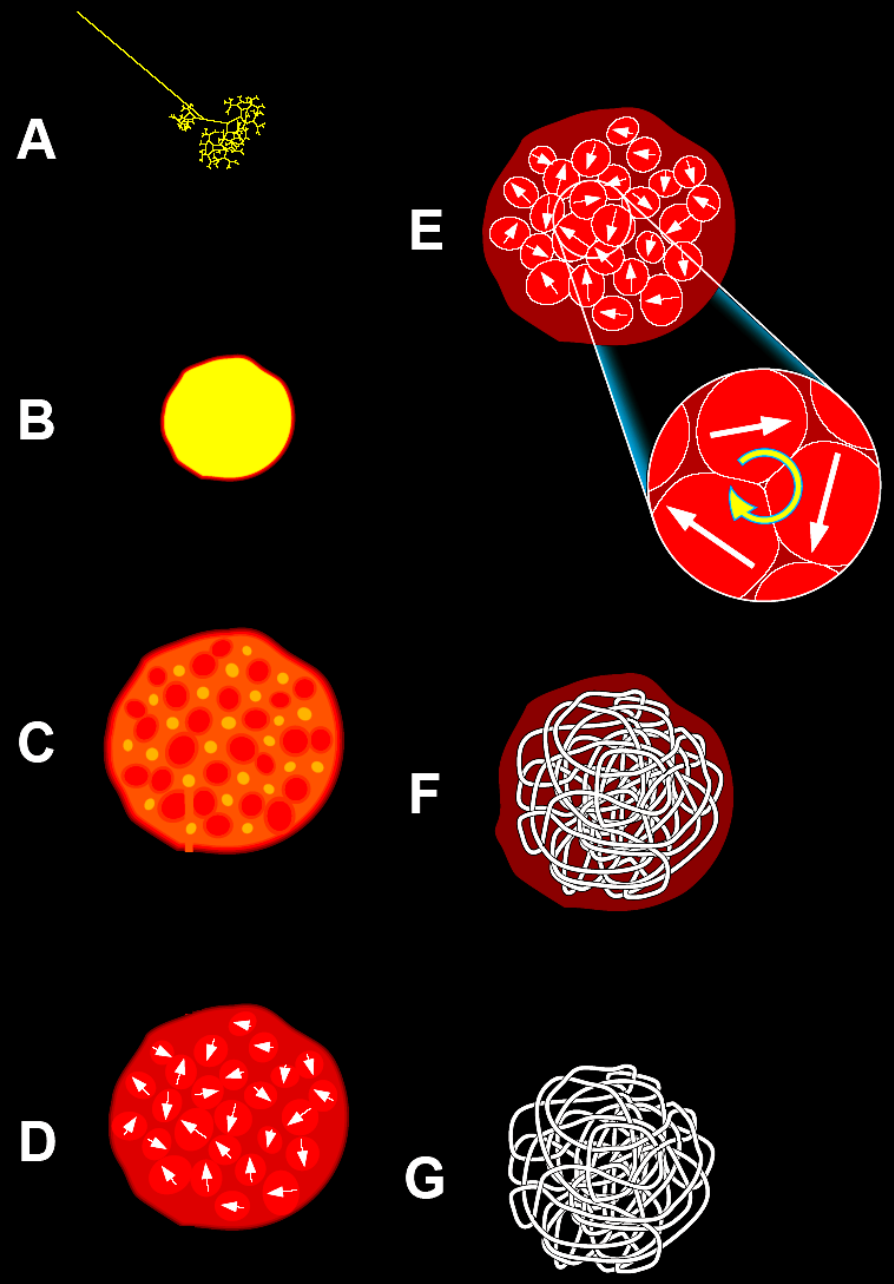
'Gauge' part

$$SO(3)^S \times SO(3)^L \times U(1)$$

Superfluid ^{3}He bolometry






A
B
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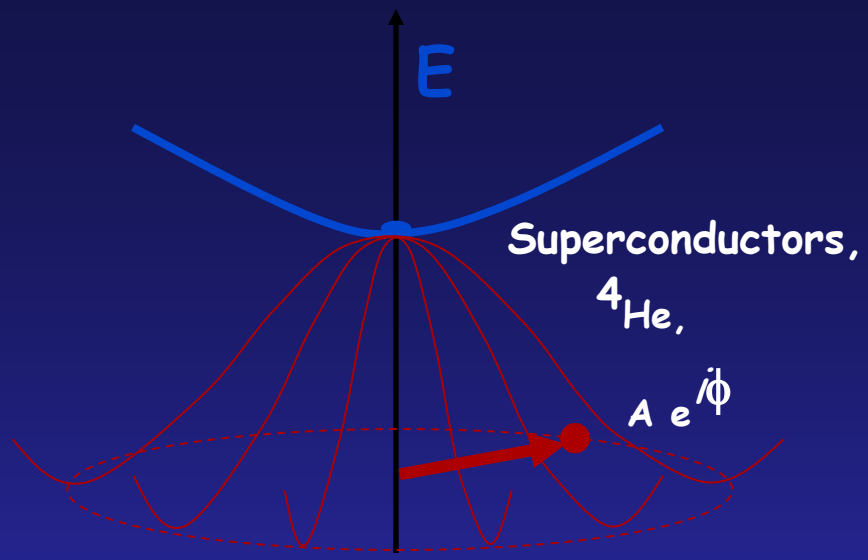
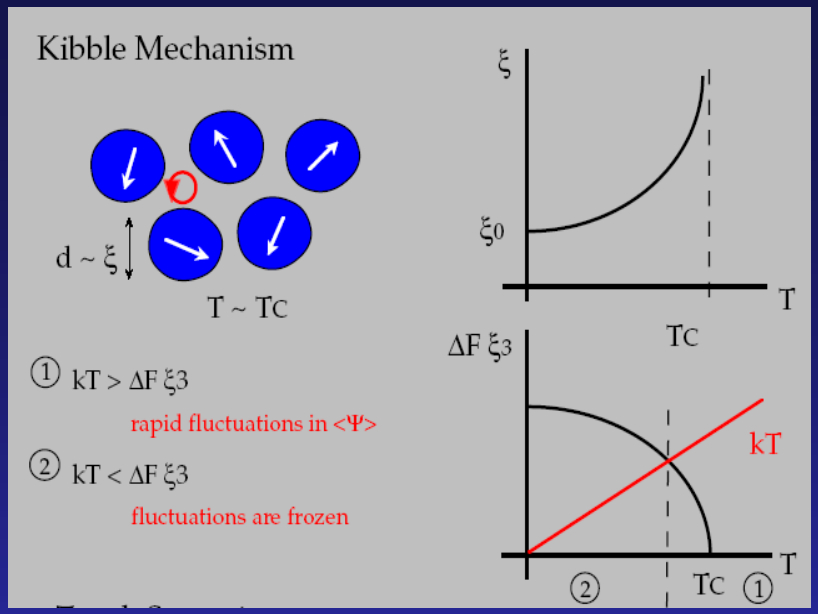
Topological defects creation and fast transition:





T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
W. H. Zurek, Nature (London) **317**, 505 (1985).

Zurek Scenario

domain size is determined by "critical slowing down"

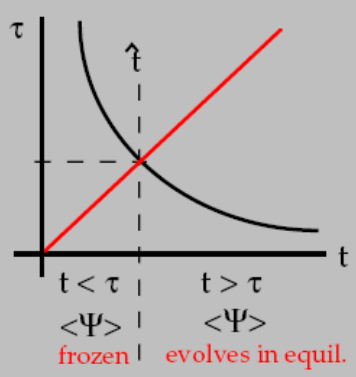
$\varepsilon = 1 - T/T_C = t/\tau_Q$

corr. length: $\xi = \xi_0\, |\varepsilon|^{-1/2}$

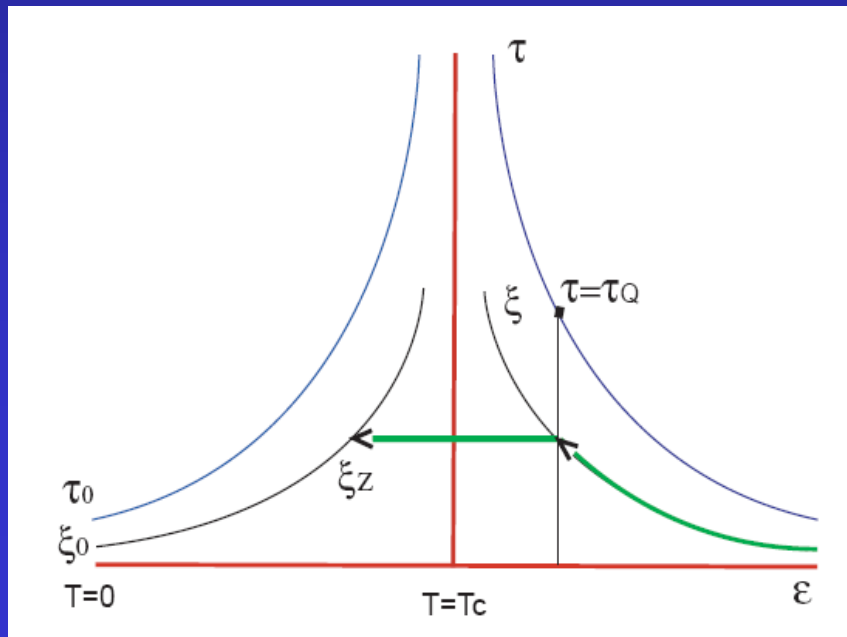
char. speed: $\upsilon = \upsilon_0\, |\varepsilon|^{1/2}$

corr. time: $\tau = \xi/\upsilon = \tau_0\tau_Q/t$

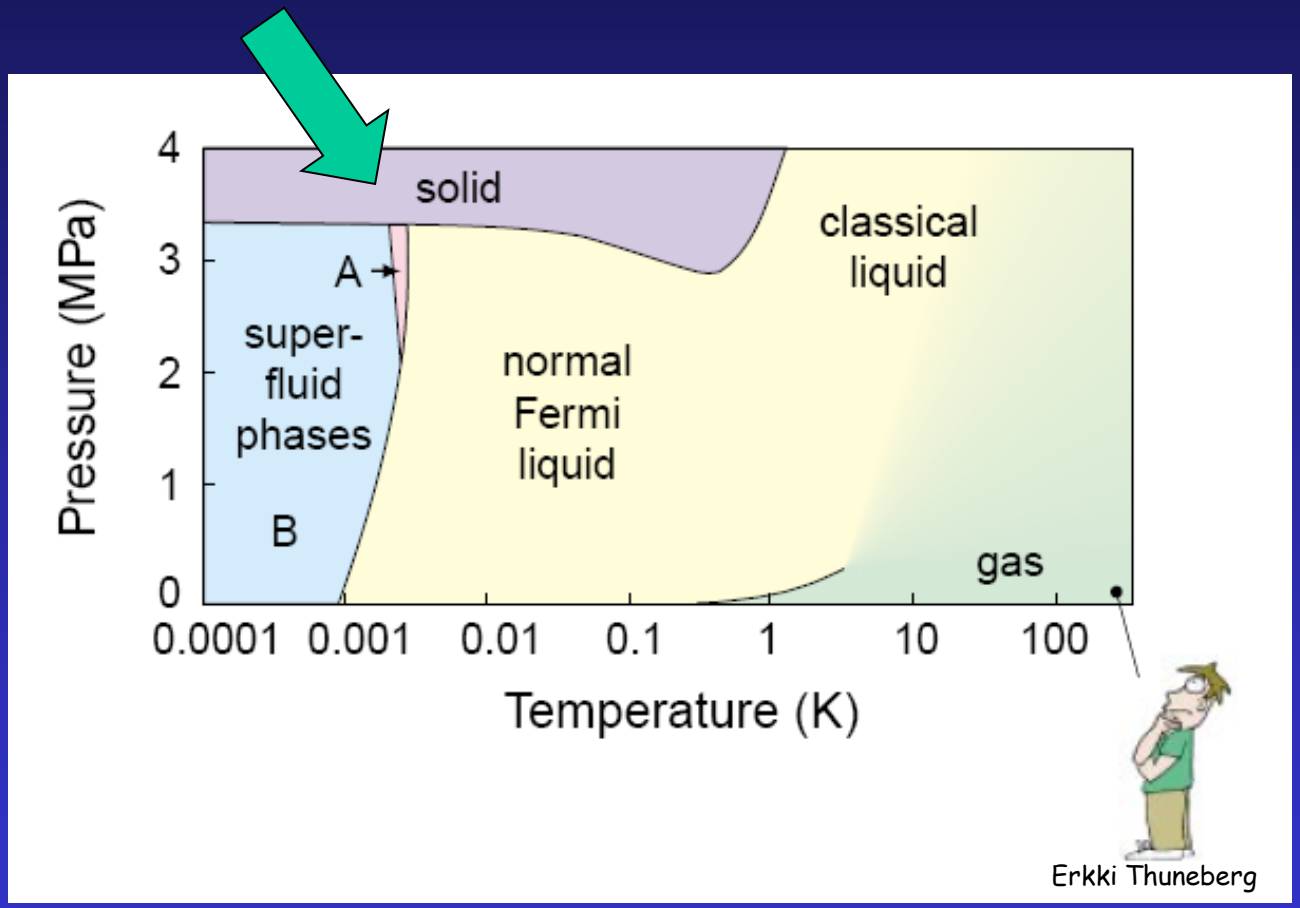
for $\hat{t} = \tau(\hat{t}) \longrightarrow \hat{t} = (\tau_0\tau_Q)^{1/2}$



$$d \sim \xi(\hat{t}) = \xi_0\left(\frac{\hat{t}}{\tau_Q}\right)^{-1/2} = \xi_0\left(\frac{\tau_Q}{\tau_0}\right)^{1/4}$$ domain size



Phase Diagram of ^{3}He



Multi-spin exchange:
of quasi-localized Fermions

- Identical particles
- Hamiltonian without explicit spin-dependent interactions

Pauli principle: the spin state is coupled to the parity of the wave function

Permutation of spins & particles: Dirac (1947) : Effective Hamiltonian on spin variables

$$H_{ex} = -\Sigma P\ (-1)^p\ Jp\ P$$

Two-particle permutations: $P_2 = (1 + \sigma i.\sigma j)$ (Heisenberg Hamiltonian)

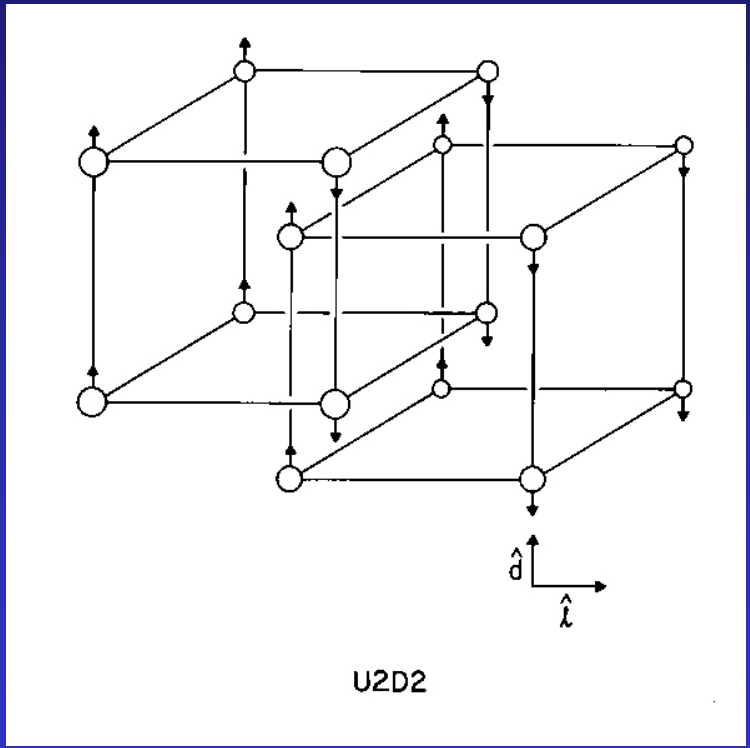
Multi-spin exchange in solid 3He (Thouless, 1965)

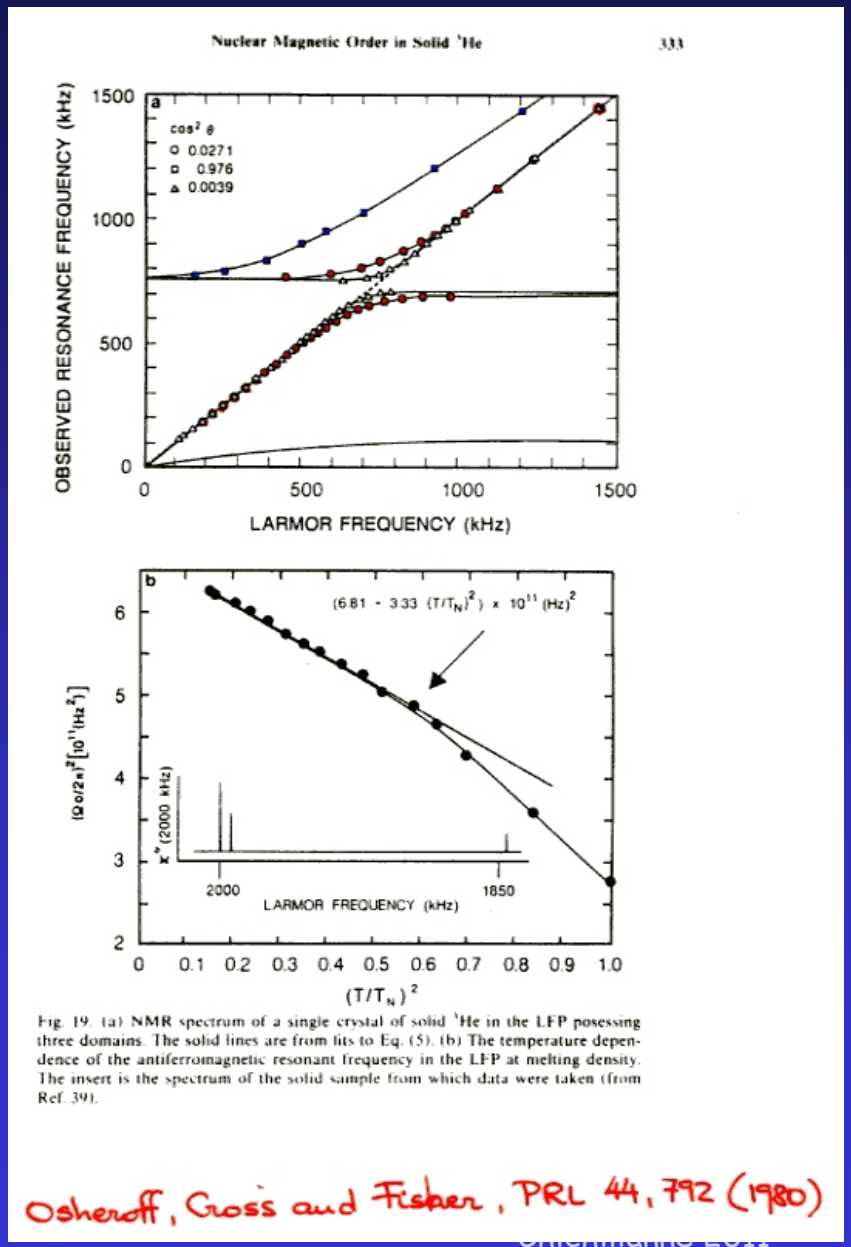
Three-particle exchange is also Heisenberg
$P3 = (1 + \sigma i.\sigma j + \sigma j.\sigma k + \sigma k.\sigma i)$

Four-spin exchange introduces new physics:
$P4 = (1 + \Sigma\ \sigma\mu.\sigma\nu + \Sigma\ ((\sigma i.\sigma j).(\sigma k.\sigma l) + (\sigma i.\sigma l).(\sigma j.\sigma k) - (\sigma i.\sigma k).(\sigma j.\sigma l)))$

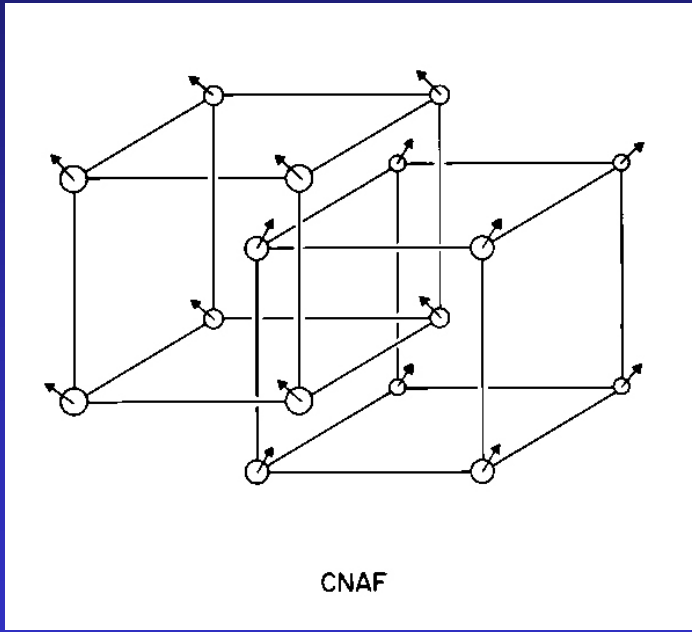
All exchange coefficients J are <u>positive</u>

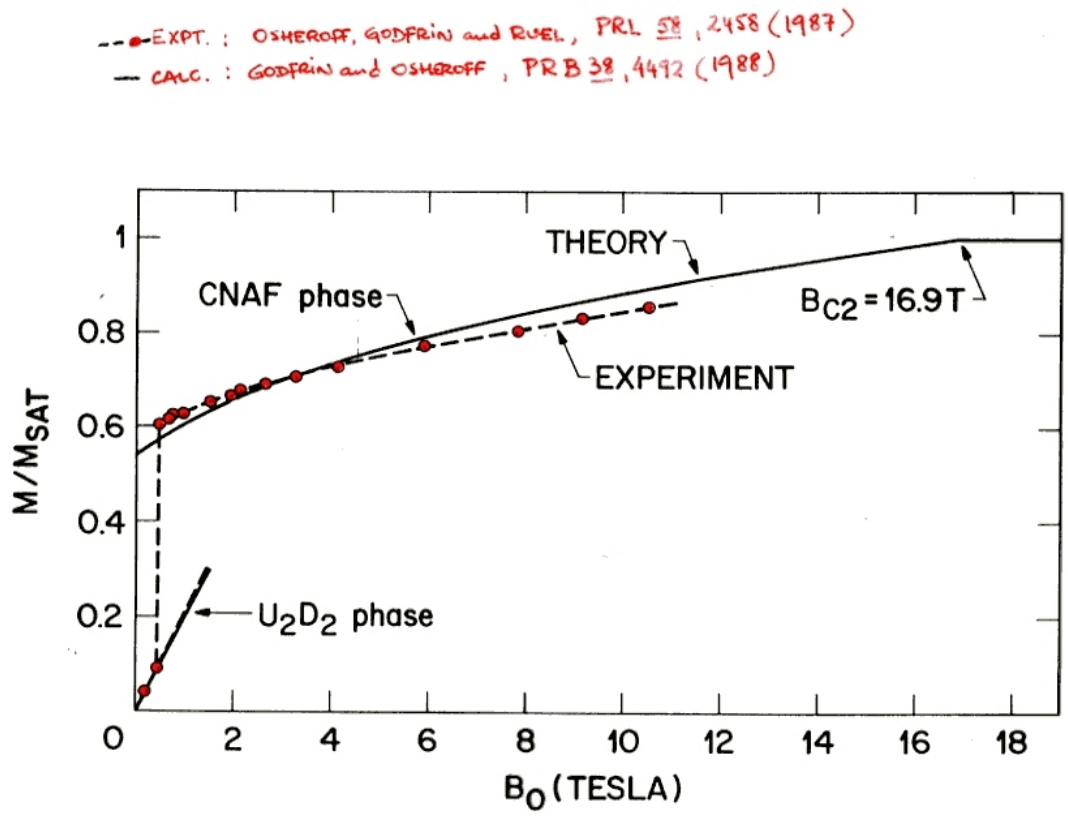
Nuclear order: U2D2 phase



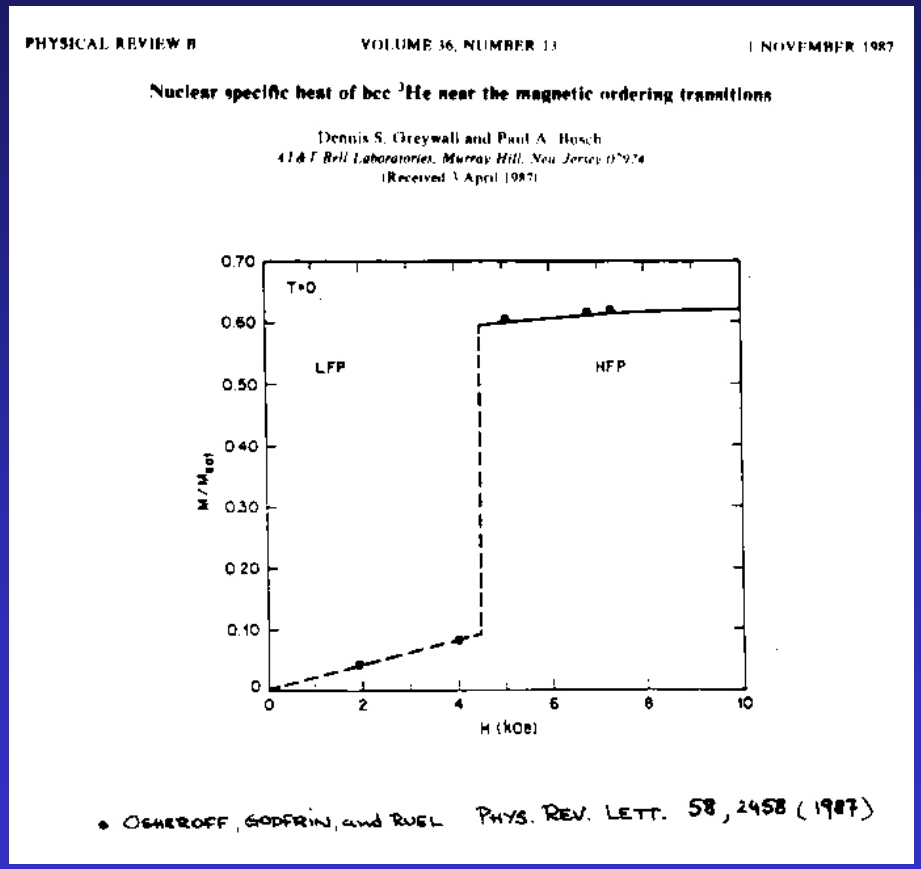


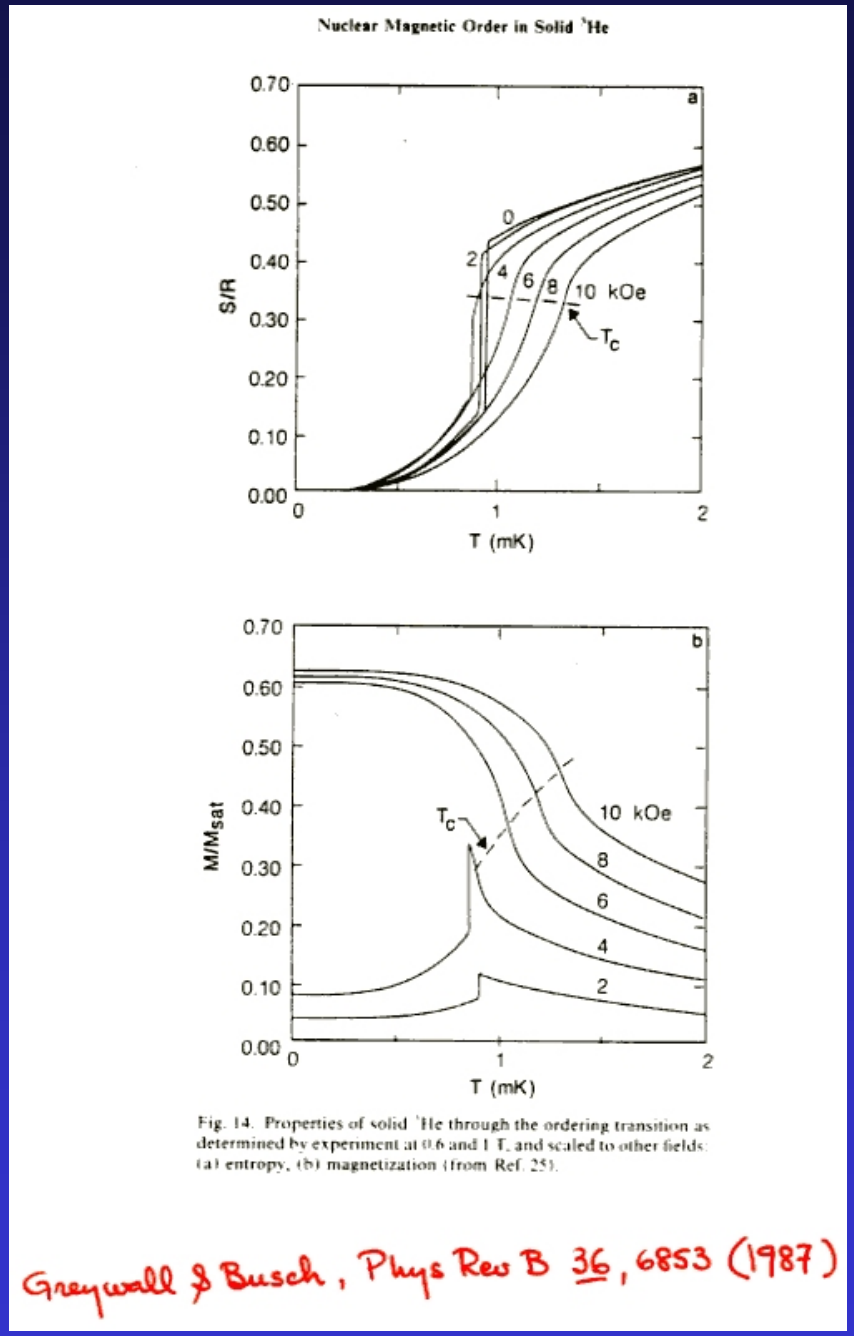
Nuclear order: CNAF phase



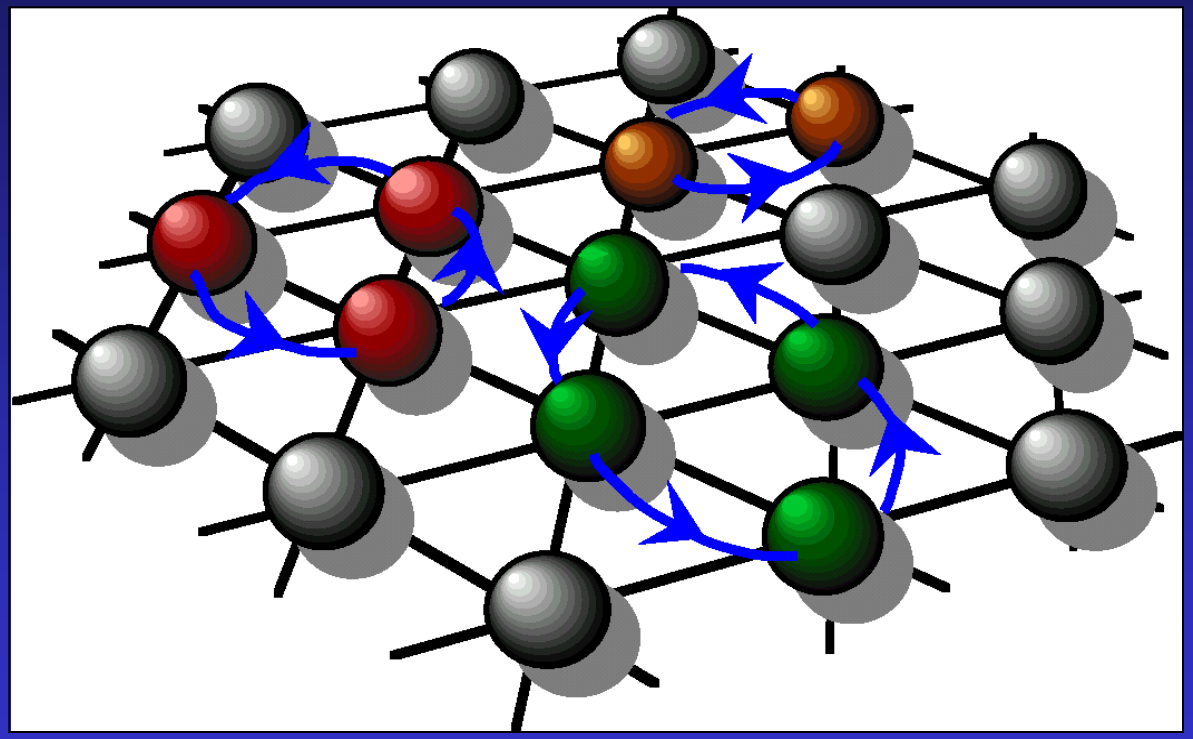


Heat capacity

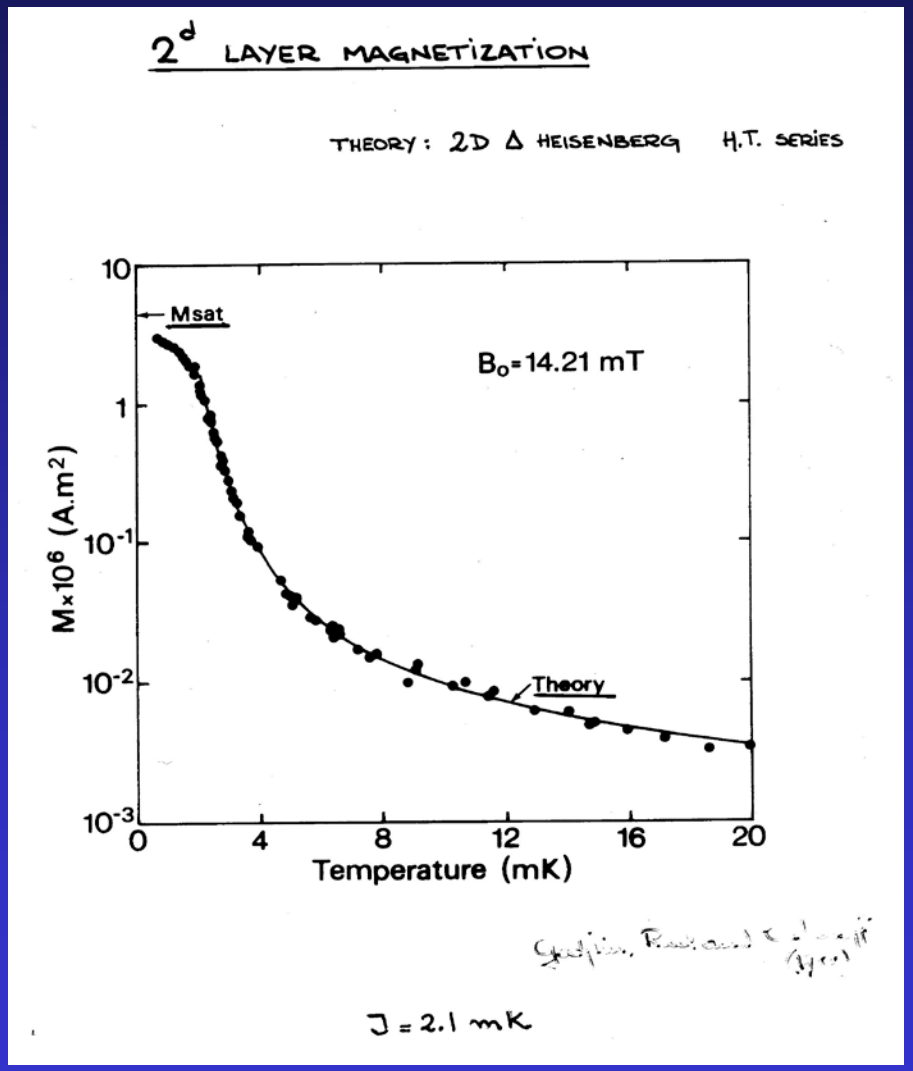




Multi-spin exchange in 2D



2D Heisenberg Ferromagnet



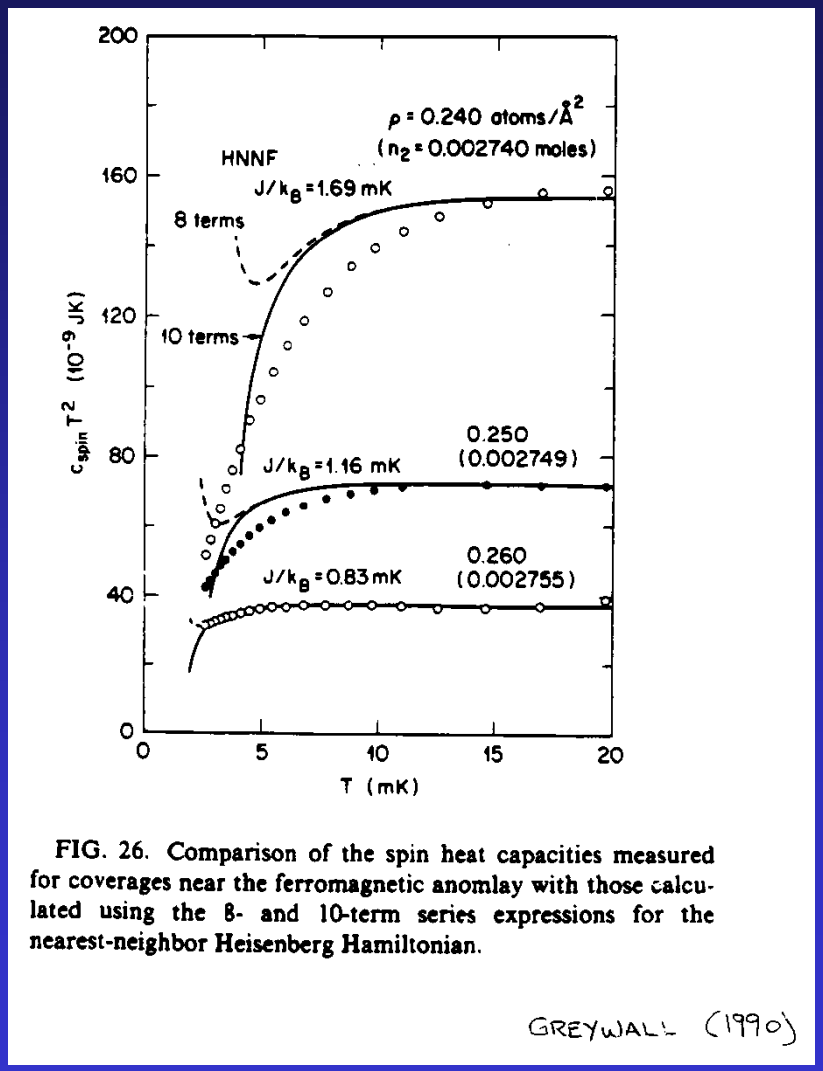


FIG. 26. Comparison of the spin heat capacities measured for coverages near the ferromagnetic anomlay with those calculated using the 8- and 10-term series expressions for the nearest-neighbor Heisenberg Hamiltonian.

Spin-liquid phase

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Low Temperature Heat-Capacity Anomalies in Two-Dimensional Solid ^{3}He

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The heat capacity of second-layer solid ^{3}He adsorbed on graphite has been measured down to extremely low temperatures below 100 μK. We observed a double-peak structure for a low-density registered solid, which strongly suggests that the system is a highly frustrated spin-1/2 two-dimensional (2D) antiferromagnet with a disordered ground state. As the density increases it approaches a 2D nearest-neighbor Heisenberg ferromagnet with a single rounded peak, which can be explained semiquantitatively by considering higher-order exchange processes up to six-spin exchange. [S0031-9007(97)04414-1]

PACS numbers: 67.70.+n, 67.80.Jd, 75.10.Jm, 75.70.Ak

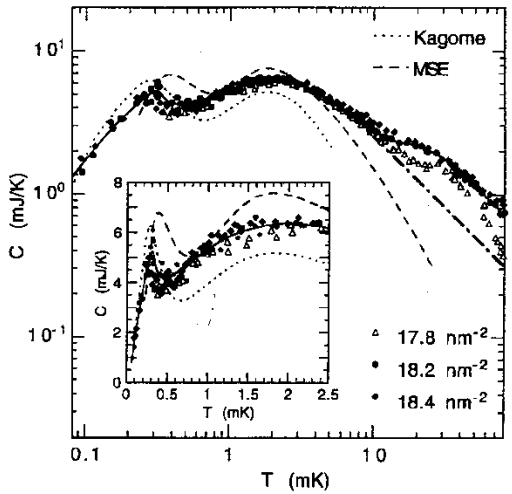

FIG. 1. Heat-capacity data at densities near the second-layer solidification. The solid line is a guide for the eye for the data at 18.2 nm^{-2}. The dotted line is from the Kagome model ($J = 2.59$ mK; Ref. [2]). The dashed line is from the MSE model ($J = -3.23$ mK, $J_4 = J_6 = 0.81$ mK; Ref. [3]). The exchange parameters were determined so that the first-peak temperatures coincide with the experiment. Note that three quarters of N_2 are assumed to contribute to the spin heat capacity in the Kagome model (see text). The dash-dotted line is proposed true high-temperature behavior. The inset shows a linear plot of the low-temperature data.

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Quantum Frustration in the "Spin Liquid" Phase of Two-Dimensional ^{3}He

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We have measured the ultralow temperature and low field magnetic susceptibility of the $\frac{4}{7}$ phase of two-dimensional ^{3}He adsorbed on graphite preplated by one layer of ^{4}He. The experiments are performed by progressively adding ^{4}He to the system, thus suppressing in a controlled way the ^{3}He atoms trapped in substrate heterogeneities. This procedure enables us to determine the intrinsic properties of this spin $\frac{1}{2}$ model magnet in the zero field limit. The results show quantitatively that the system is strongly frustrated by multiple spin exchange interactions. A characteristic gapped spin liquid behavior is observed at ultralow temperature.

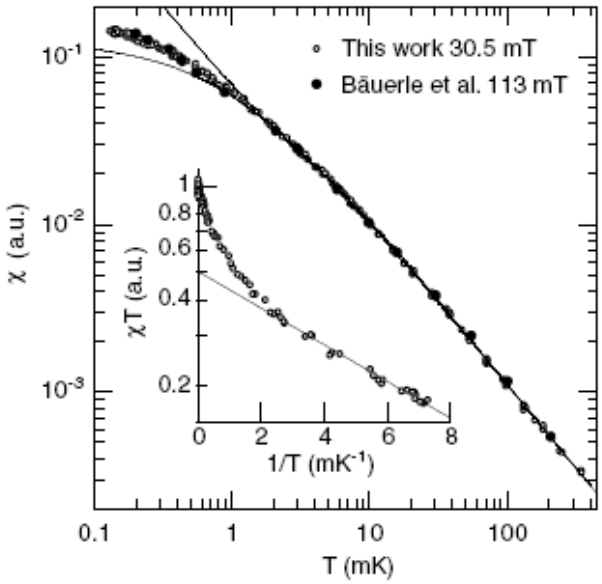

FIG. 3. Magnetic susceptibility of the pure $\frac{4}{7}$ phase. The solid lines correspond to the [2,3] Padé approximant of the HTSE of the MSE Hamiltonian [23] (upper curve), and to a Curie-Weiss fit (lower curve). The inset emphasizes the low temperature behavior of the susceptibility multiplied by temperature. The solid line shows an exponential decrease.

Conclusions

- Quantum fluids constitute interesting, simple, quantum many-body systems
- Elementary excitations
- Superfluidity
- Crystallisation
- Magnetism
- Supersolidity?