

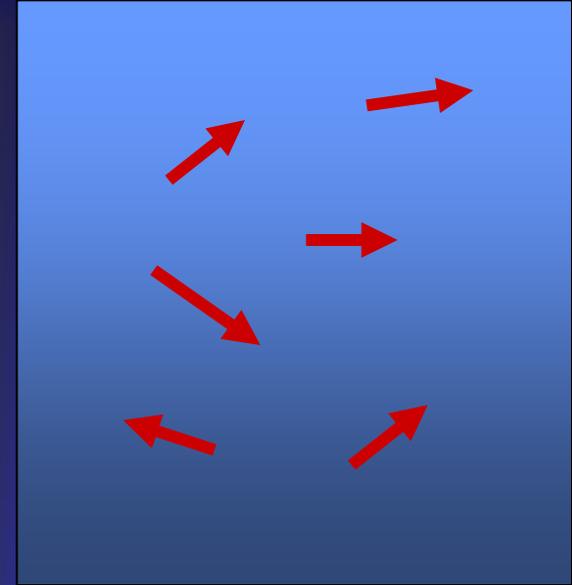
Disordered Solids



real crystals

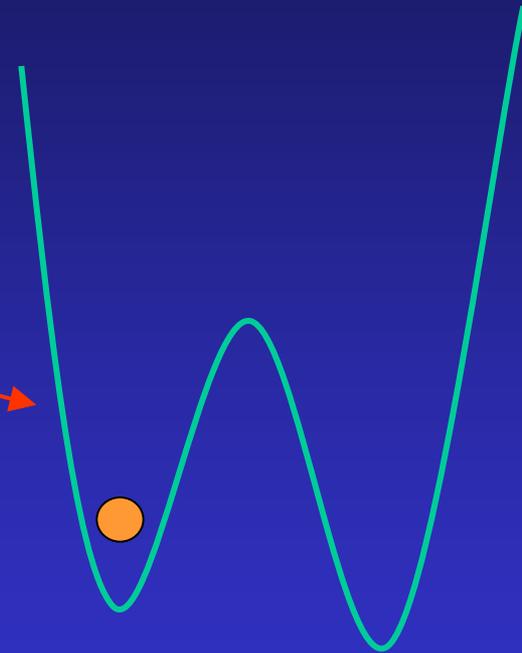
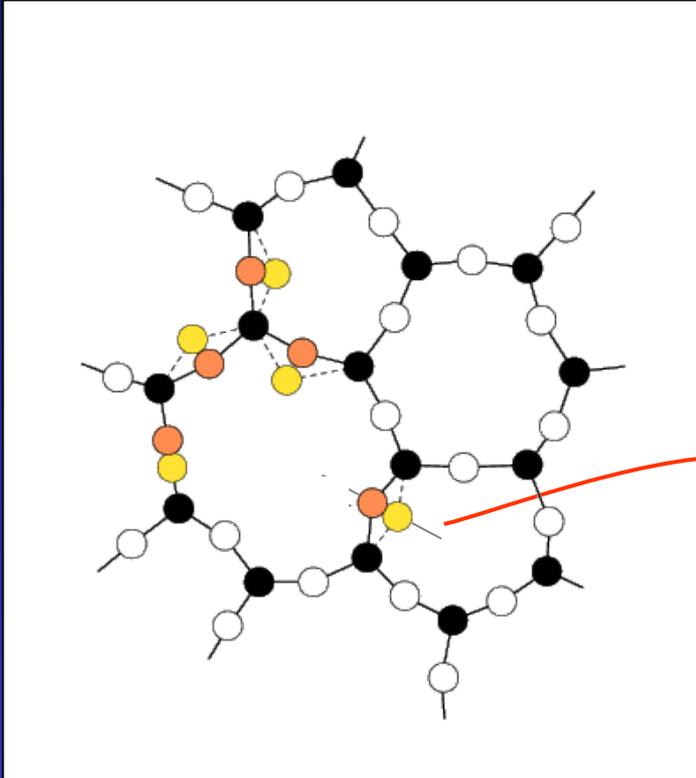


glasses

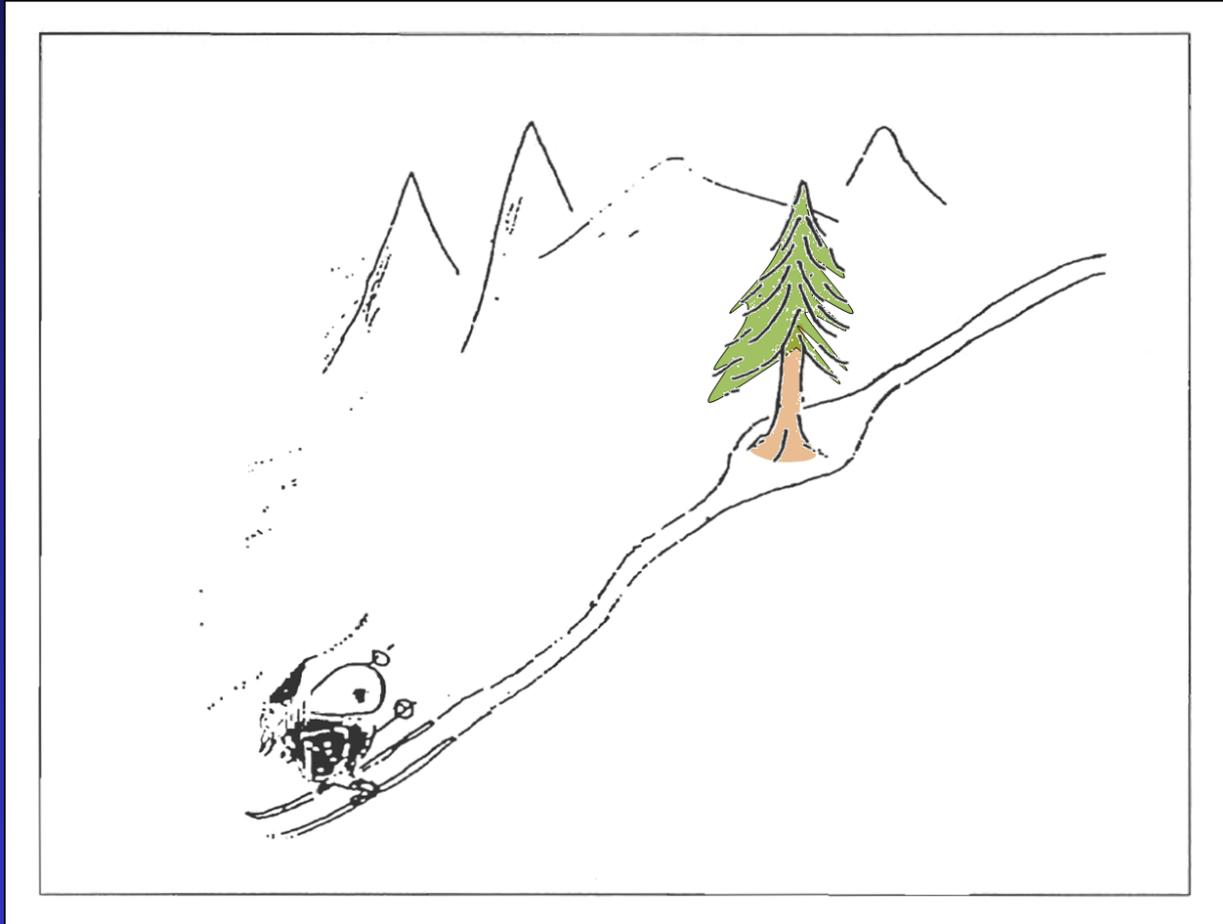


spin glass

Tunneling of Atoms in Solids

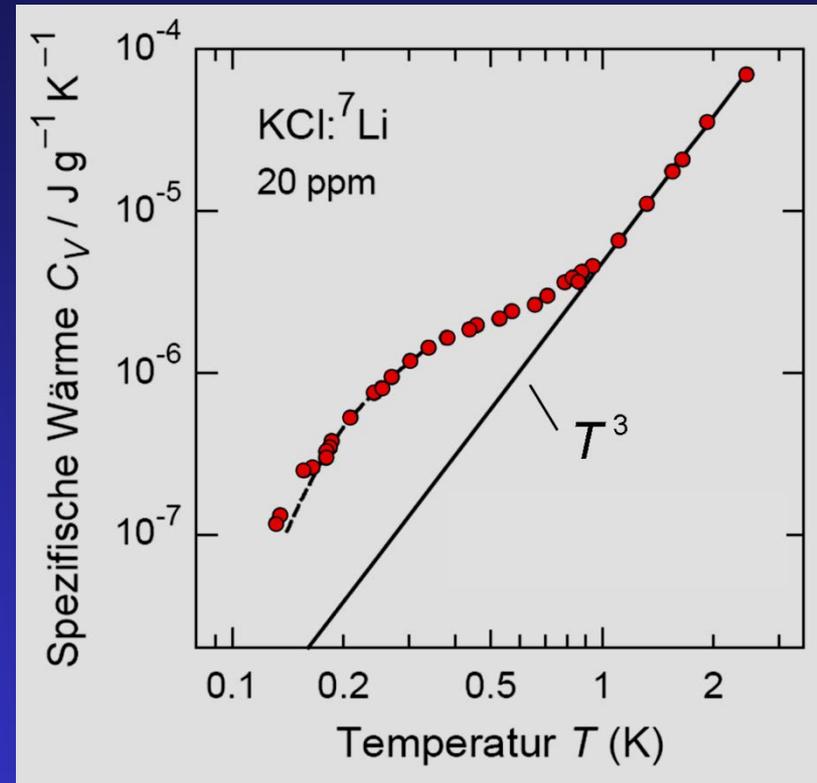


Tunneln



KCl:Li Specific Heat

specific heat roughly a factor of 10 higher at 0.5 K

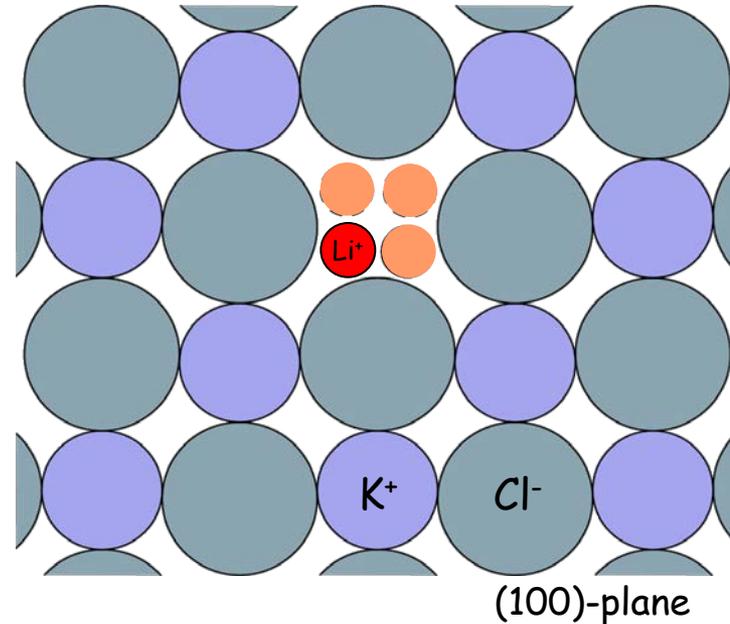


Li-Tunneling Systems in KCl-Crystals

Li⁺ substitutes K⁺

ionic radius: $r_{\text{Li}^+} < r_{\text{K}^+}$

→ 8 off-center positions
in $\langle 111 \rangle$ direction



tunnel splitting:

$$\Delta_0 = \hbar\Omega e^{-\lambda} \quad \text{with} \quad \lambda = \frac{d}{2\hbar} \sqrt{2mV}$$

$${}^7\text{Li} : \quad {}^7\Delta_0/k_B = 1,1 \text{ K}$$

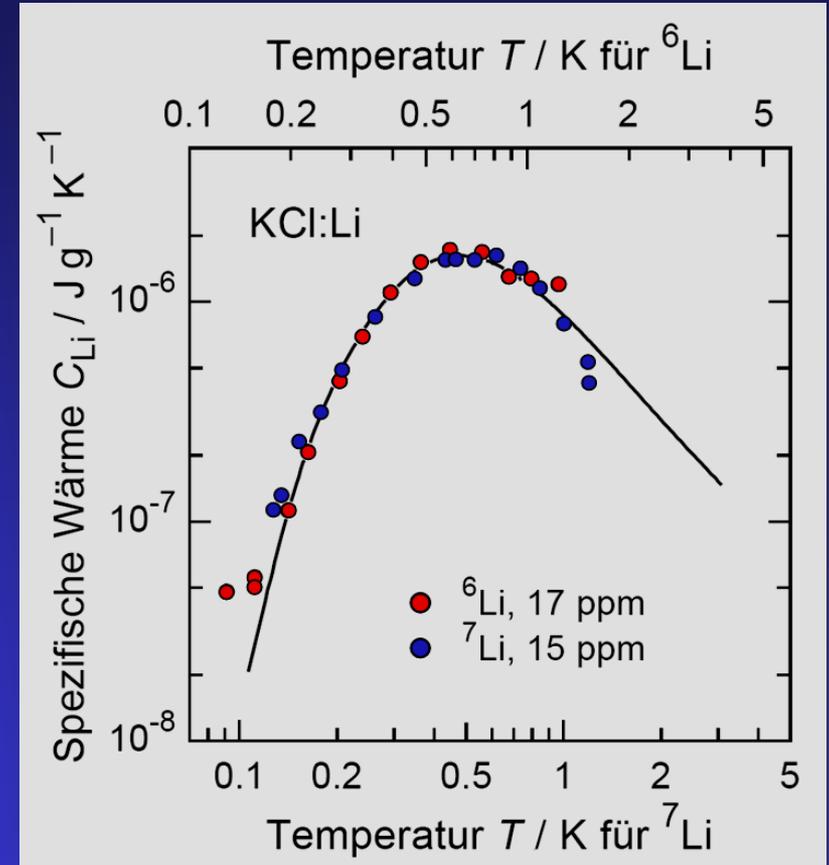
$${}^6\text{Li} : \quad {}^6\Delta_0/k_B = 1,6 \text{ K}$$

Isotope effect

Schottky-Anomaly

$$C_{\text{Li}} = \frac{3nk_{\text{B}}}{\varrho} \left(\frac{\Delta_0}{2k_{\text{B}}T} \right)^2 \text{sech}^2 \left(\frac{\Delta_0}{2k_{\text{B}}T} \right)$$

$$\text{number density } n = \frac{N}{V}$$

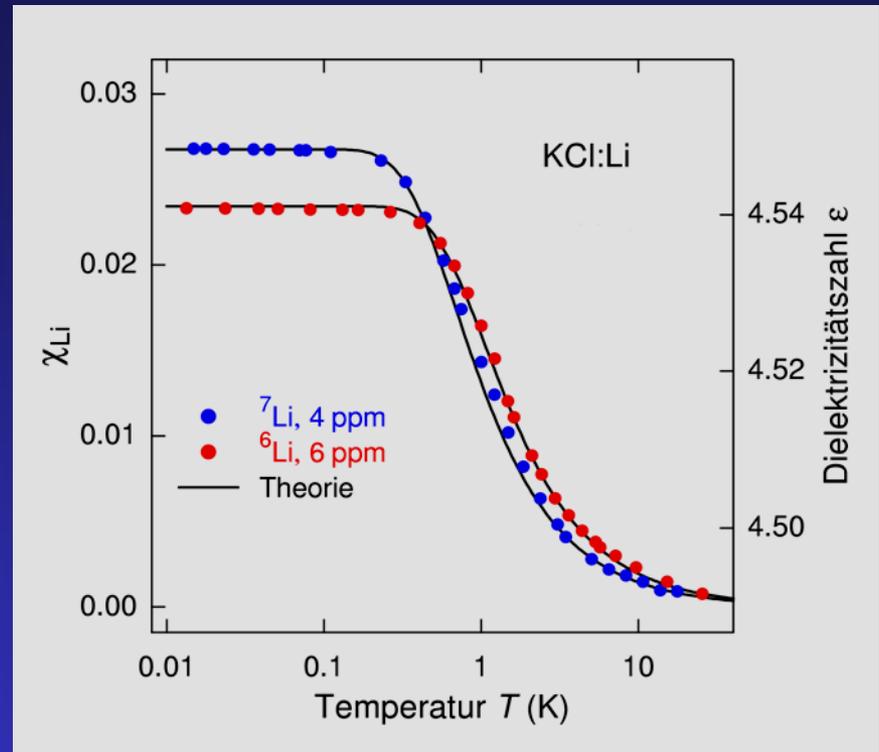


Dielectric Susceptibility

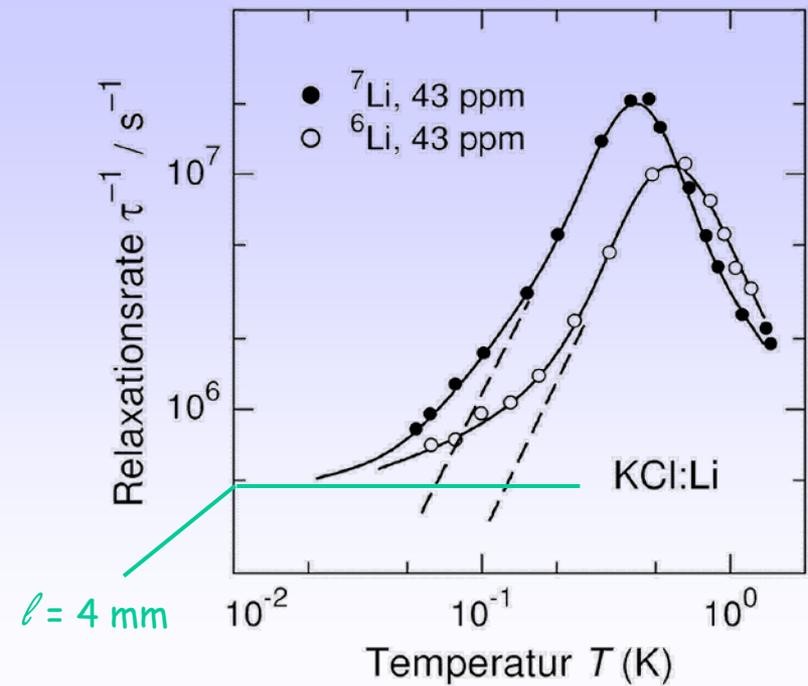
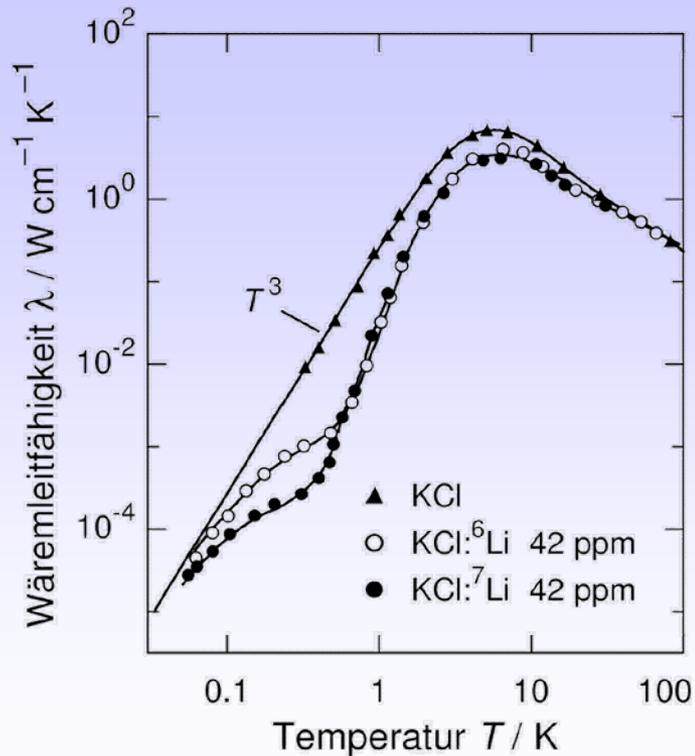
$$\chi_{\text{Li}} = \frac{2}{3} \frac{np^2}{\epsilon_0 \Delta_0} \tanh\left(\frac{\Delta_0}{2k_{\text{B}}T}\right)$$

number density $n = \frac{N}{V}$

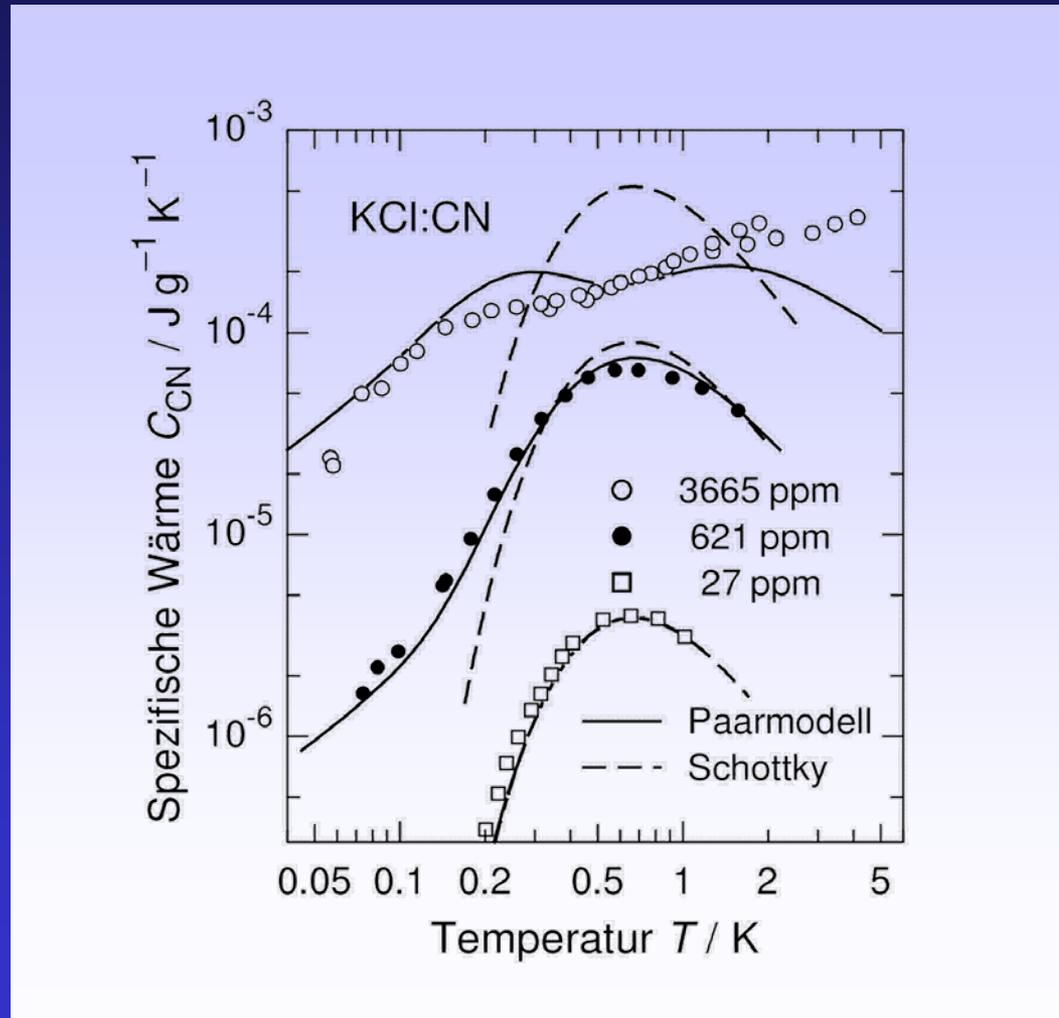
dipole moment p



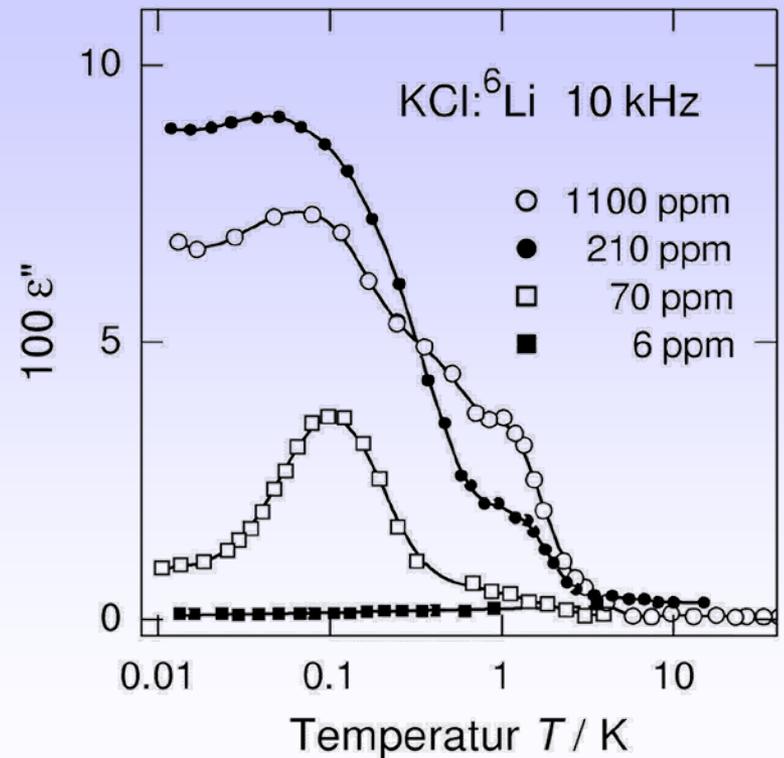
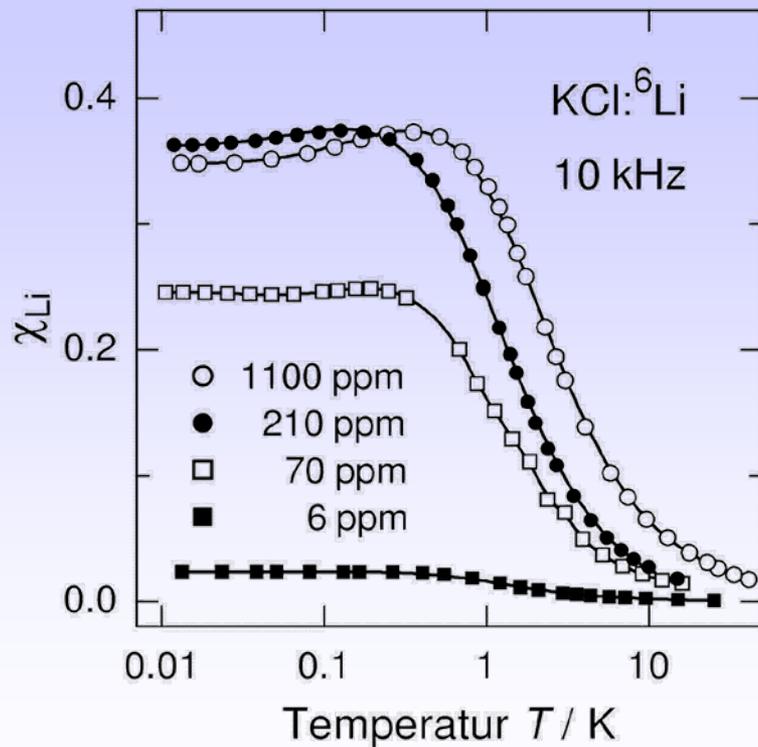
Thermal Conductivity of KCl:Li Isotope effect



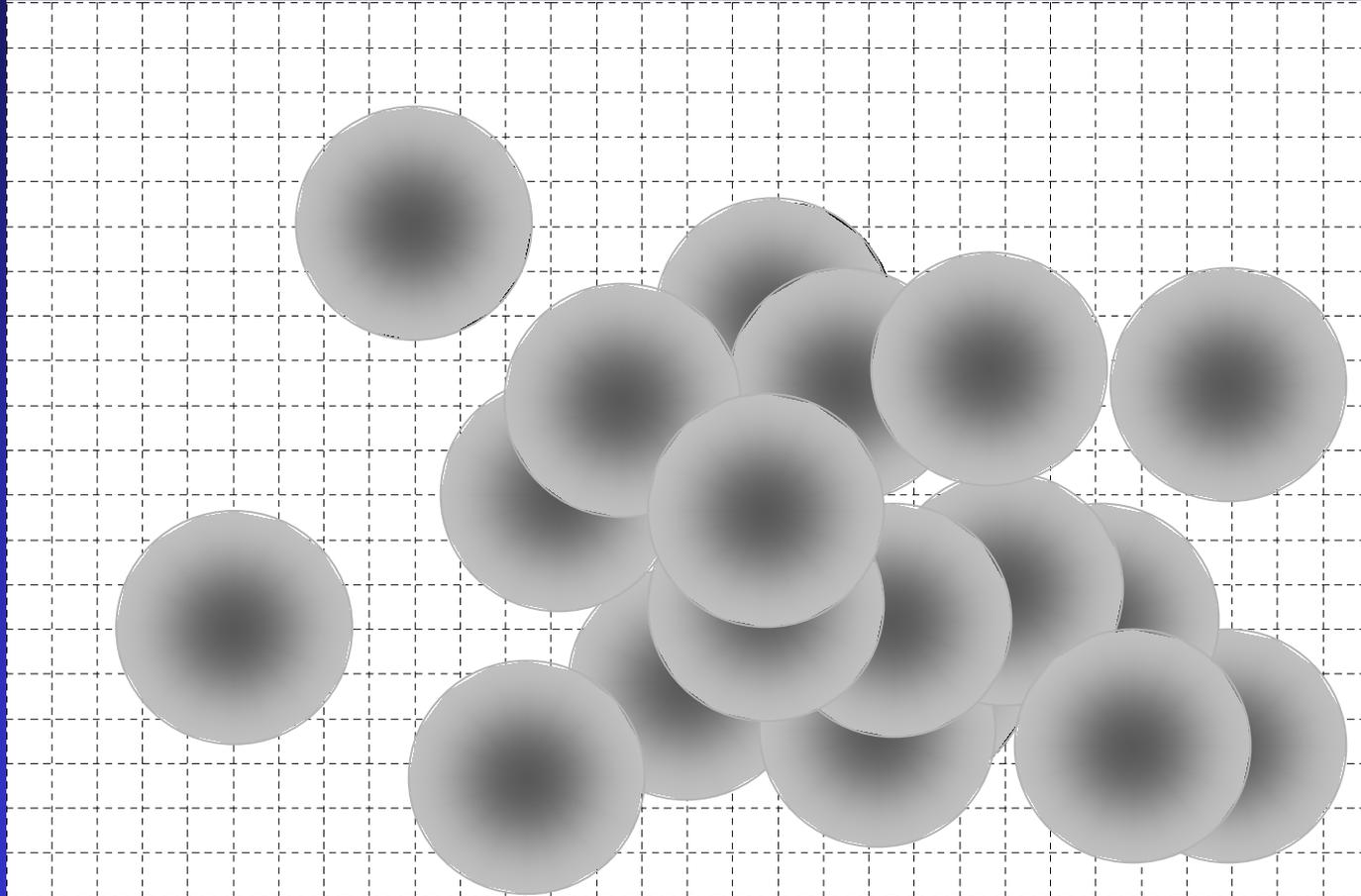
KCl:Li Specific Heat, Concentration Dependence



Dielectric Susceptibility, Concentration Dependence



Interacting Tunneling Systems



Transition to Incoherent Tunneling

- defects in crystals:
at high concentrations \longrightarrow cross over to incoherent tunneling

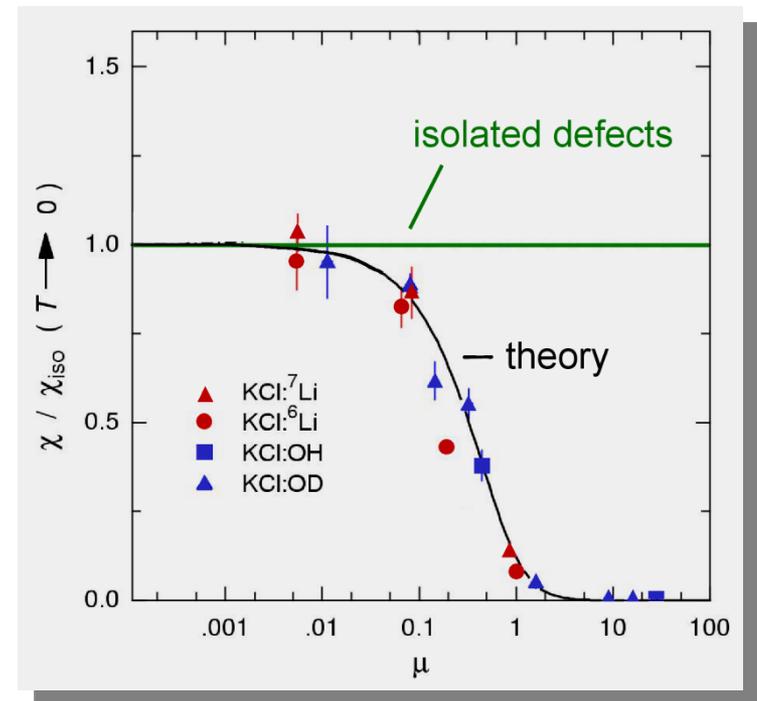
consequences:

reduced resonant contribution
new phononless relaxation channel

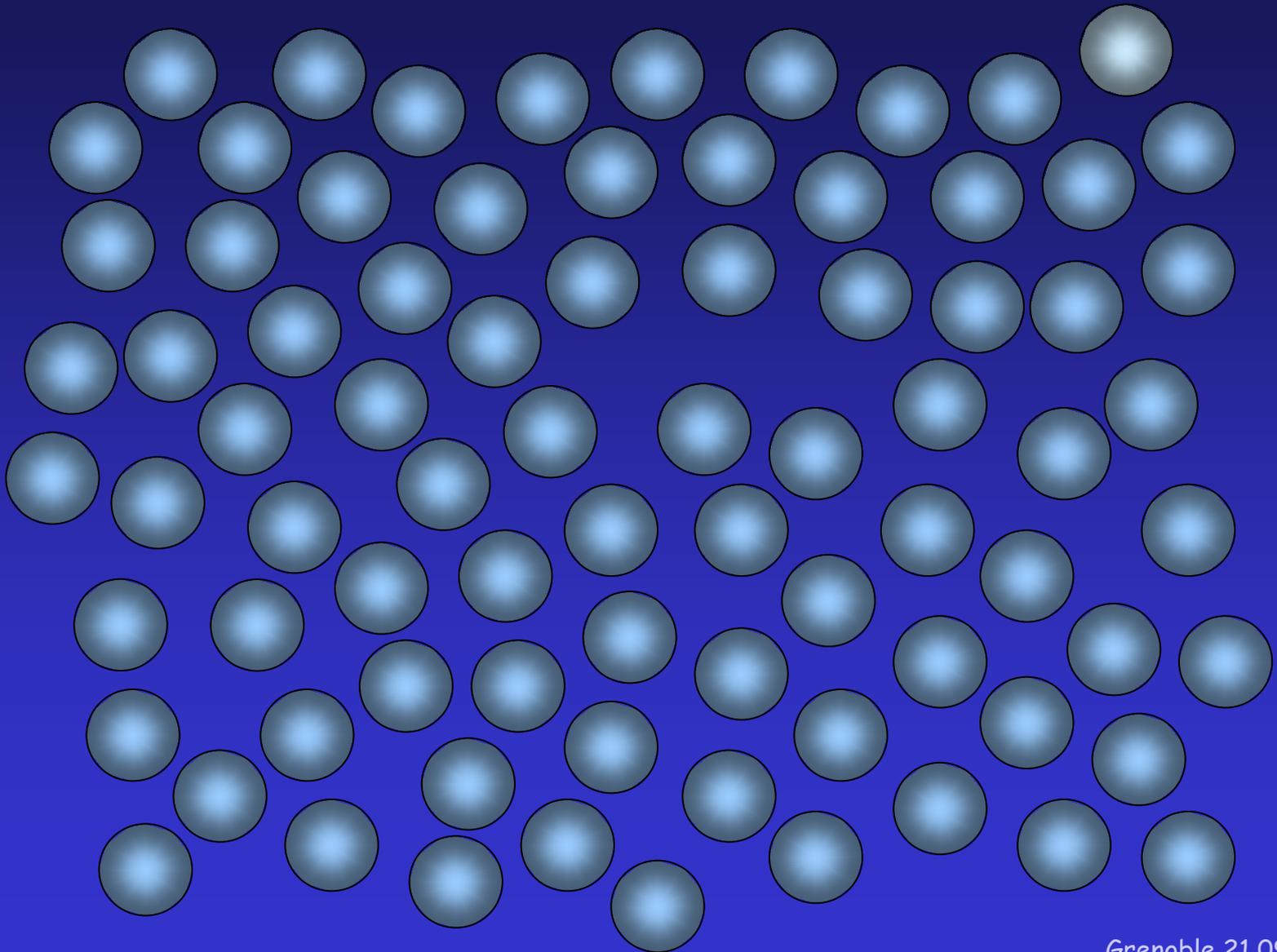
$$\mu = \frac{\bar{J}}{\Delta_0}$$

- incoherent tunneling in glasses at very low temperatures?

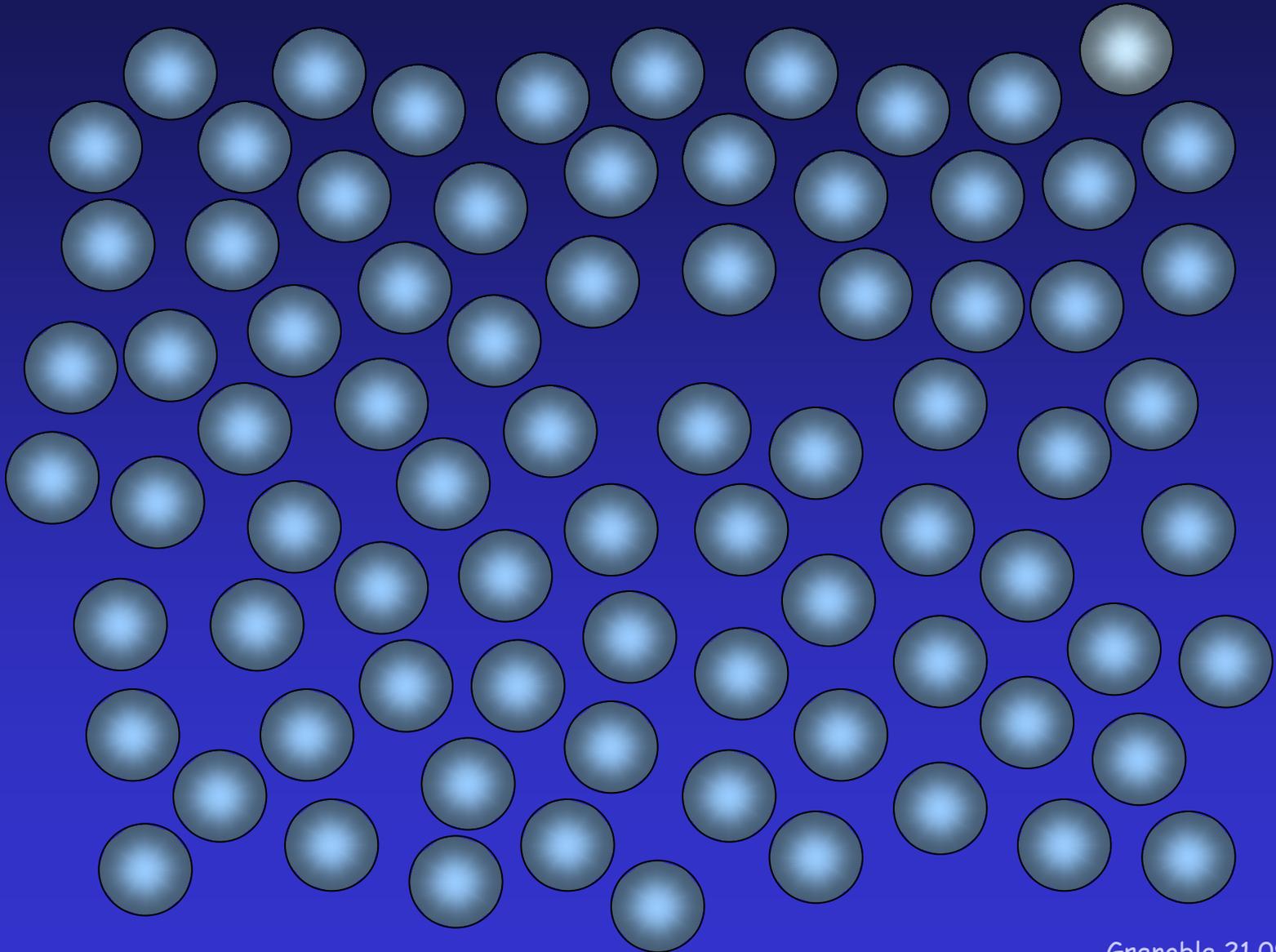
$$\bar{J} > \Delta_0 \approx k_B T$$



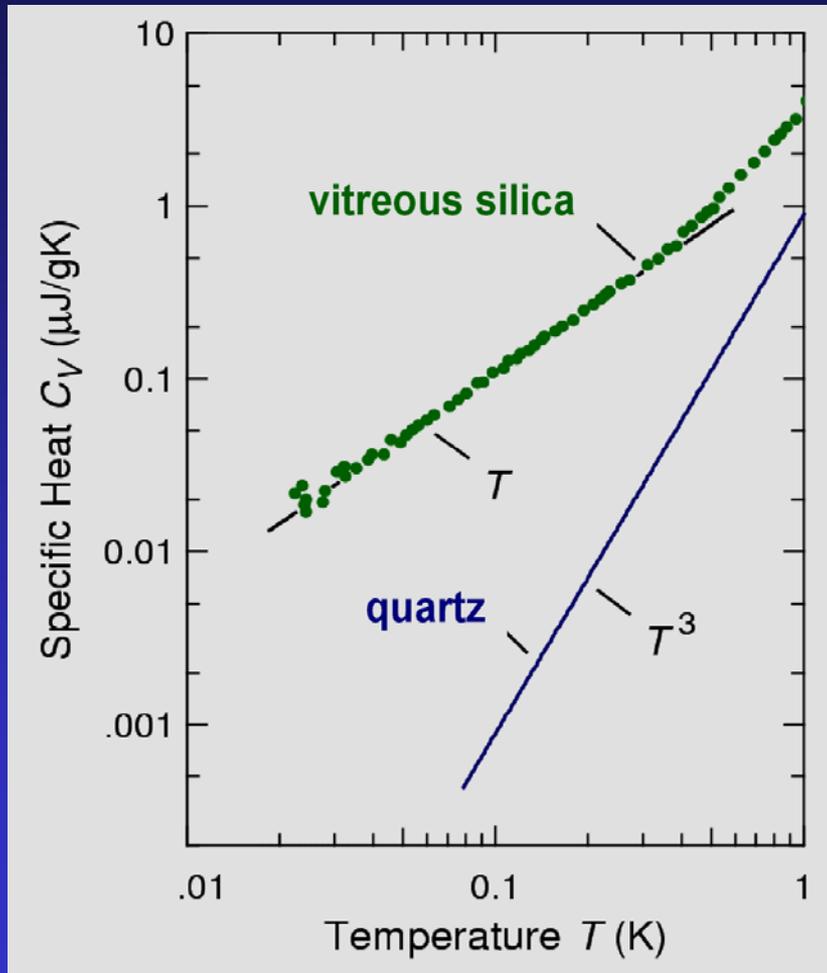
Atomic Tunneling Systems in Glasses



Atomic Tunneling Systems in Glasses

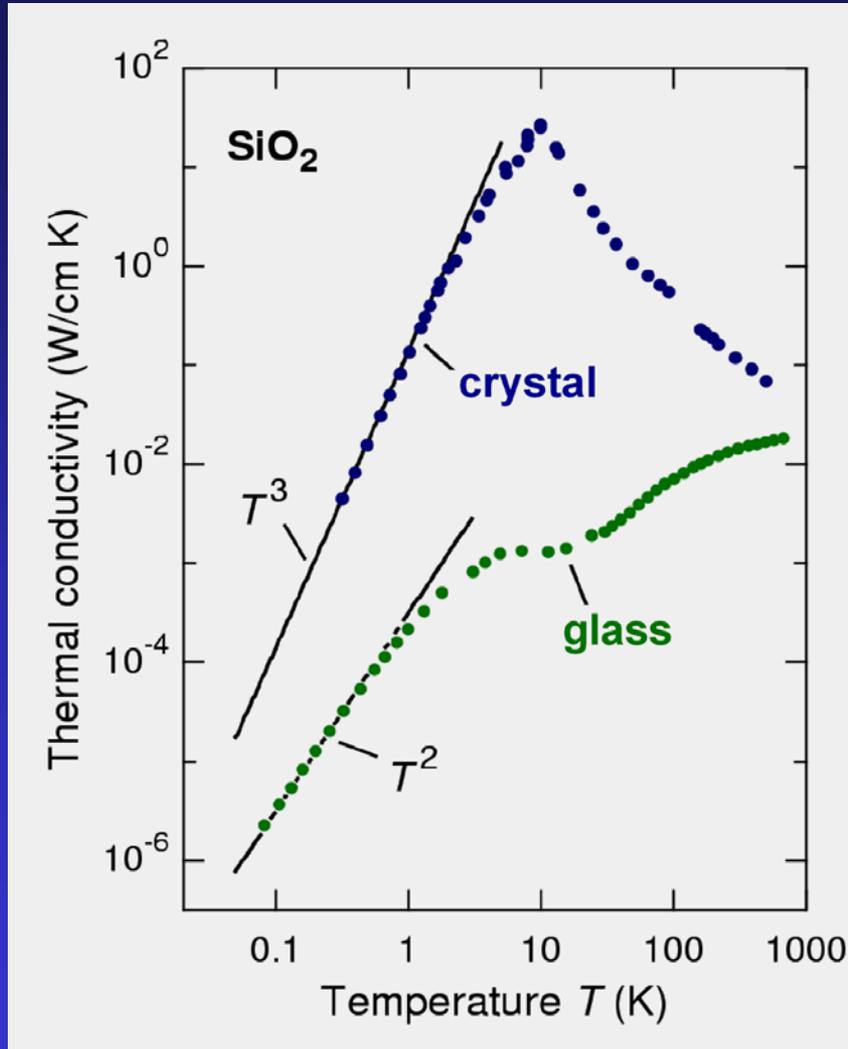


Specific Heat



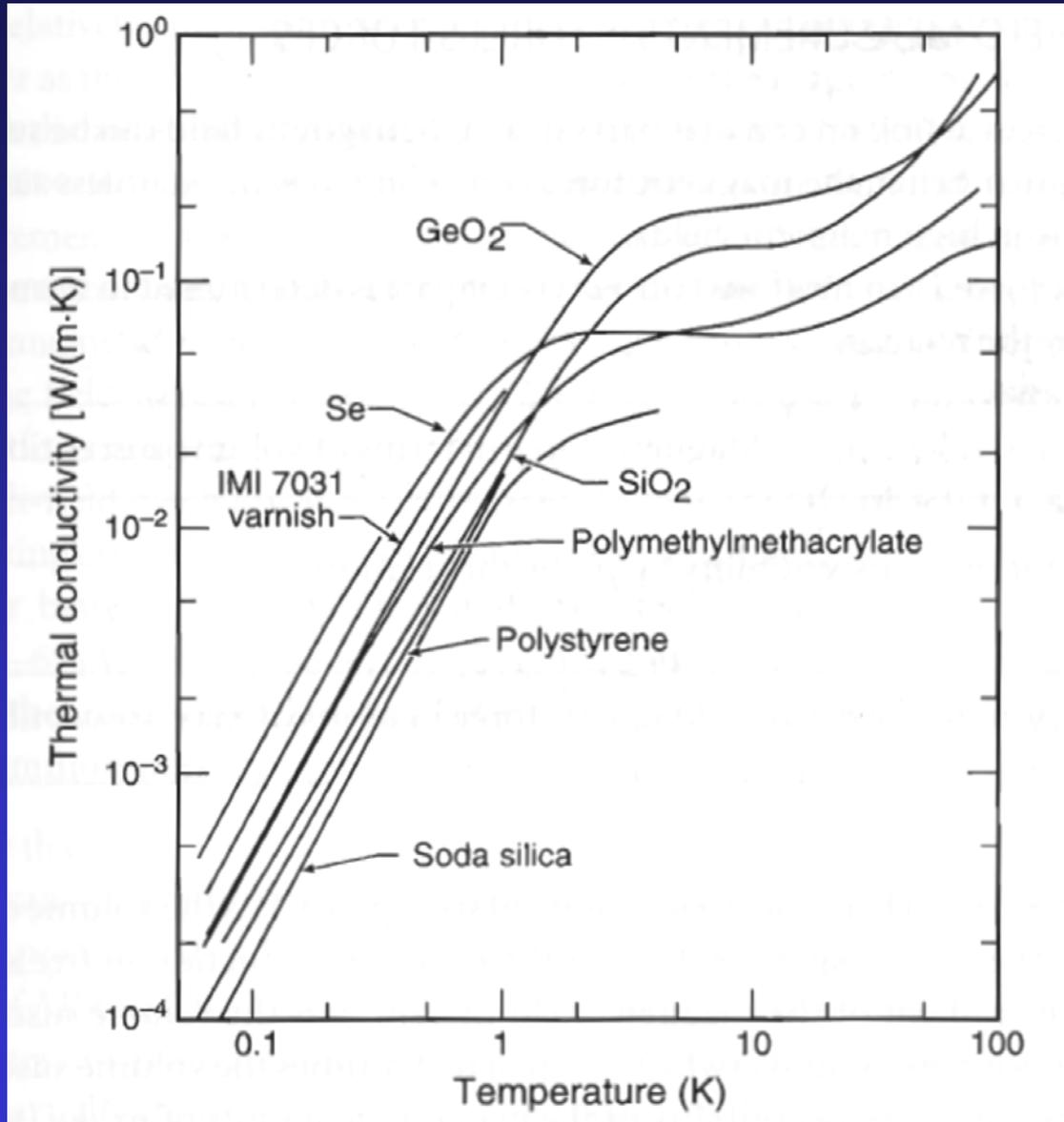
broad distribution of
low-energy excitations

Thermal Conductivity

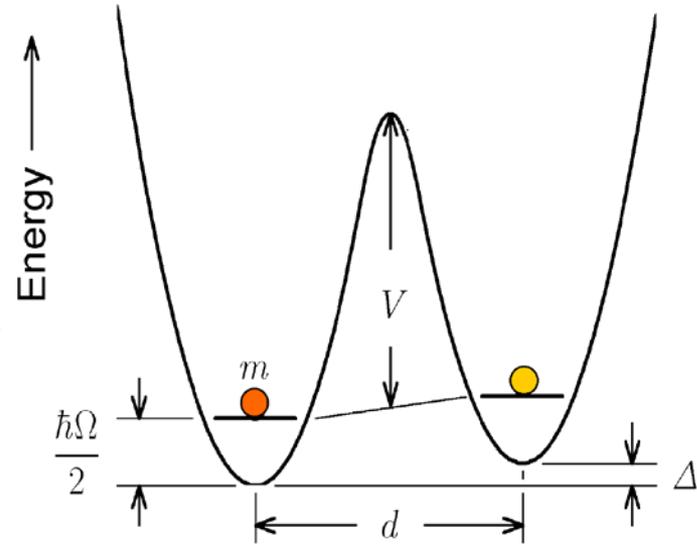
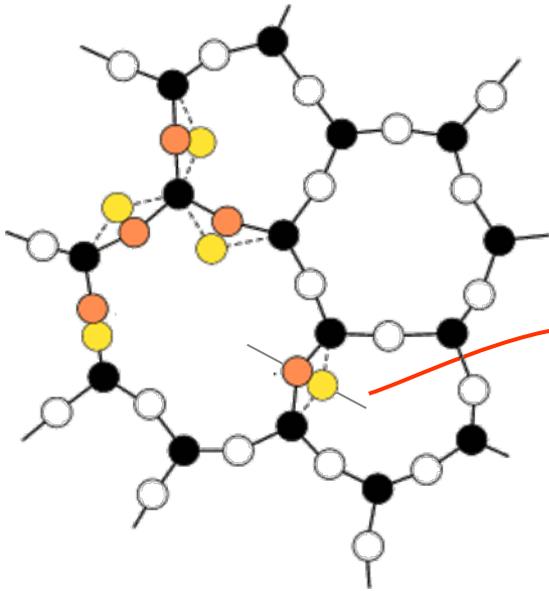


strong coupling to phonons
systems are localized

Universality

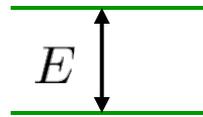


Atomic Tunneling Systems in Glasses



energy splitting

$$E = \sqrt{\Delta_0^2 + \Delta^2}$$

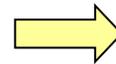


tunnel splitting

$$\Delta_0 = \hbar\Omega e^{-\lambda} \quad \lambda = \frac{d}{2\hbar} \sqrt{2mV}$$

distribution function

$$P(\lambda, \Delta) d\lambda d\Delta = \bar{P} d\lambda d\Delta$$



elastic, dielectric und thermal properties

Thermal Properties in the Tunneling Modell

Spezifische Wärme:

$$D(E) = \int_0^{\lambda_{\max}} P(E, \lambda) d\lambda = \bar{P} \lambda_{\max} \ln \frac{2E}{\hbar\Omega} \approx D_0 = \text{const.}$$
$$P(E, \lambda) dE d\lambda = P(\Delta, \lambda) \frac{\partial \Delta}{\partial E} dE d\lambda$$

$$C_V = \frac{1}{6} D_0 \pi^2 k_B^2 T \propto T$$

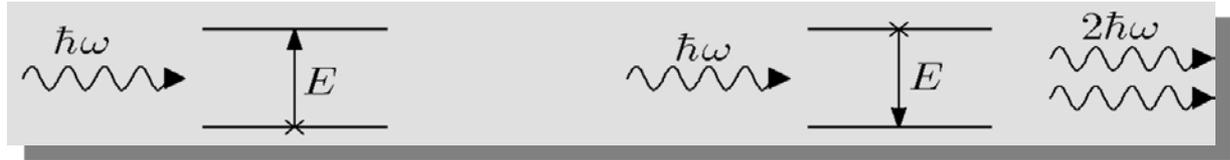
Wärmeleitung:

$$\Lambda = \frac{1}{3} C_V v \ell \quad \ell^{-1} \propto E \tanh\left(\frac{E}{2k_B T}\right) \propto \bar{\omega} \propto T$$
$$E = \hbar\bar{\omega} = k_B T$$

$$\Lambda \propto T^2$$

Elastic and Dielectric Properties

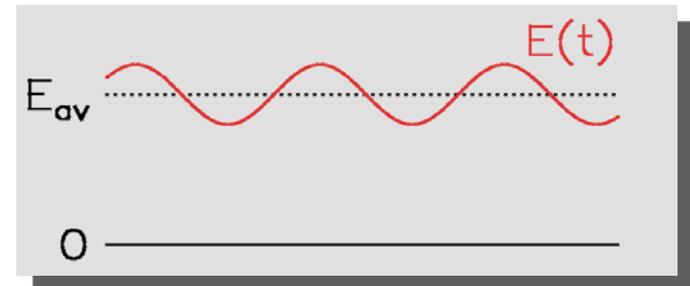
resonant processes



relaxational processes

→ modulation of Δ $\delta\Delta = 2\gamma e$

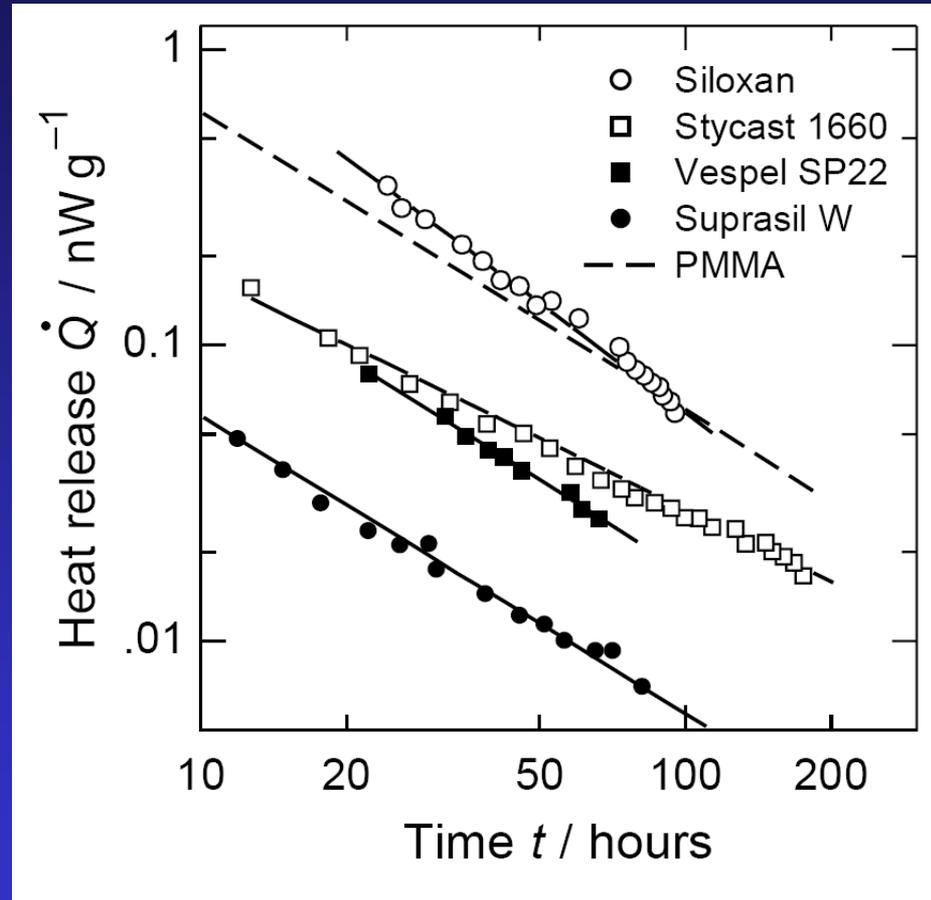
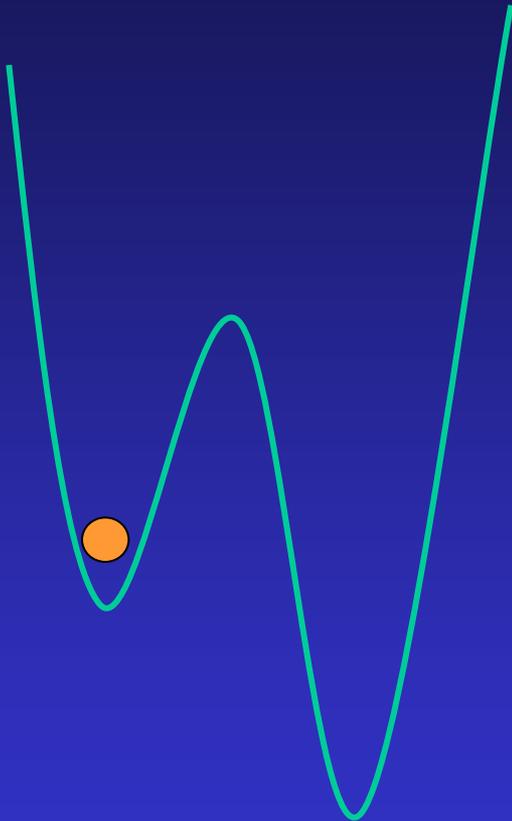
$$\delta\Delta = 2\mathbf{p}\cdot\mathbf{F}$$



$T < 1$ K one-phonon relaxation → wide distribution even for fixed E

$$\tau_1 = \mathcal{A} \left(\frac{E}{\Delta_0} \right)^2 \frac{1}{E^3} \tanh \left(\frac{E}{2k_B T} \right)$$

Heat Release

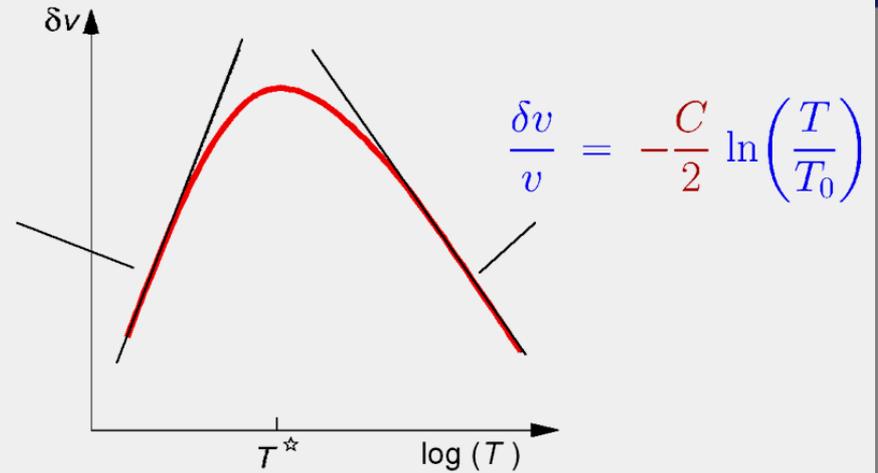


Sound Velocity and Internal Friction

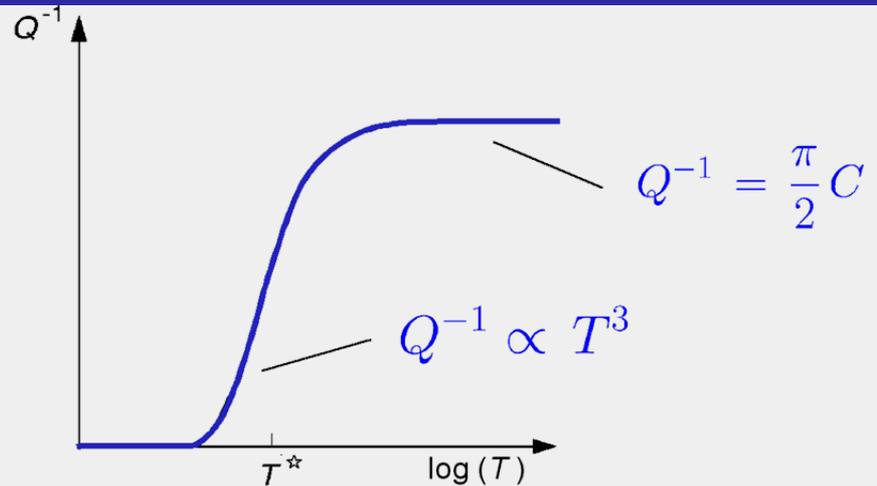
sound velocity $\delta v/v$

$$\frac{\delta v}{v} = C \ln\left(\frac{T}{T_0}\right)$$

$$C = \frac{\bar{P}\gamma^2}{\rho v^2}$$



internal friction Q^{-1}



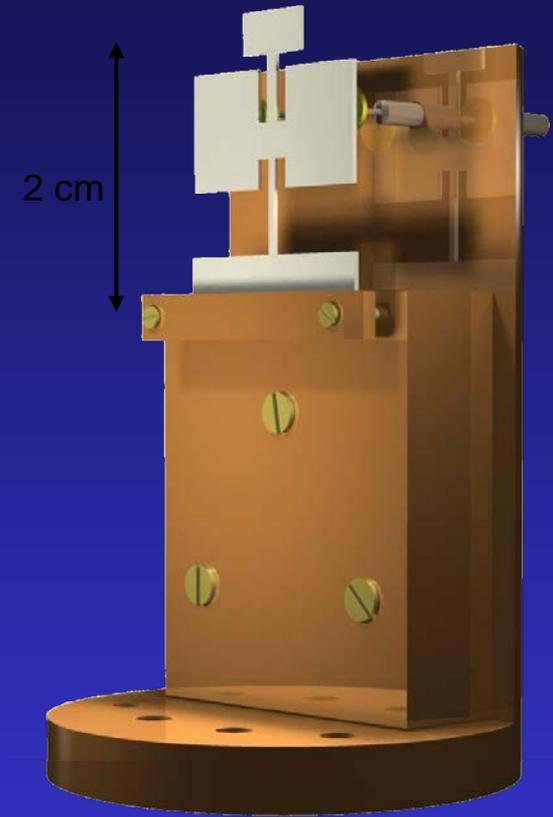
Elastic Measurements with Mechanical Oscillators



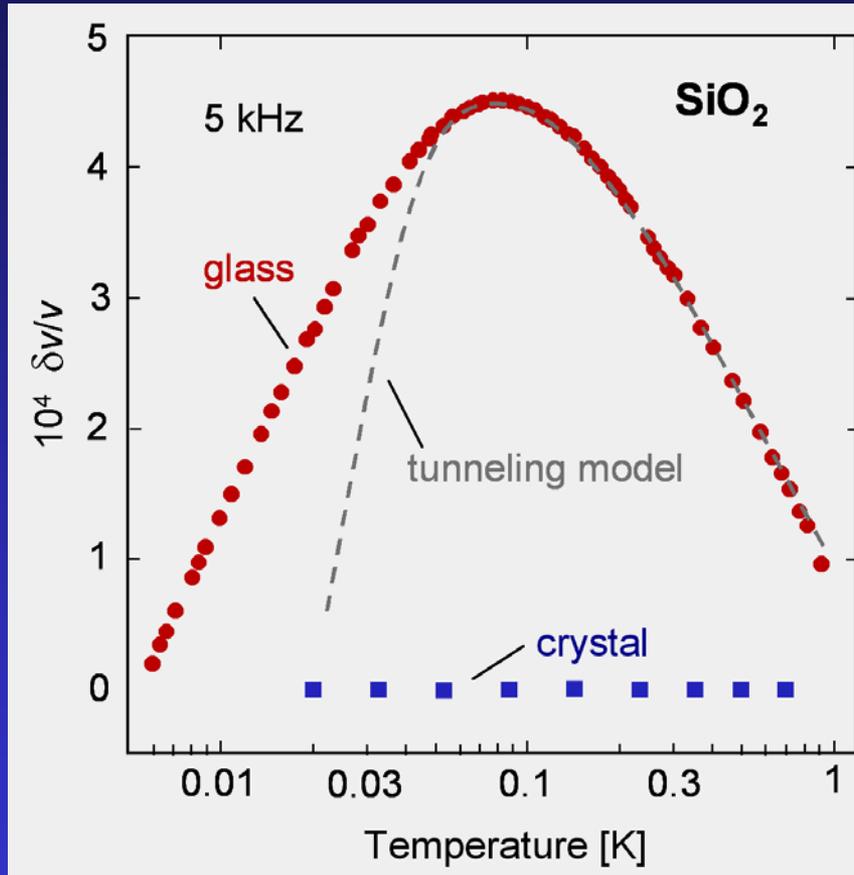
1 μm silver film

→ good thermalization

laser-cut glass
neck

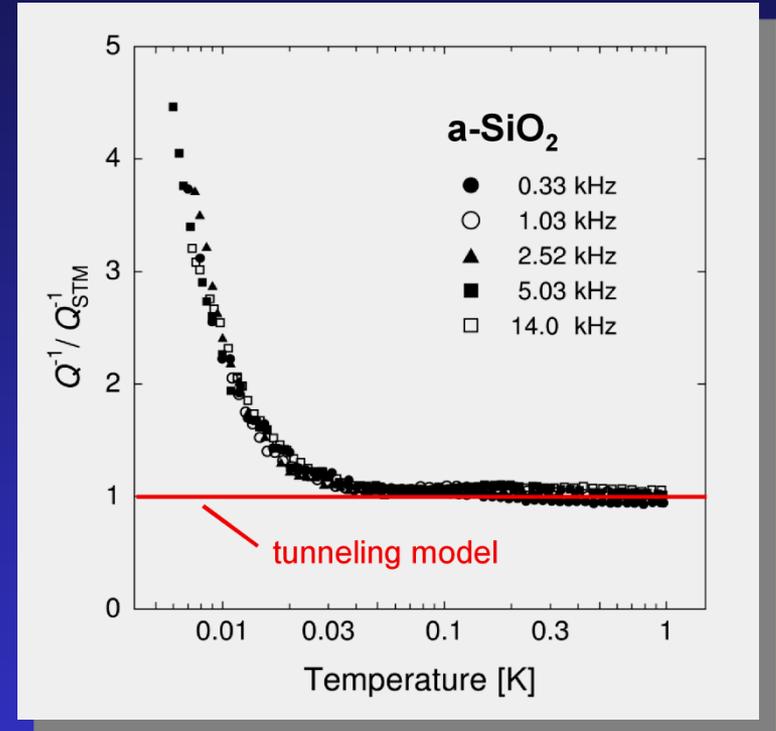
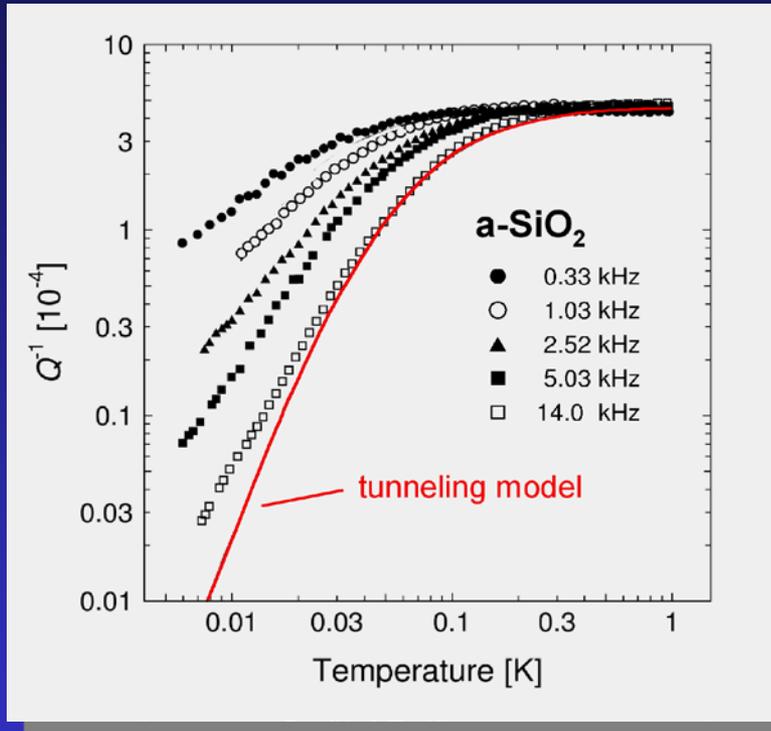


Sound Velocity



→ discrepancy at low temperatures

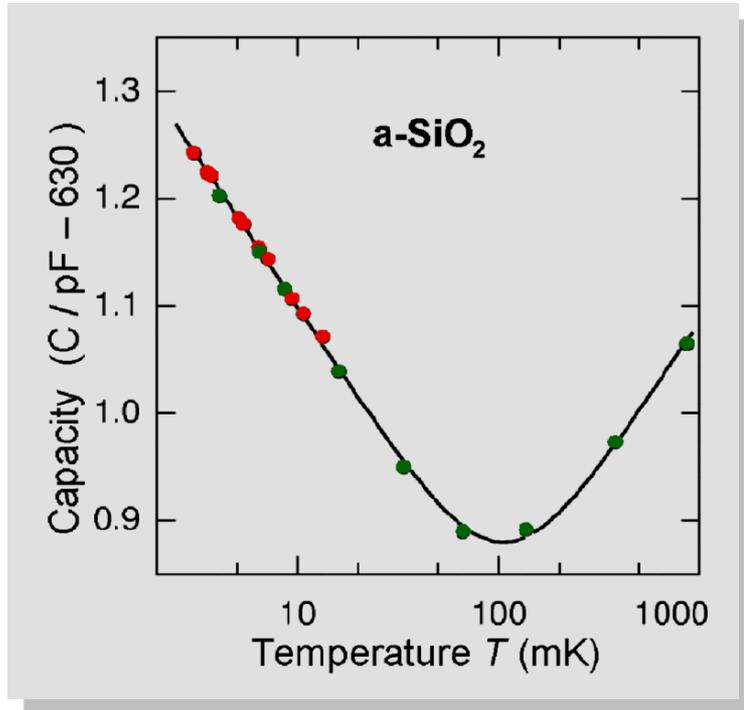
Internal Friction



$T < 30$ mK

➔ additional relaxation channel

Dielectric Constant - Magnetic Field Independent?



● E polar vector, B axial vector

● inversion symmetry of glasses

→ no linear terms !

$$\epsilon = 1 + \chi_1 + \chi_2 B^2$$

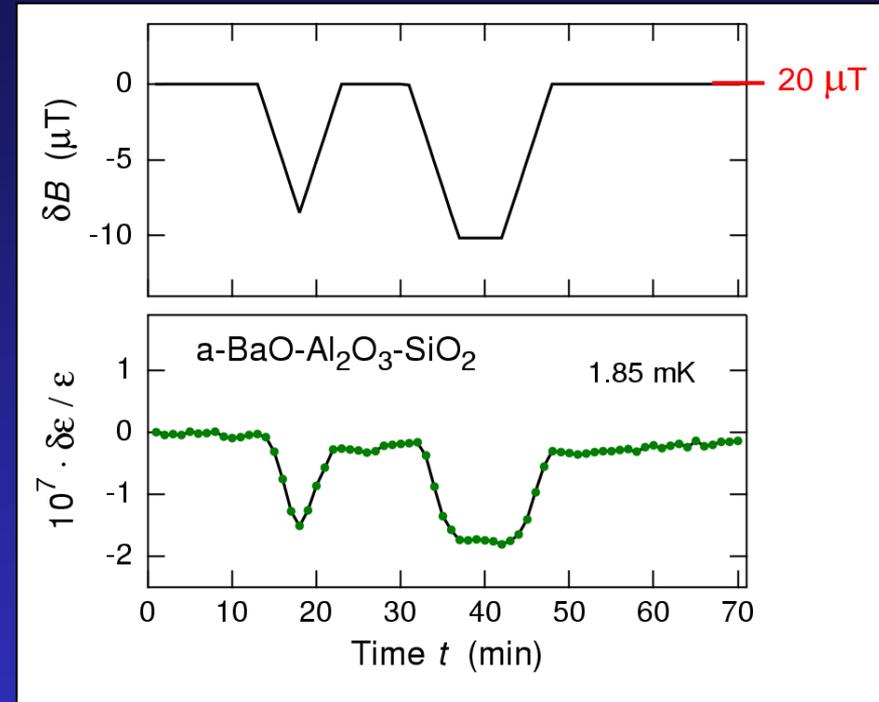
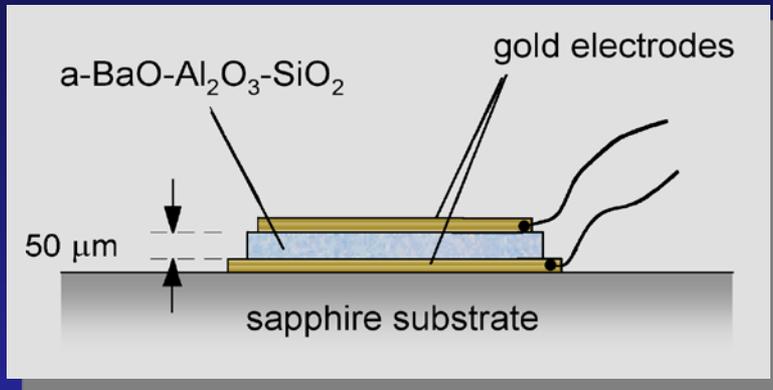
→ $\epsilon' \propto B^2$

Naughton et al. (4.2 K, 16 T)

→ $\epsilon' \propto B^0$

Wiegers et al. (2 mK, 9 T)

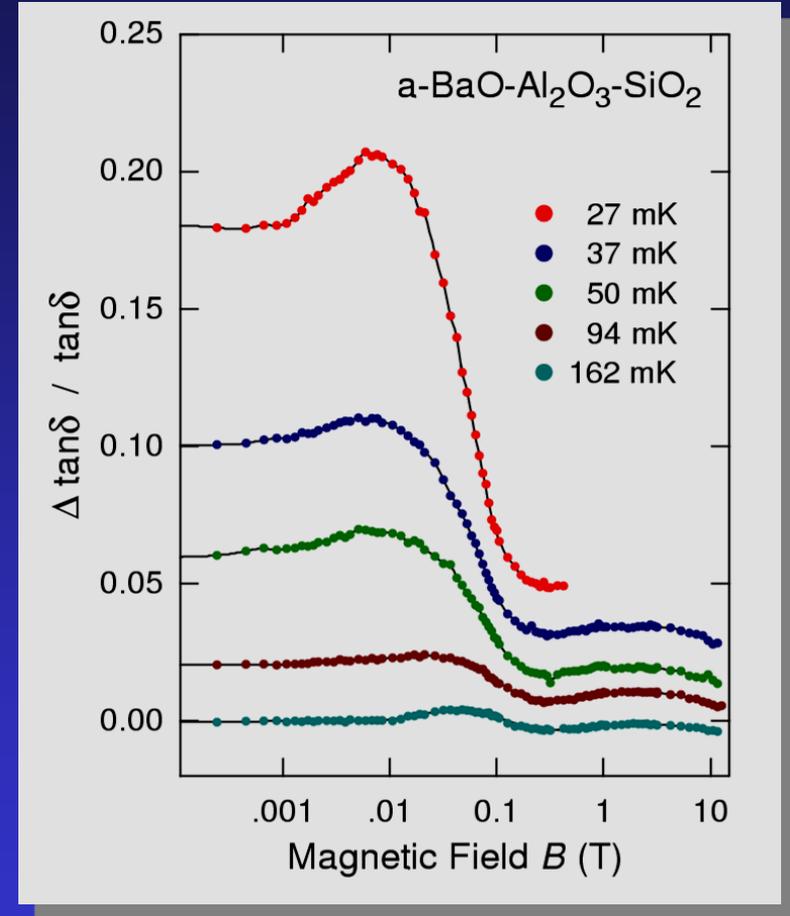
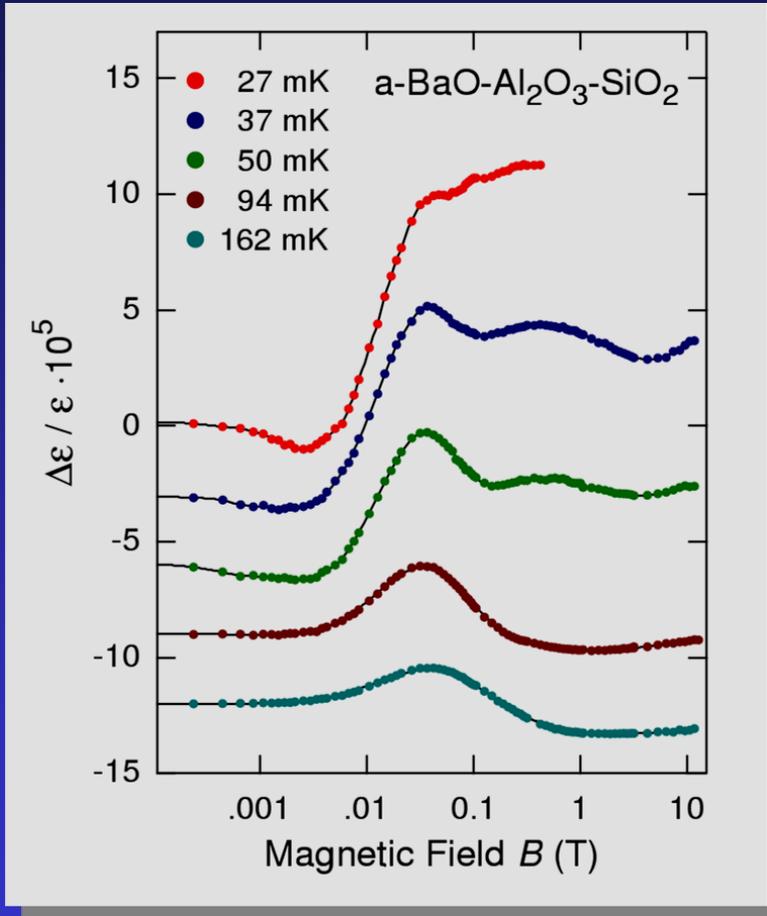
Dielectric Constant at Ultra-low Temperatures



- dielectric constant follows field variations
- extremely high sensitivity to magnetic fields

$$B = 0.1 \text{ T} \longrightarrow \delta\epsilon/\epsilon \approx 0.01$$

Temperature Dependence



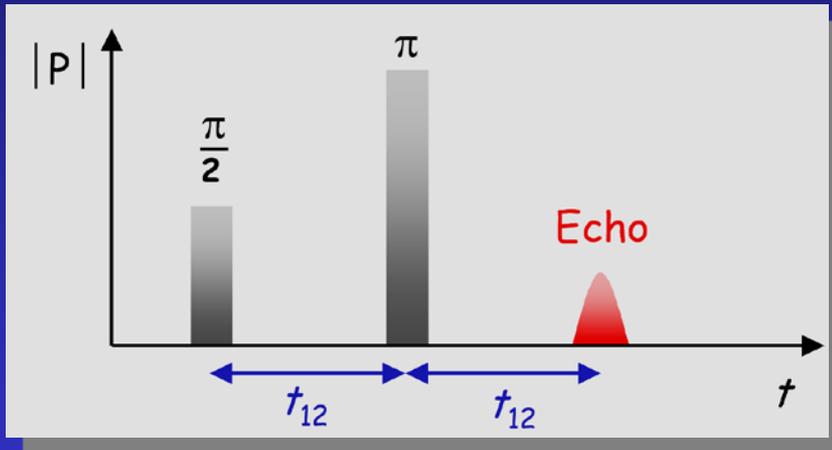
Coherent Properties

$$t \ll \tau_1, \tau_2 \rightarrow \infty$$



coherent regime

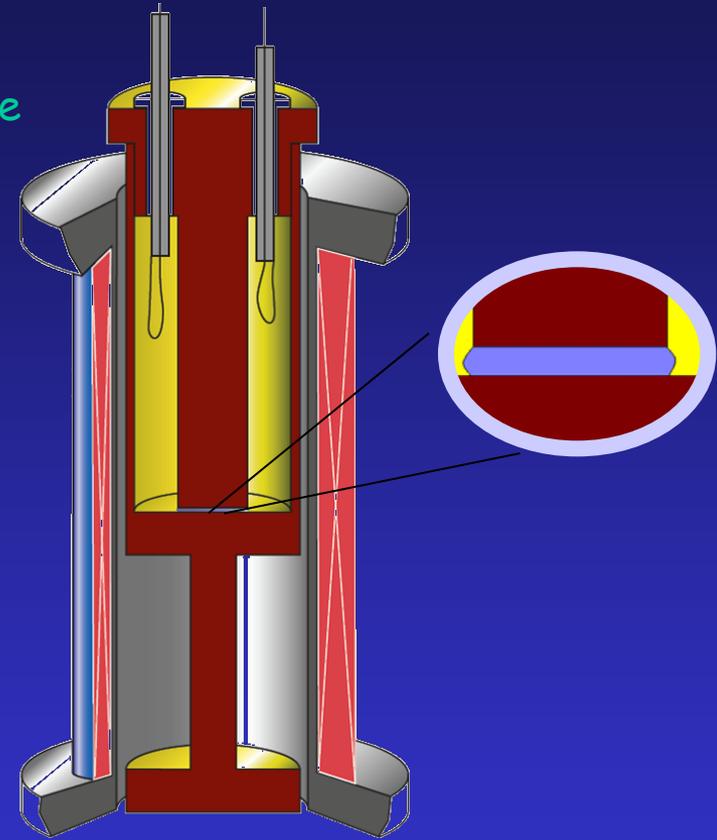
two-pulse polarization echoes:



$$\Theta_p = \Omega_R t_p$$

Rabi frequency

$$\Omega_R = \frac{1}{\hbar} \frac{\Delta_0}{E} \mathbf{p} \cdot \mathbf{F}$$

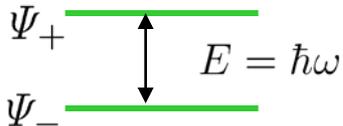


microwave cavity

1 GHz \rightarrow 50 mK

Echo – Theoretical Background I

coherent regime: $t \ll \tau_1, \tau_2 \rightarrow \infty$

two level approximation: 

applied field: $\mathbf{F} = \mathbf{F}_0 (e^{i\omega_r t} + e^{-i\omega_r t})$

Schrödinger equation: $i\hbar \frac{\partial \Psi}{\partial t} = [H_0 + H_S] \Psi$ mit $H_S = 2 \frac{\Delta_0}{E} \mathbf{p} \cdot \mathbf{F}$

ansatz: $\Psi(t) = a_1(t) \Psi_- + a_2(t) \Psi_+$ $\left. \begin{array}{l} a_1(t) = \cos(\Omega_R t) e^{-i\omega_r t} \\ a_2(t) = -i \sin(\Omega_R t) e^{-i\omega_r t} \end{array} \right\}$

Rabi frequency: $\Omega_R = \frac{1}{\hbar} \frac{\Delta_0}{E} \mathbf{p} \cdot \mathbf{F}_0$

Echo – Theoretical Background II

polarisation vector:
$$\mathbf{P} = \begin{pmatrix} ab^* + ba^* \\ i(ab^* - ba^*) \\ aa^* - bb^* \end{pmatrix} = \begin{pmatrix} -\sin(\Omega_R t) \sin(\omega t) \\ \sin(\Omega_R t) \cos(\omega t) \\ \cos(\Omega_R t) \end{pmatrix} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$$

Bloch equations:

$$\frac{d\langle S_x \rangle}{dt} = -\frac{2}{\hbar} \left(\frac{E}{2} + \frac{\Delta}{E} \mathbf{p} \cdot \mathbf{F} \right) \langle S_y \rangle - \frac{\langle S_x \rangle}{\tau_2}$$

$$\frac{d\langle S_y \rangle}{dt} = \frac{2}{\hbar} \left(\frac{E}{2} + \frac{\Delta}{E} \mathbf{p} \cdot \mathbf{F} \right) \langle S_x \rangle - \frac{2}{\hbar} \left(\frac{\Delta_0}{E} \mathbf{p} \cdot \mathbf{F} \right) \langle S_z \rangle - \frac{\langle S_y \rangle}{\tau_2}$$

$$\frac{d\langle S_z \rangle}{dt} = \frac{2}{\hbar} \left(\frac{\Delta_0}{E} \mathbf{p} \cdot \mathbf{F} \right) \langle S_y \rangle - \frac{[\langle S_z \rangle - S_z^0(\mathbf{F})]}{\tau_1}$$

τ_1 : energy relaxation

$T < 1\text{K} \longrightarrow$ one phonon process

..... ?

τ_2 : phase coherence time

τ_1 processes

spectral diffusion



spin diffusion

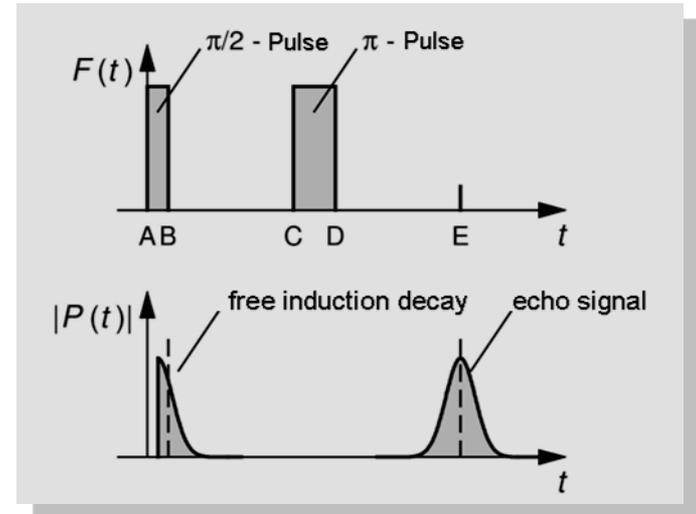
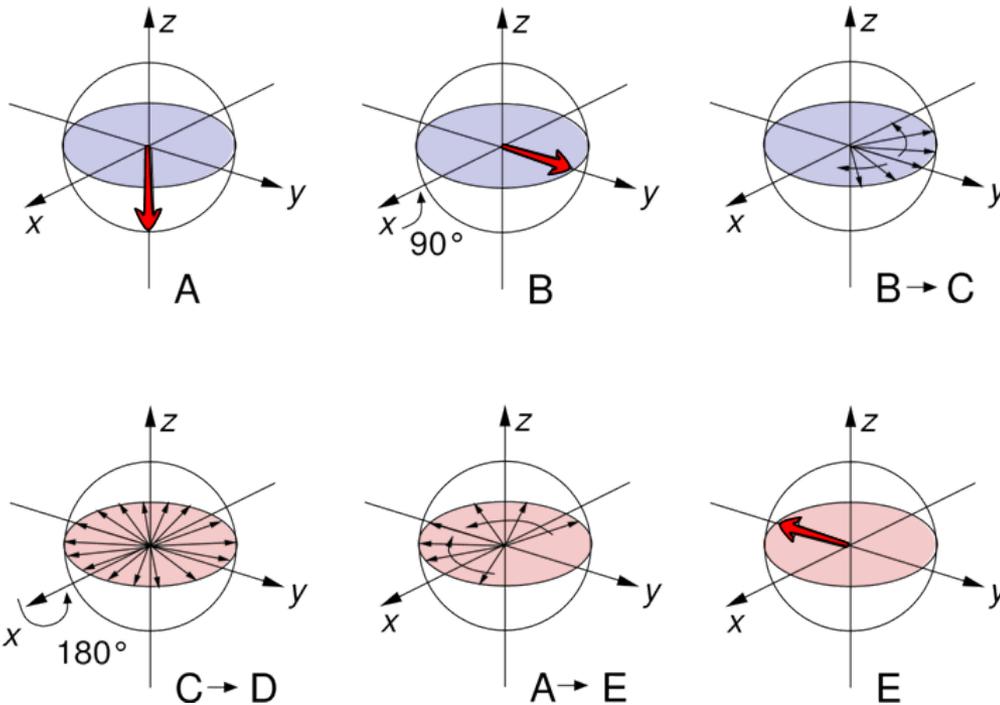
..... ?

Two Pulse Echo I

polarization vector

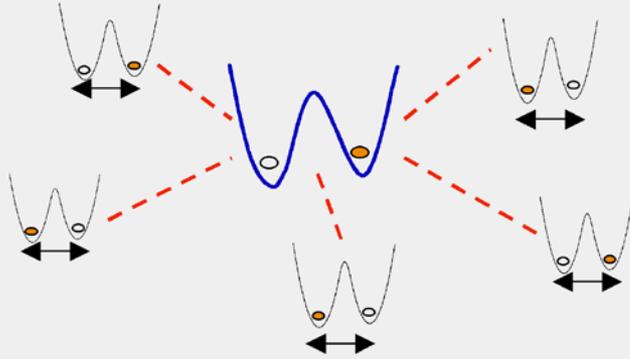
$$P = (S_x, S_y, S_z)$$

rotating frame

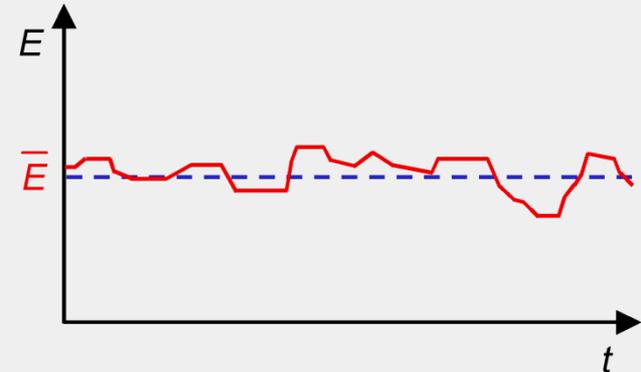


Spectral Diffusion

interaction between resonant TS and thermally fluctuating TS



energy splitting of single TS fluctuating with time



- short time limit (no flip limit): $t_{12} \ll \tau_{\min}$

→ $A(2t_{12}) = A(0) e^{-(2t_{12}/\tau_2)^2}$ Gaussian decay

- long time limit (multiple flip limit): $t_{12} \gg \tau_{\min}$

→ $A(2t_{12}) = A(0) e^{-2t_{12}/\tau_2}$ exponential decay

Temperature Dependence

short time limit (no flip limit): $t_{12} \ll \tau_{\min}$

$$A(2t_{12}, T) \propto \tanh\left(\frac{E}{2k_B T}\right) e^{-m_0 T^4 (\Delta/E) t_{12}^2}, = A_0(T) e^{-m(T) (\Delta/E) t_{12}^2}$$

$$\tau_2 \propto 1/\sqrt{m(T)} \propto T^{-2}$$

long time limit (multiple flip limit):

$$A(2t_{12}, T) \propto A_0(T) e^{-2t_{12}^2/\tau_2}$$

$$\tau_2 \propto T^{-1}$$

Theoretical papers:

P. Hu, S.R. Hartmann, PRB **9**, 1 (1974)

J.L. Black, B.I. Halperin, PRB **16**, 2879 (1976)

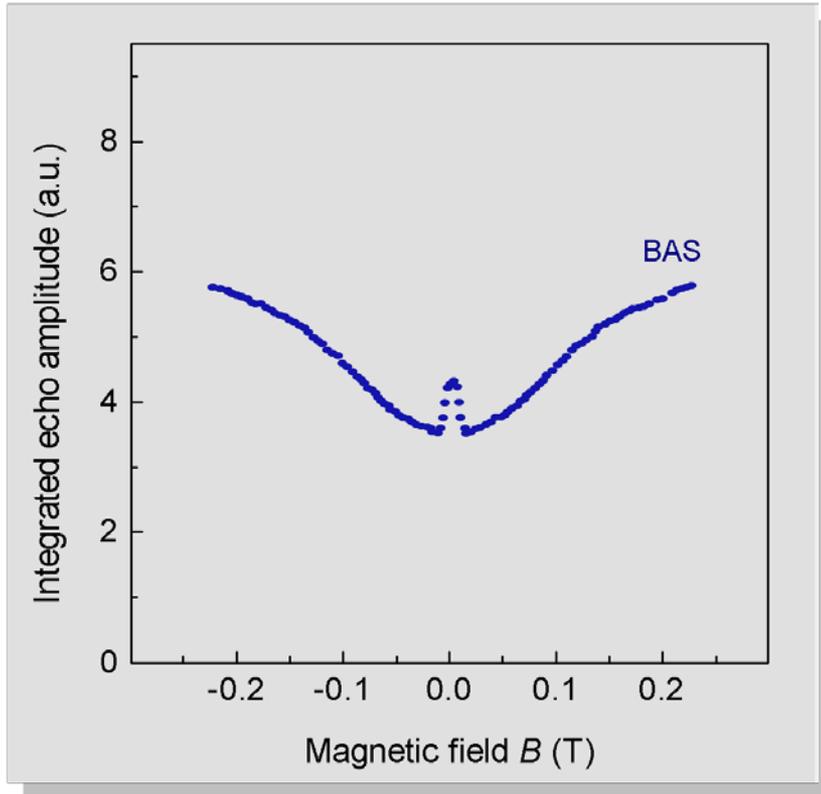
P. Hu, L.R. Walker, PRB **18**, 1300 (1978)

R. Maynard, R. Rammal, R. Suchail, J. Phys. Paris Lett. **41**, L-291 (1980)

B.D. Laikhtman, PRB **31**, 400 (1985)

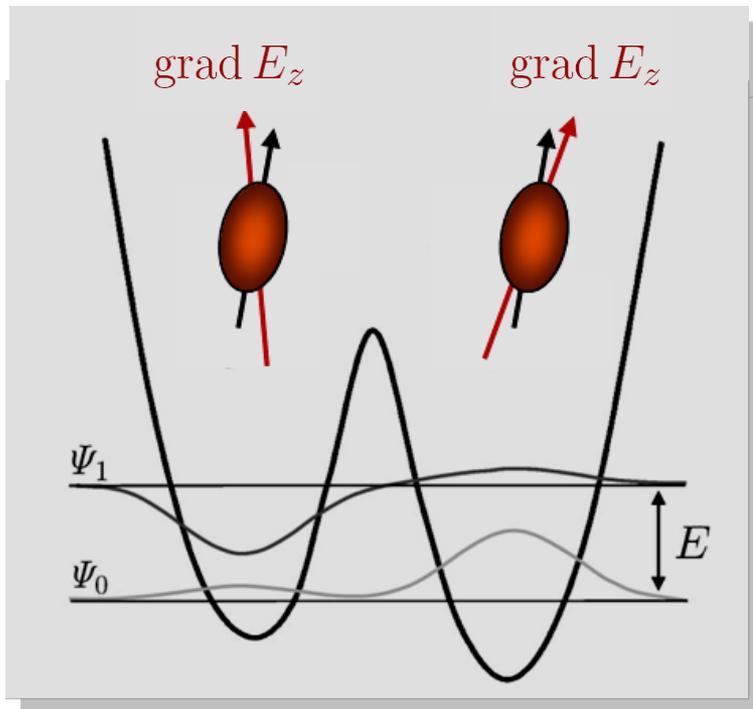
Yu.M. Galperin, V.L. Gurevich, D.A. Parshin, PRB **37**, 10339 (1988)

Echo Amplitude: Magnetic Field Dependence



- Tunneling systems couple to magnetic fields
- What is different in case of α -SiO₂?

Nuclear Quadrupole Moment is Important



- nuclear quadrupole moment of tunneling particle sees the electric field gradient in the two wells

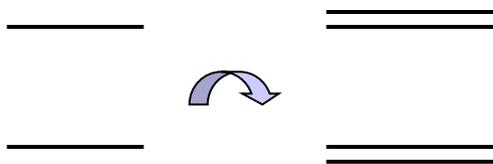


splitting of tunneling levels



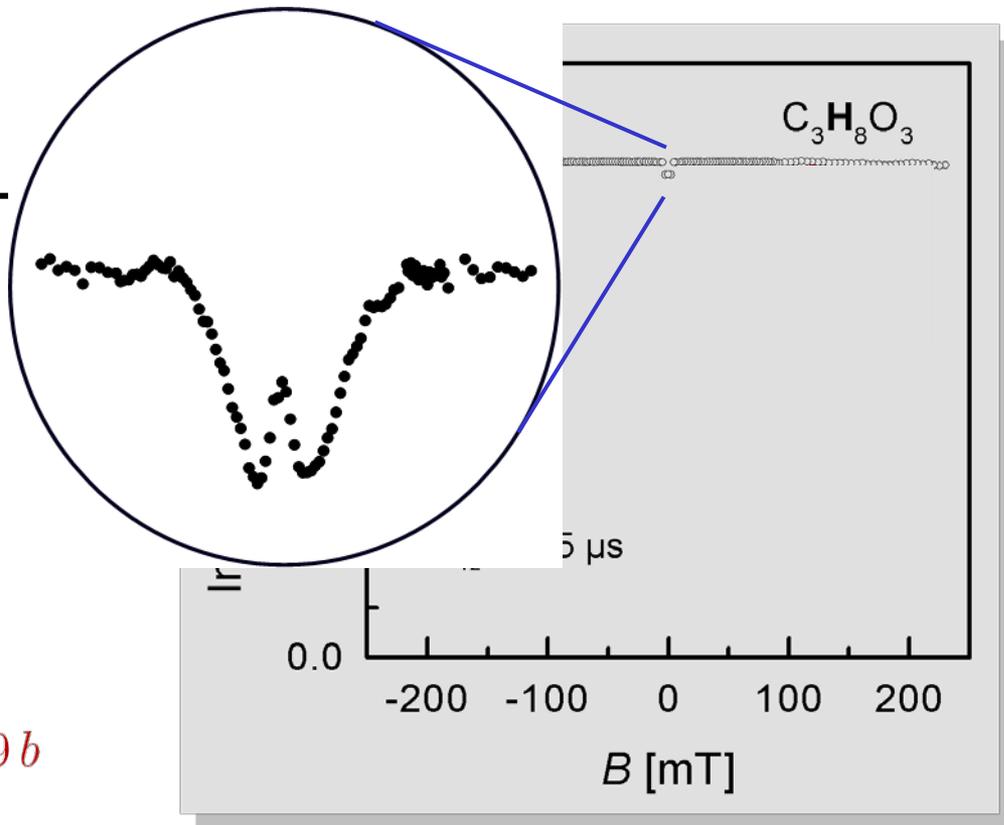
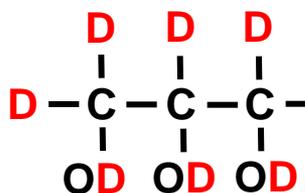
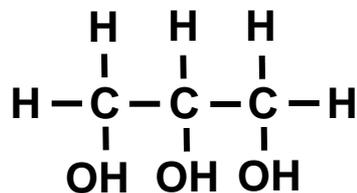
multi-level systems

- magnetic field causes an additional Zeeman splitting of nuclear levels



Isotope Effect $H \leftrightarrow D$

Glycerol



hydrogen

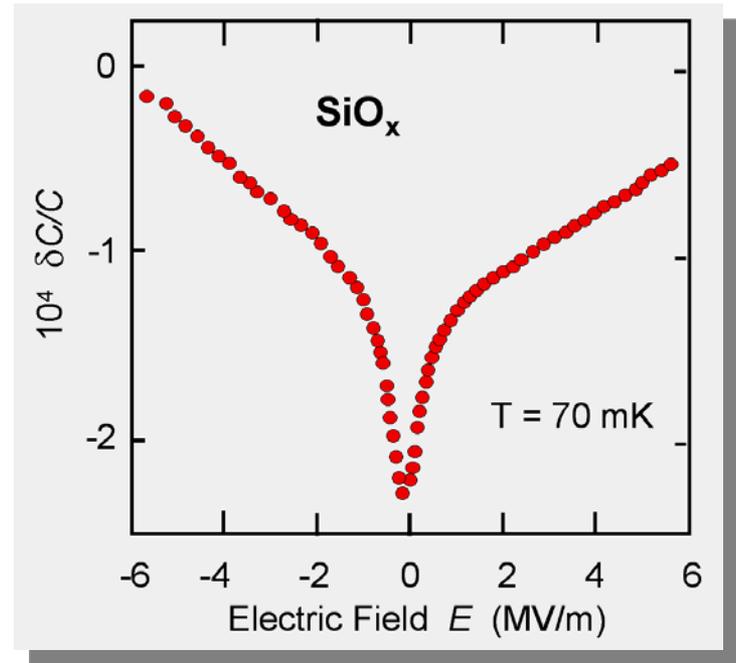
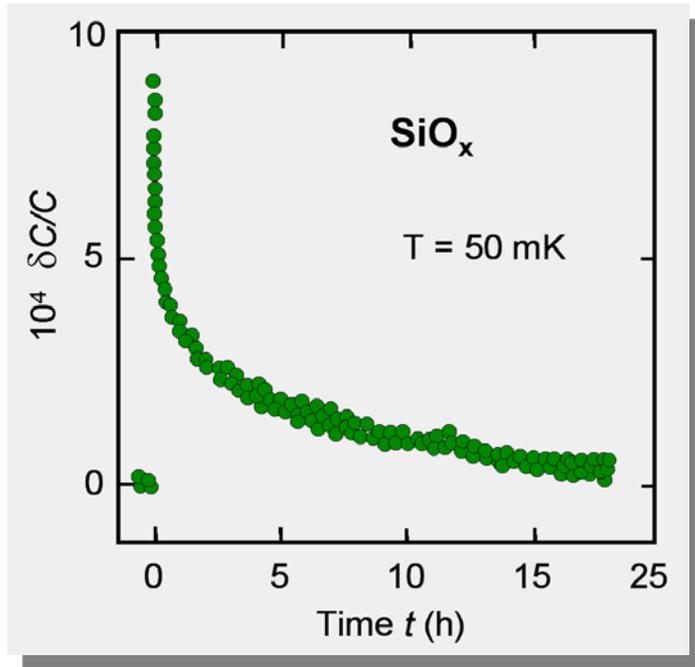
$$I = 1/2, \quad \mu = 2.79 \mu_N, \quad Q = 0$$

deuterium atom

$$I = 1, \quad \mu = 0.86 \mu_N, \quad Q = 0.0029 b$$

→ proof of the quadrupole model

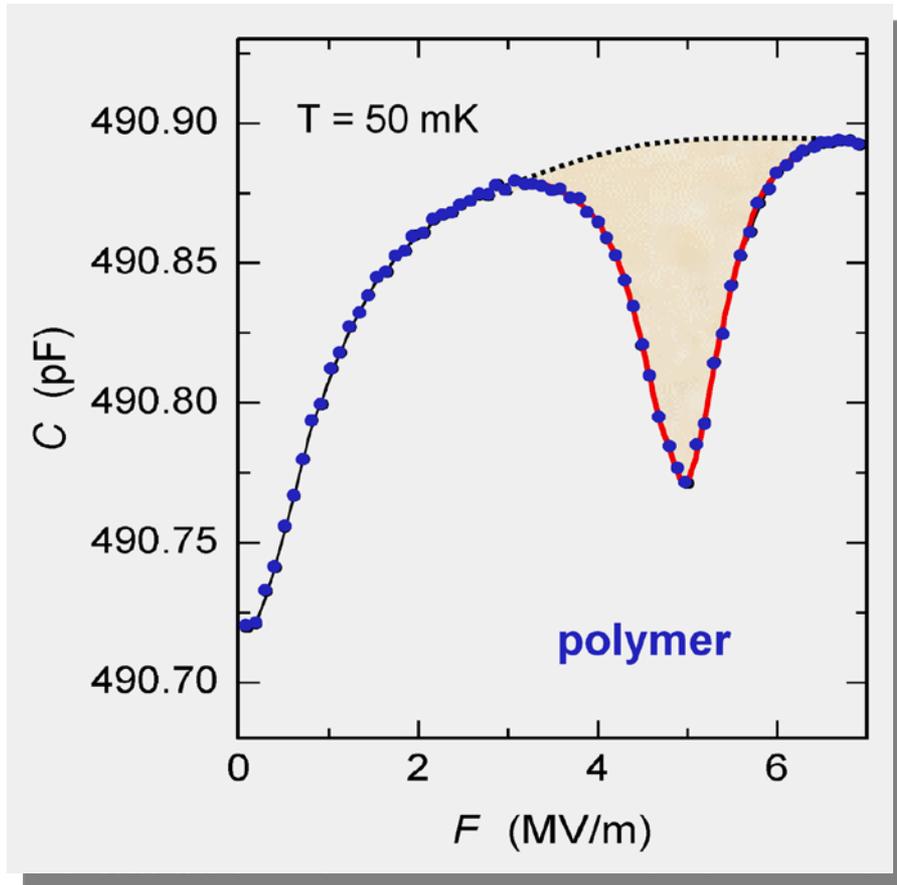
Evidence for a Dipole Gap in Glasses



→ modification of density of states: **dipole gap**

↑
slow sweep experiment

Memory Effect



slow sweep after applying 5 MV/m for 2 h



dielectric constant remembers previous dc-field