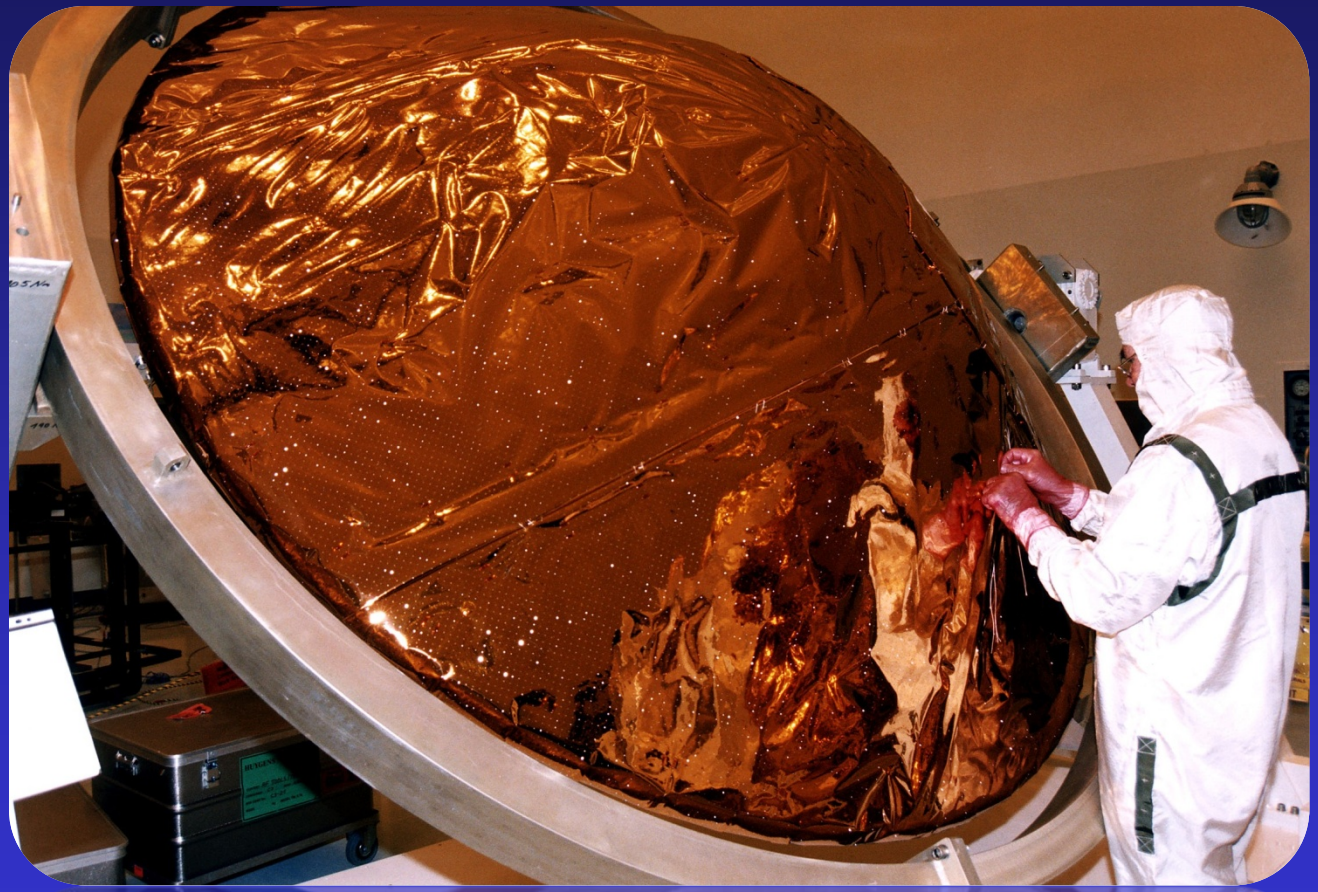
Heat Transfer, Thermal Isolation, Thermal Contact



Thermal insulation of Huygens Satellit

Overview

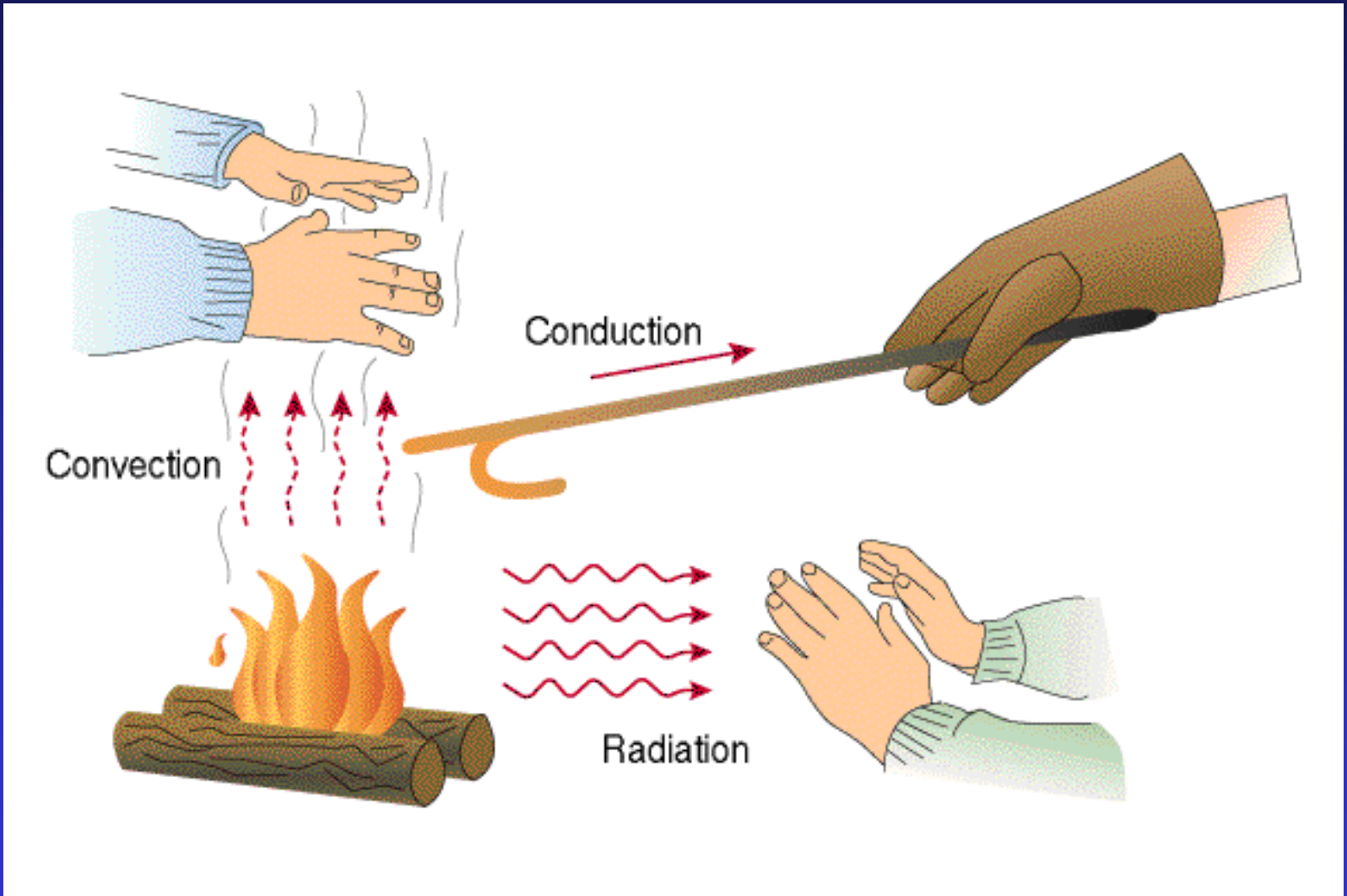
Forms of Heat Transfer

- conduction
- convection
- radiation

Thermal Isolation and Contact

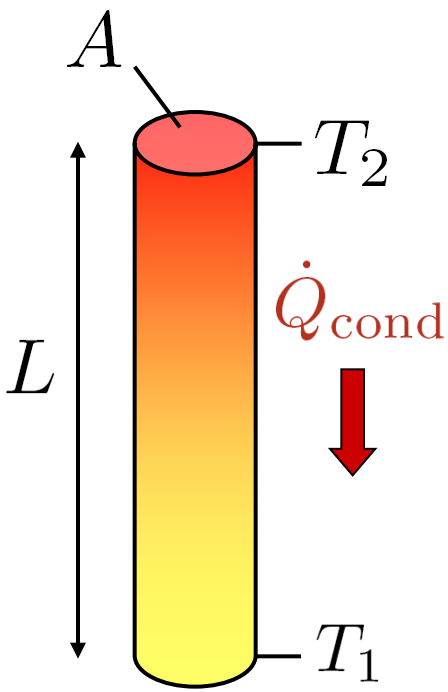
- vacuum
- superinsulation
- thermal anchoring
- heat exchanges
- heat switches

Forms of Heat Transfer



Heat Conduction Through Solids

Fourier's Law $$j = -\lambda \nabla T$$

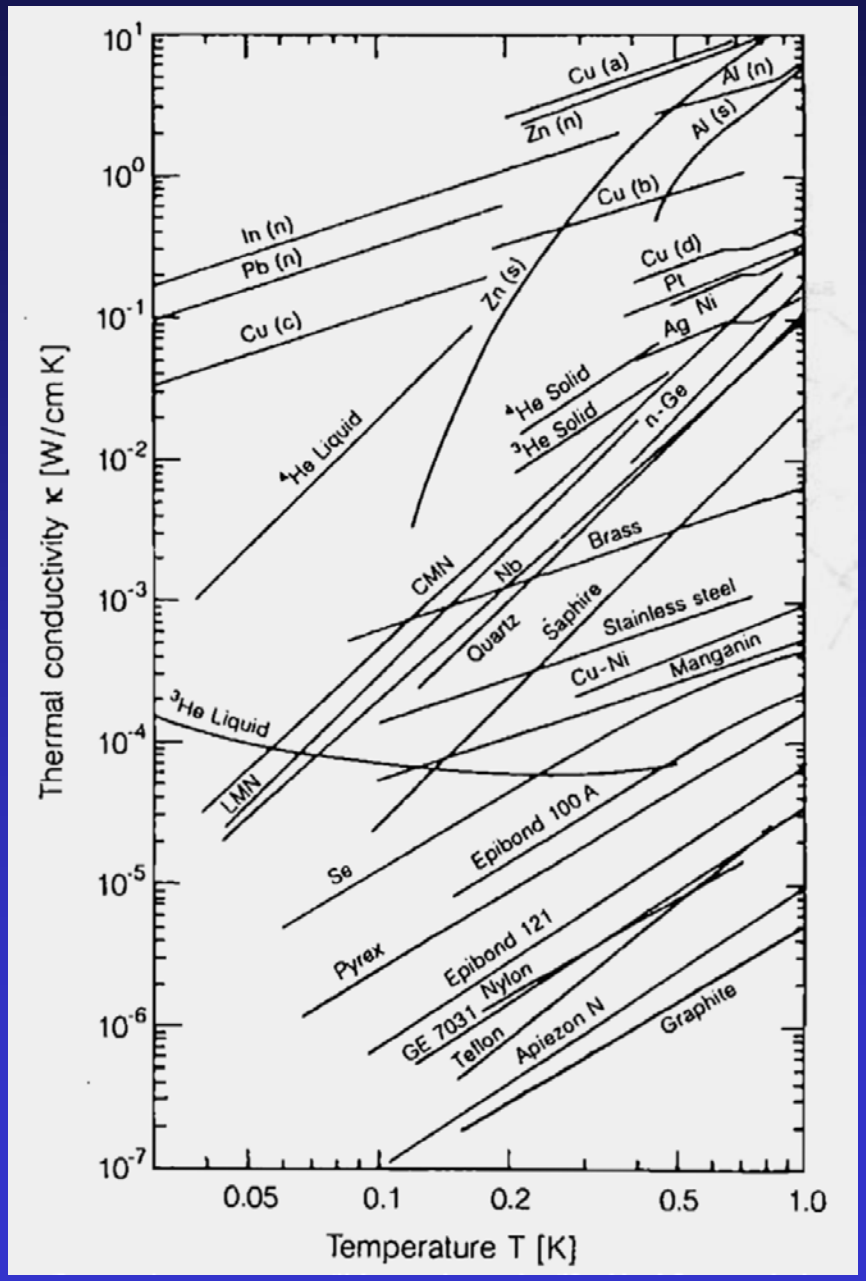


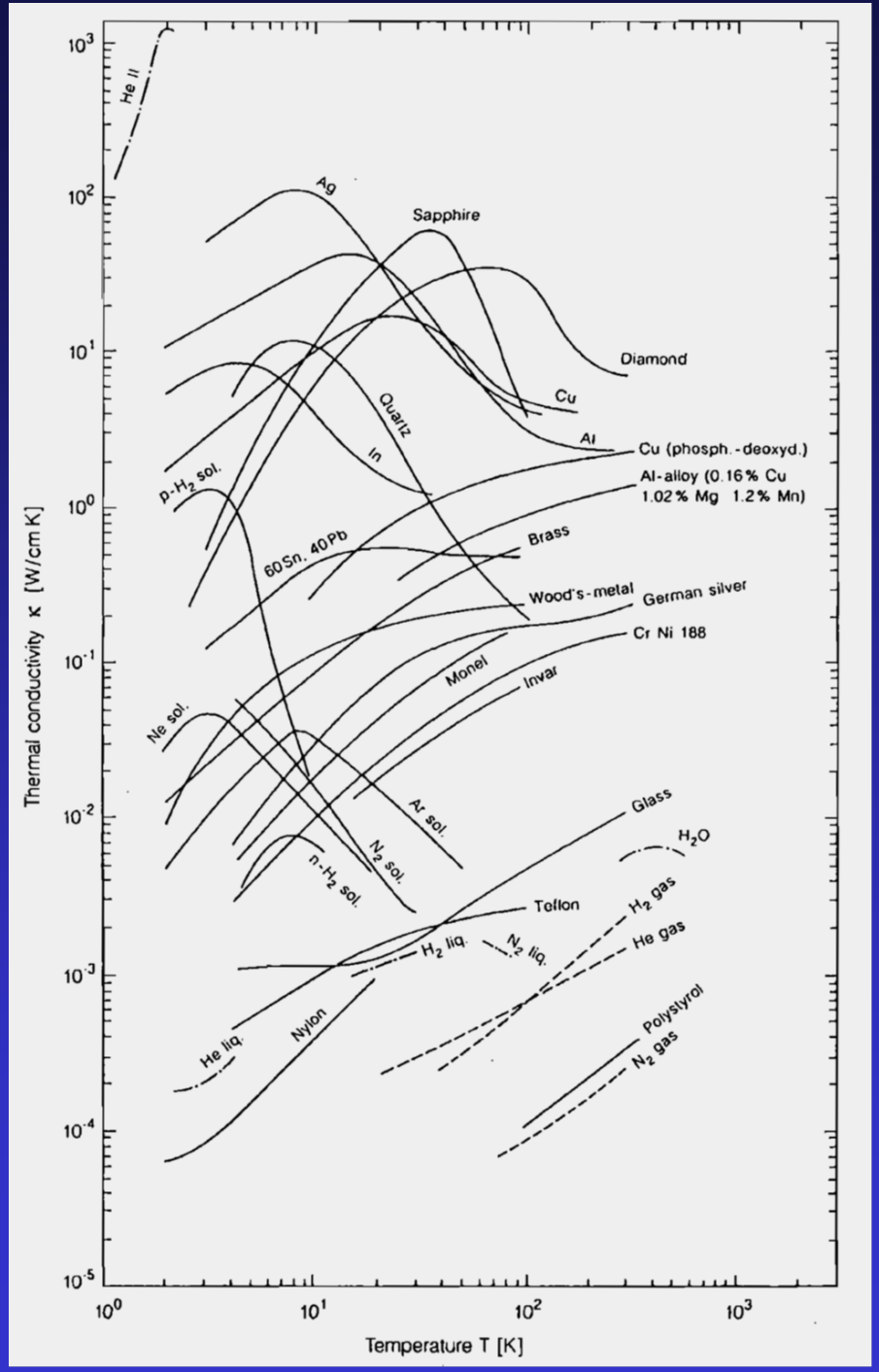
$$\dot{Q}_{\mathrm{cond}} = \lambda(T) A \frac{\mathrm{d}T}{\mathrm{d}x}$$

$$\dot{Q}_{\mathrm{cond}} = \frac{A}{L} \int_{T_1}^{T_2} \lambda(T)\, \mathrm{d}T = \frac{A}{L} \overline{\lambda}\, \Delta T$$

mean thermal conductivity

Thermal Conductivity Data





Thermal Conductivity Integrals

$$\int_{4\,\mathrm{K}}^{T} \lambda \, \mathrm{d}T \quad [\mathrm{W/m}]$$

T(K)	Copper (wire)	Stainless Steel	Glass	Teflon
6	800	0.63	0.211	0.113
10	3 320	2.93	0.681	0.44
20	14 000	16.3	2.0	1.64
50	50 800	135	8.46	7.16
77	68 600	317	17.5	13.0
100	80 200	528	29.2	18.7
140	97 600	939	54.2	28.7
200	122 000	1 660	103	44.2
300	162 000	3 060	199	70.2

$$\dot{Q}_{\mathrm{cond}} = \frac{A}{L} \int_{T_1}^{T_2} \lambda(T) \, \mathrm{d}T = \frac{A}{L} \left\{ \int_{4\,\mathrm{K}}^{T_2} \lambda(T) \, \mathrm{d}T - \int_{4\,\mathrm{K}}^{T_1} \lambda(T) \, \mathrm{d}T \right\}$$

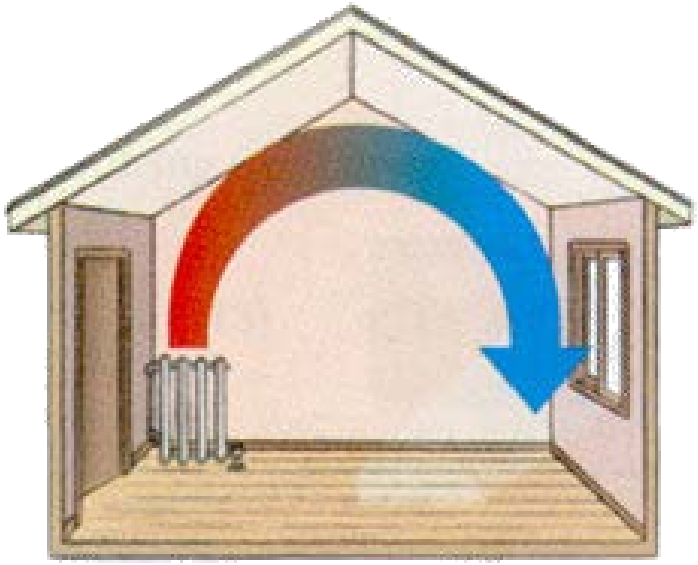
For Metals: Wiedemann-Franz Law

Useful relation between electrical conductivity and thermal conductivity

$$\frac{\lambda}{\sigma} = \underbrace{\frac{\pi^2}{3}\left(\frac{k_\mathrm{B}}{e}\right)^2}_{\mathcal{L} = 2.45 \times 10^{-8}\,\mathrm{V}^2\,\mathrm{K}^{-2}} T$$

Only valid if the mean free path is limited by the same process for both electrical resistance and thermal resistance

Convective Heat Transfer through Gases and Liquids



Mechanism of Convection

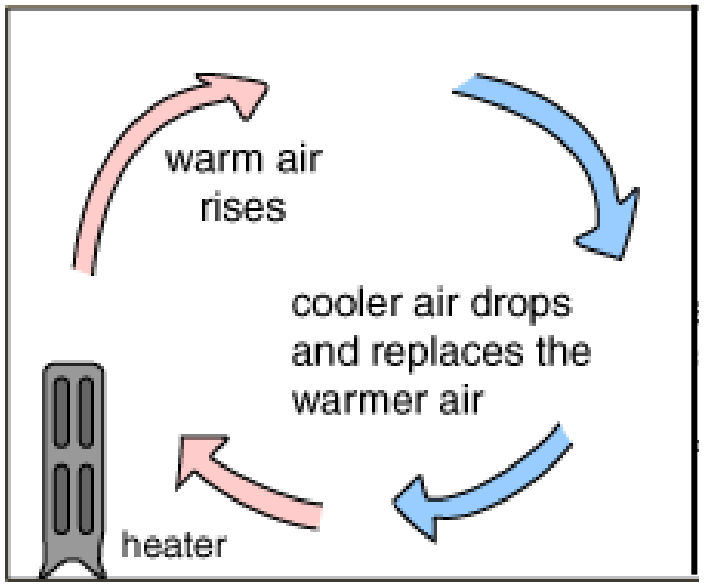
ideal gas at constant pressure

$$\frac{V}{T} = \text{const}$$

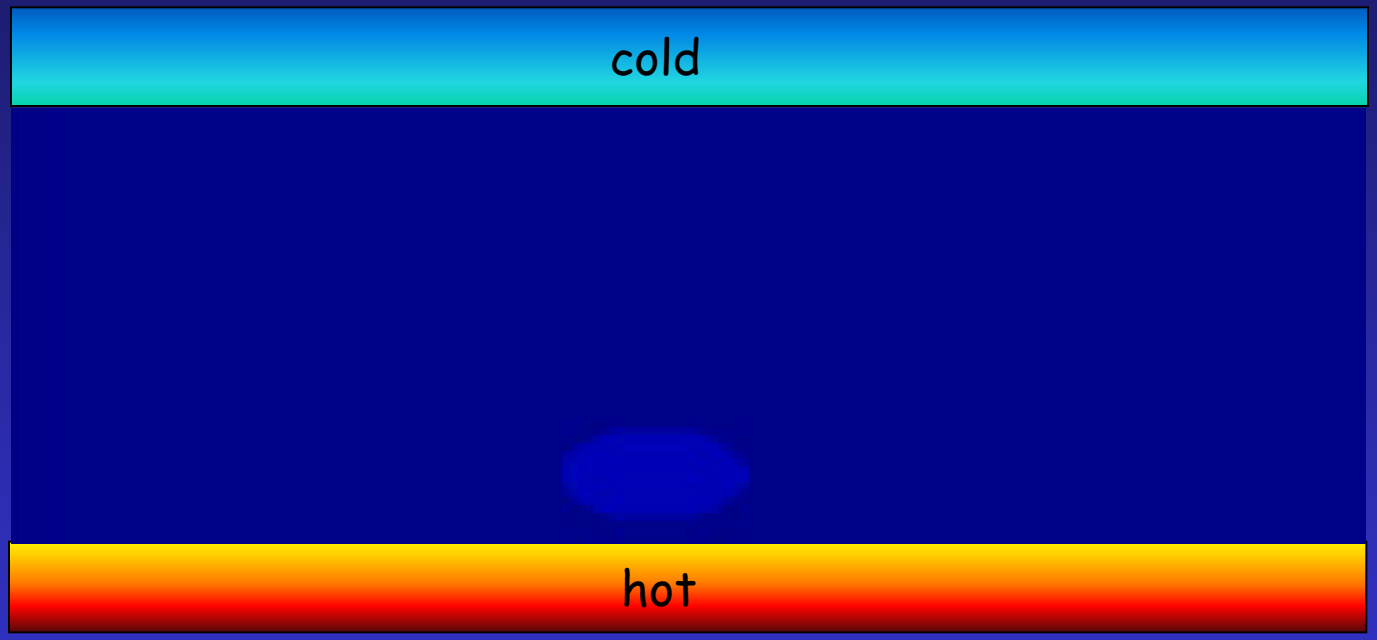
$$\varrho = \frac{m}{V}$$

Temperature rise

- ➡ volume rises
- ➡ density decreases



Simulation For Two Parallel Plates



Convective Heat Transfer through Gases and Liquids

two limiting cases: $\ell \ll d$ hydrodynamic regime

$\ell \gg d$ Knudsen regime (free molecule regime)

mean free path:

$$\ell = \frac{1}{\sqrt{2} n \sigma}$$

ideal gas:

$$\ell = \frac{RT}{\sqrt{2} \pi d^2 N_\mathrm{A}\, p}$$

empirical law for real cryogenic gases:

$$\ell = 2.87 \times 10^{-3} \frac{T^{j+1}}{p}$$

[K]

[Pa]

[cm]

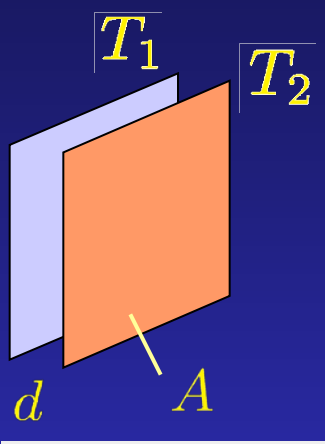
Helium: $j + 1 = 1.147$

Mean Free Path Depending on Pressure (Ideal Gas Law)

Vacuum range	*p* [mbar]	Molecules / cm^3	mean free path
Ambient pressure	1013	2.7×10^{19}..	68 nm
Low vacuum	300..1	$10^{19}..10^{16}$	0.1 ... 100 µm
Medium vacuum	$1..10^{-3}$	$10^{16}..10^{13}$	0.1 ... 100 mm
High vacuum	$10^{-3}..10^{-7}$	$10^{13}..10^{9}$	10 cm ...1 km
Ultra high vacuum	$10^{-7}..10^{-12}$	$10^{9}..10^{4}$	1 km ... 10^5 km
Extremely high vacuum	$<10^{-12}$	$<10^{4}$	$> 10^5$ km

Hydrodynamic Regime $\ell \ll d$

thermal conductivity independent of pressure: $\lambda \propto \ell n\,, \quad \ell \propto \frac{1}{n}$

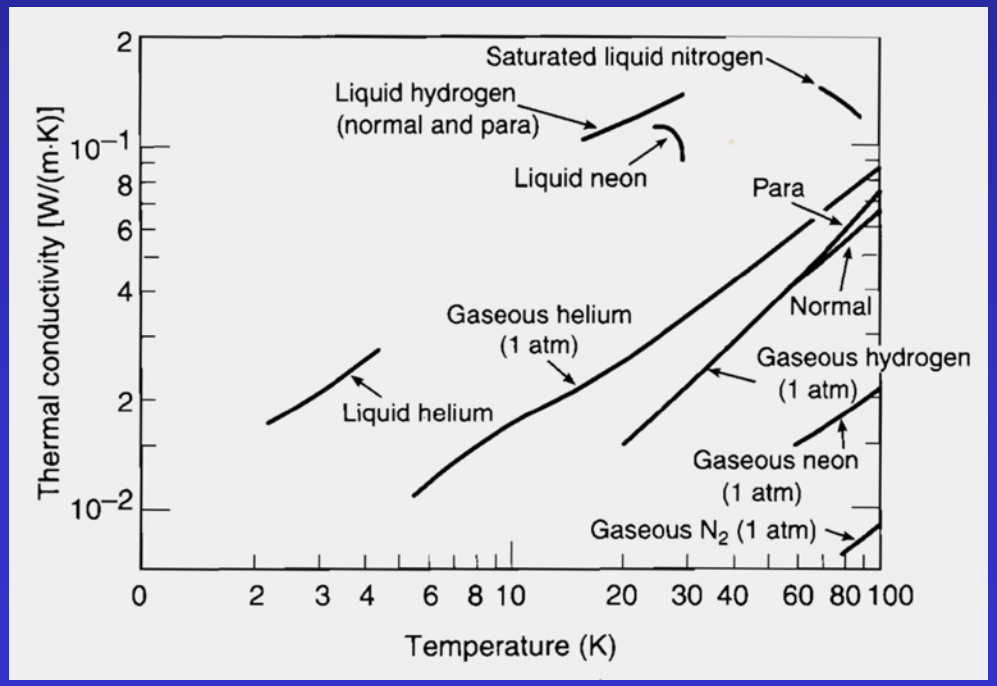


$$\dot{Q}_{\text{gas}} = \frac{\bar{\lambda}\, A\,(T_1 - T_2)}{d} = \frac{\bar{\lambda}\, A\, \Delta T}{d}$$

mean thermal conductivity

for gases: $\lambda = c\, C_V$

constant 1.5 … 2.5



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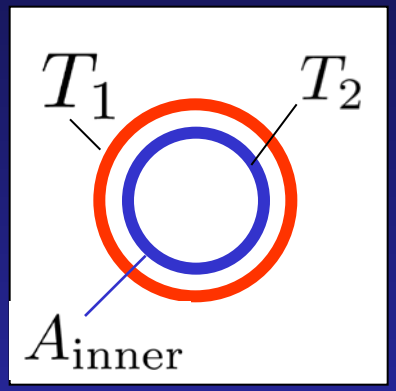
Knudsen Regime $\ell \gg d$



material dependent constant

air	1.2
helium	2.1
hydrogen	4.4

$$\dot{Q}_{\text{gas}} = k\, a_0\, p\; A_{\text{inner}} \Delta T [W]$$

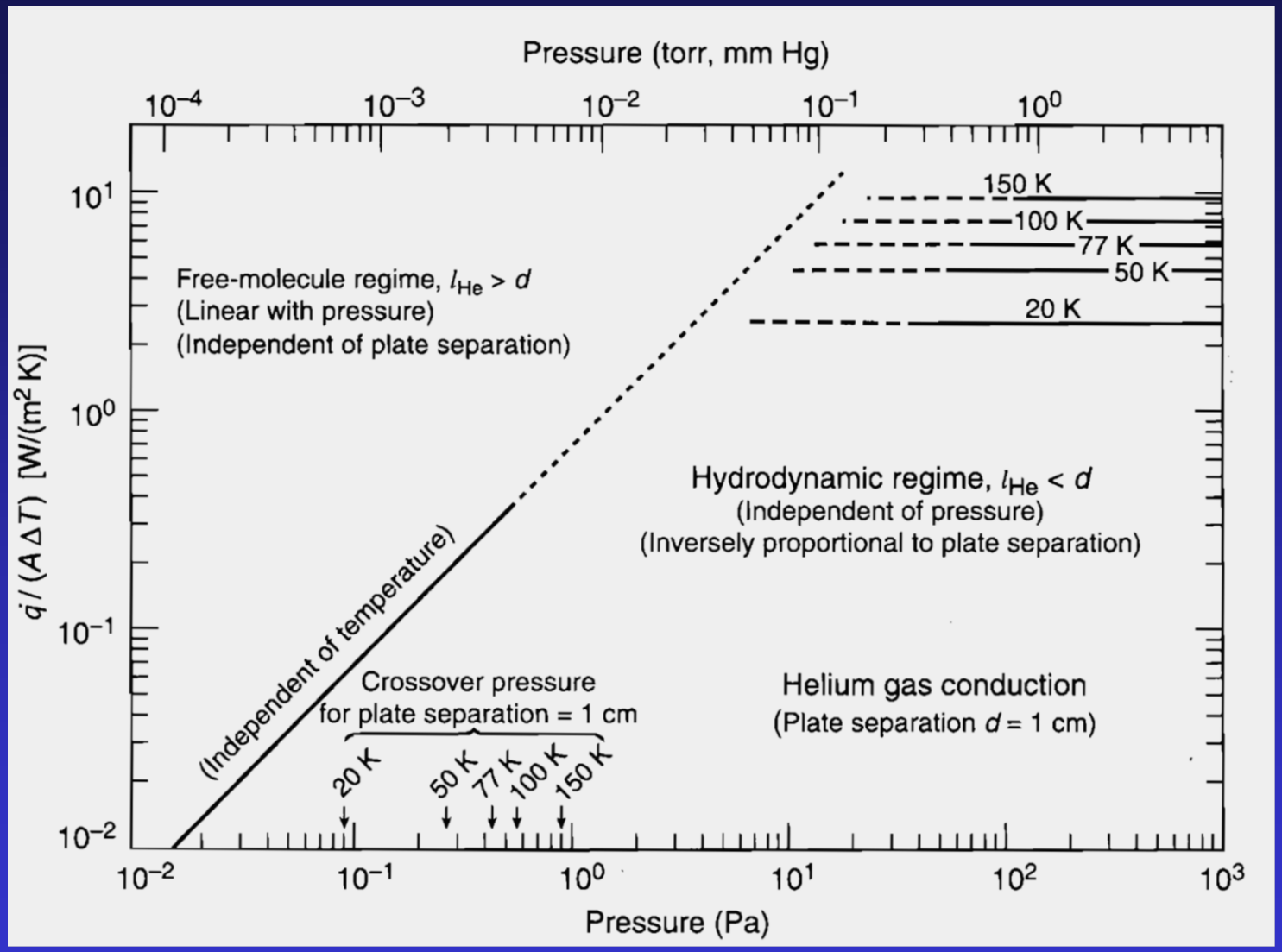
[10^{-2} mBar] [cm^2] [K]

$$a_0 = \frac{a_1 a_2}{[a_2 + (A_1/A_2)(1-a_2)a_1]}$$

$$a_1 \approx a_2 \approx 0.5 \quad \rightarrow \quad a_0 = \frac{0.5}{1 + (A_1/A_2)0.5}$$

parallel plates $\quad A_1 = A_2 \quad \rightarrow \quad a_0 = 0.33$

He Gas



Ekin 2005

Radiative Heat Transfer

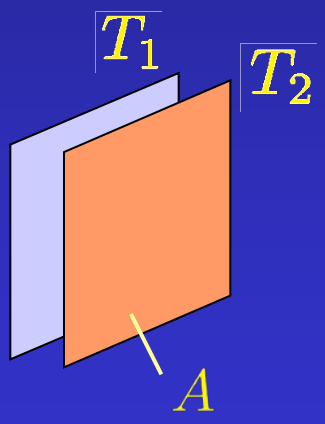
radiative heat flow from a body with surface A

$$\dot{Q}_{\mathrm{rad}} = \sigma\, \varepsilon\, A\, T^4$$

Stefan-Boltzmann Law

emissivity 0.01 ... 1

$$\sigma = 5.67 \times 10^{-8}\, [\mathrm{W}/(\mathrm{m}^2\mathrm{K}^4)]$$



$$\dot{Q}_{\mathrm{rad}} = \sigma\, \mathcal{E}\, A\, \left(T_2^4 - T_1^4\right)$$

Wien's law $\lambda T = 2900$ [µm K]

wave lenght	300 K	0.01 mm
	3 K	1 mm

Radiation Shields – silver coating

Dewar flask

Radiation Shields - Superinsulation

multiple radiation shields → smaller steps → reduction of heat flow

30 to 80 layers of low conductivity
high reflection material → aluminized Mylar

apparent thermal conductivity
~ 10^{-4} to 10^{-5} W/(m K)

A few Numbers

twisted pair of copper wire: diameter 125 µm, 1,5 m long

heat transfer between 300 K to 4 K: 0.14 mW

stainless steel tube: diameter 2 cm, wall thickness 0.4 mm, 1,5 m long

heat transfer between 300 K to 4 K: 51 mW

radiation: two plates with 30 cm^2 surface area

heat transfer between 300 K to 4 K: 26 W

exchange gas: two plates with 30 cm^2 surface area, 1 mBar, 1 cm

heat transfer between 300 K to 4 K: 45 W

Thermal Anchoring:
Samples, Sample Holders, Wires and Cables

Welding

Soldering

Varnisch and glue joints

Press contacts

Exchange gas

Metal to Metal Contact: Welding

Oxygen-acetylene welding	very hot flame, porous joint, surface oxidation Large surface area for trapping gas
Electric-arc welding	conducts electric current to heat, same problems as oxygen-acetylene welding → inert gas welding
Laser-beam welding	similar to electric are welding, heat sourse laser beam
Electric resistance welding	Joule heating melts the metal locally usually restricted to small areas: spot welding
Pressure welding	soft metals, very cleans surfaces, very high pressures, cold welding
Friction welding	used for joining dissimilar metals: Steel and Al, …

Solder Joints

Hard solders: silver solder ~ 700 °C

Soft solders: lead and tin based solders 200 to 400 °C

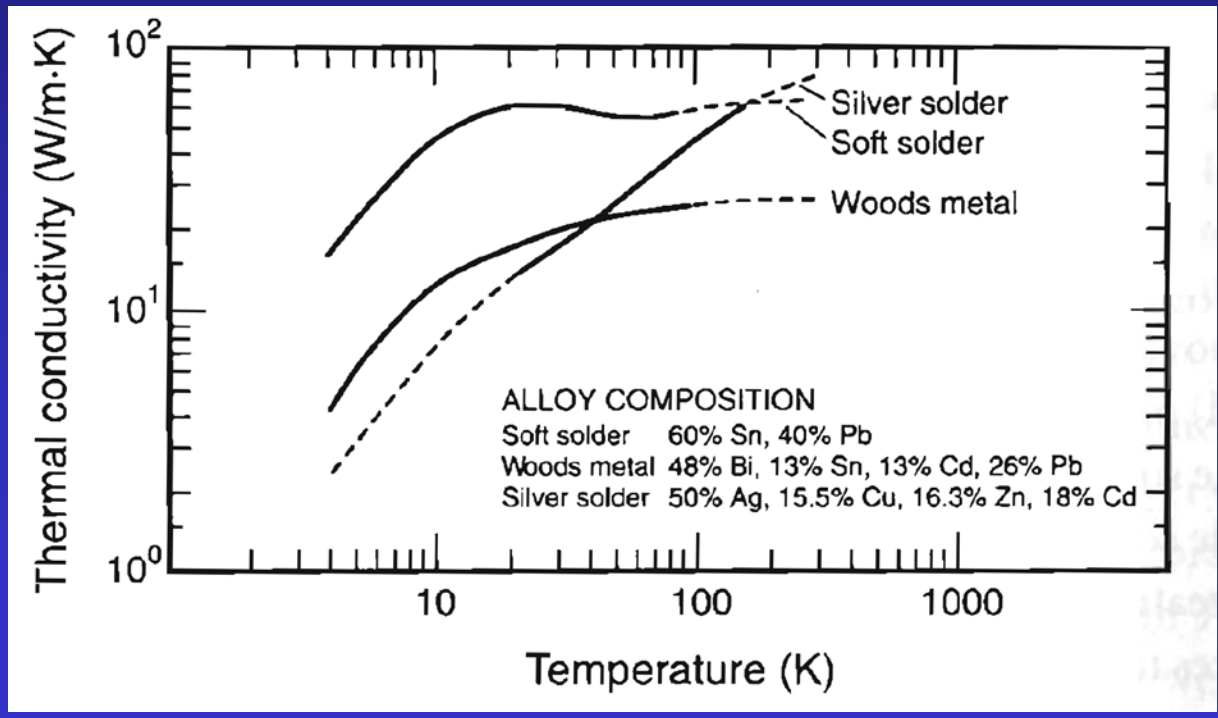
Very low temperature solders: Bismuth based below 100 °C

general problem at very low temperatures:

solder Joints become Superconductive

use the right flux

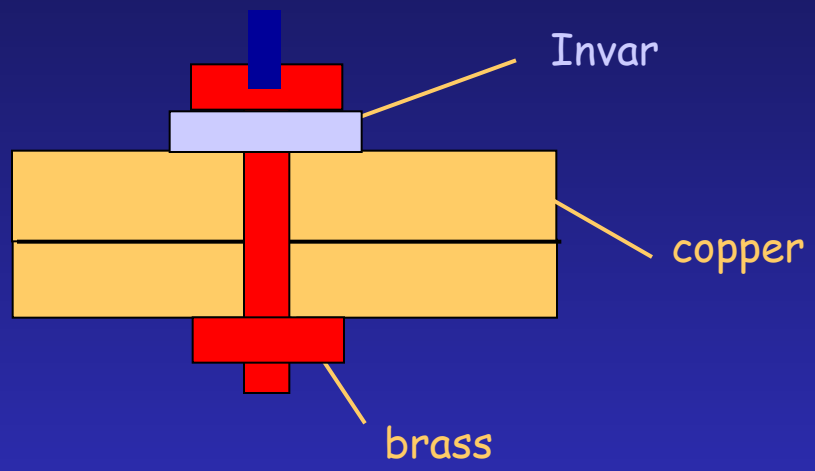
acid flux can be dangerous by creating pin holes



Press Contacts

Thermal conductance increase roughly linearly with pressure

Gold plating before bolting together



Look at thermal expansion

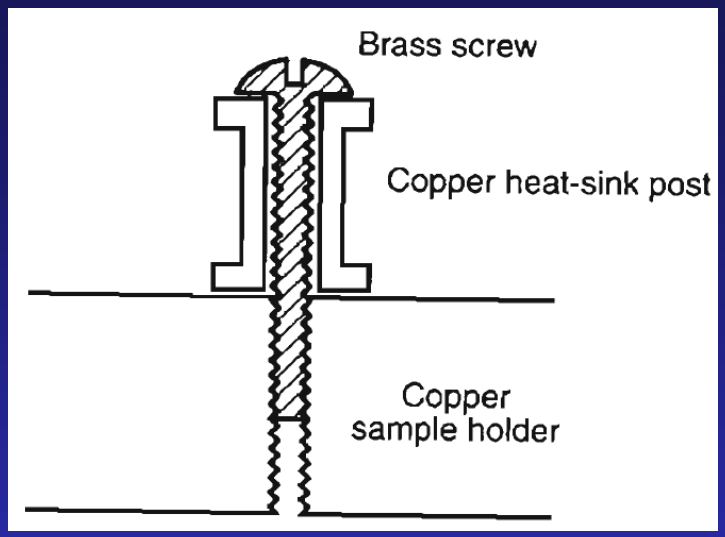
Empirical formula

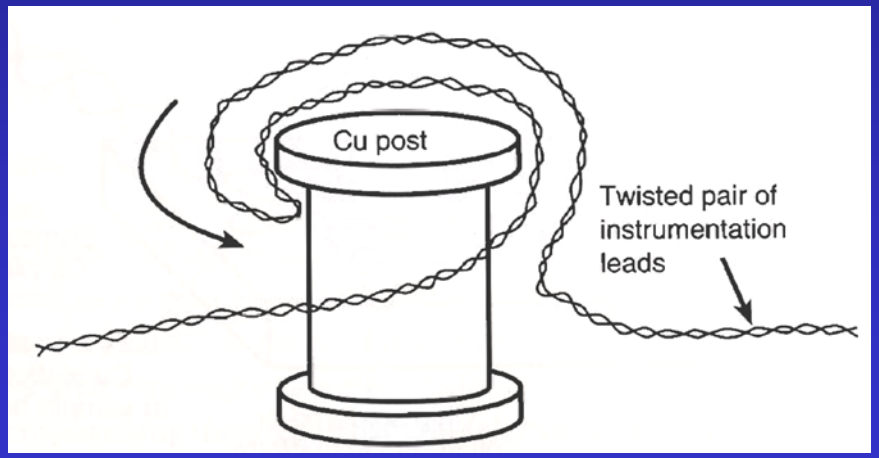
$$\dot{Q}(T) = \dot{Q}(445\,\mathrm{N}, 4\,\mathrm{K}) \left(\frac{F}{445N}\right) \left(\frac{T}{4.2\mathrm{K}}\right)^{\gamma}$$

To grease or not to grease?

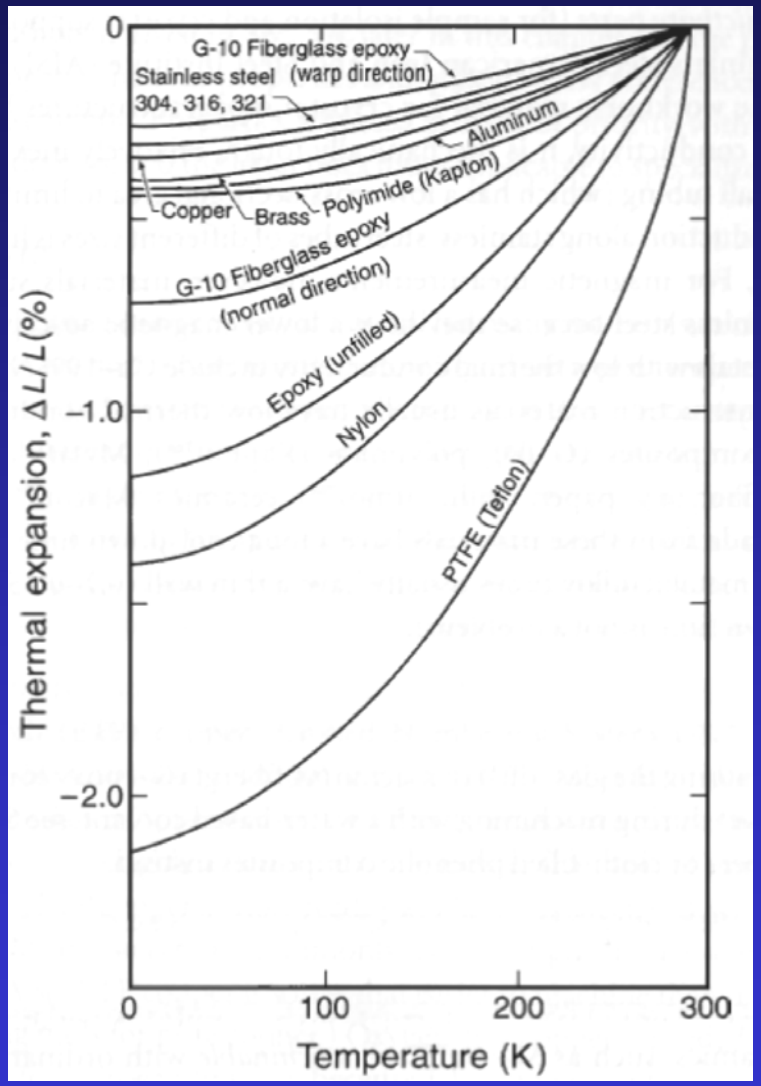
Large area and low pressure → grease (Apiezon N)

Thermal Anchoring of Wires and Cables

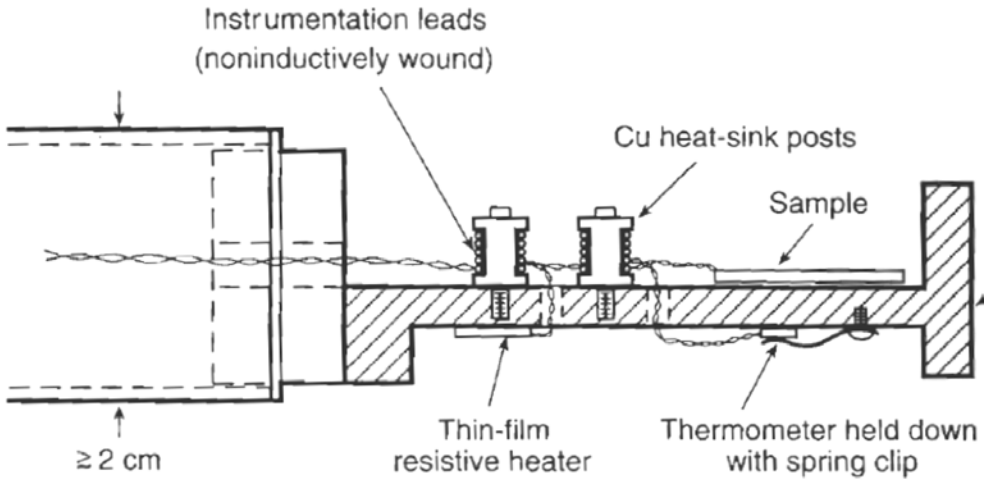


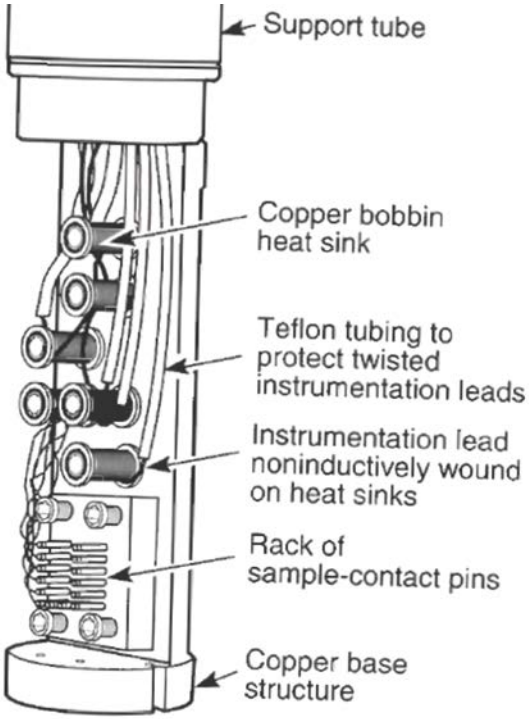


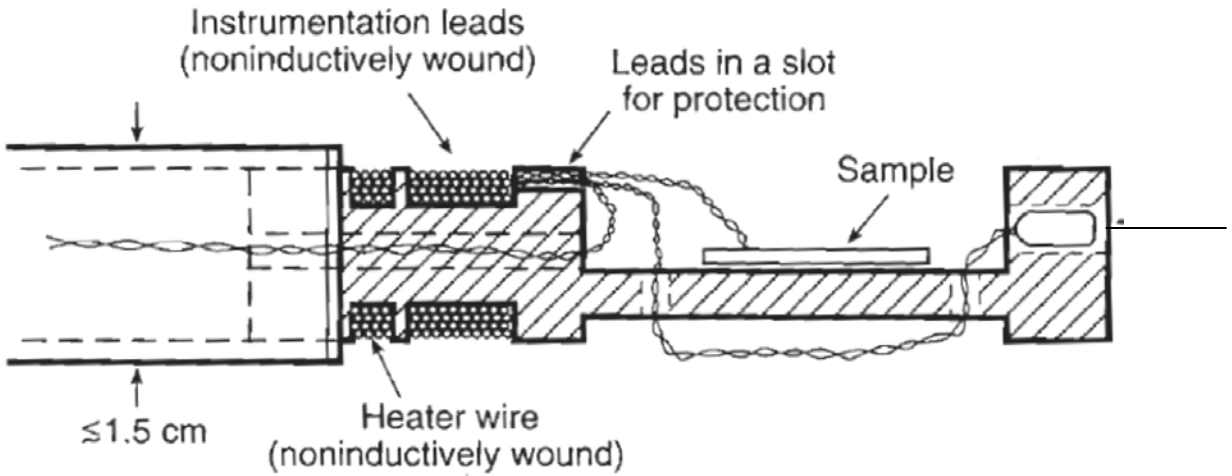
Non-inductactive wire anchoring



Thermal Anchoring of Wires and Cables



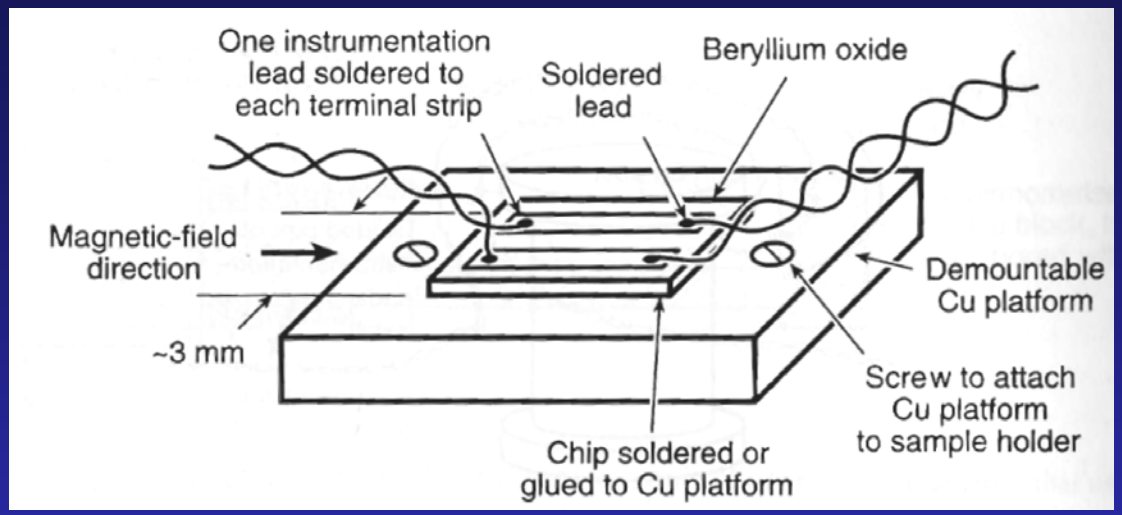




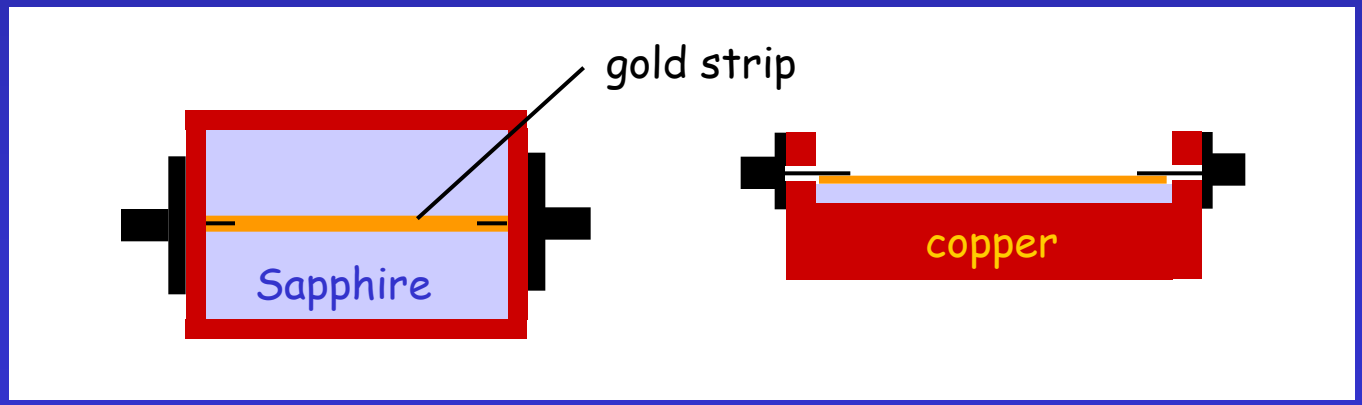
Thermometer attached
with grease

Thermal Anchoring of Wires and Cables

Heat sink of wires

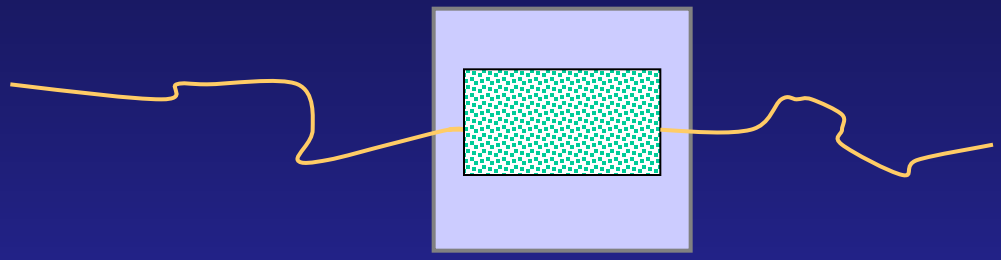


Heat sinking coaxial cables for high frequencies: Mircostripline

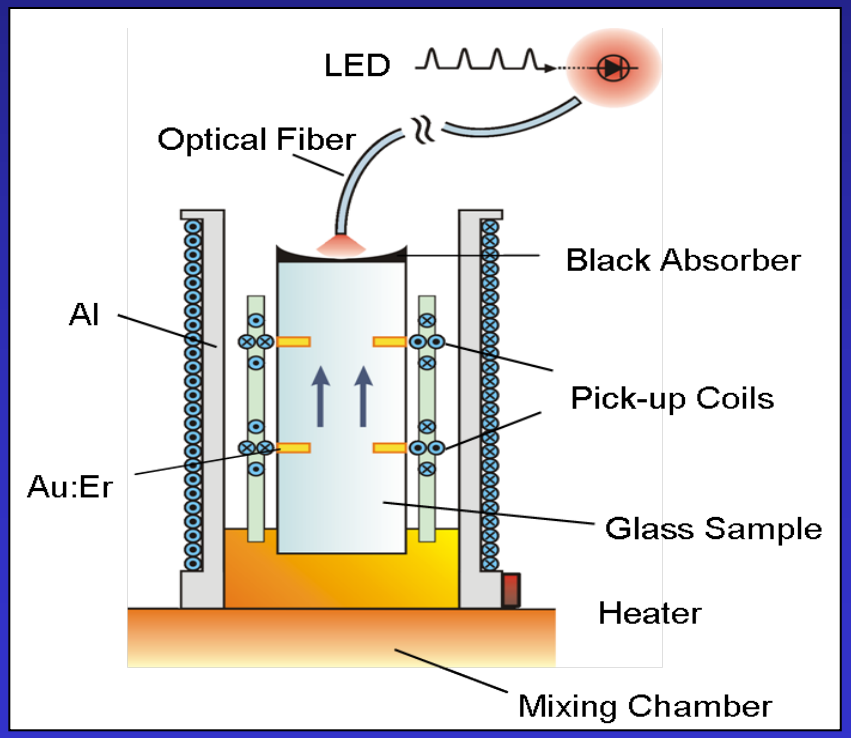


Ultra low temperature (T < 10 mK)

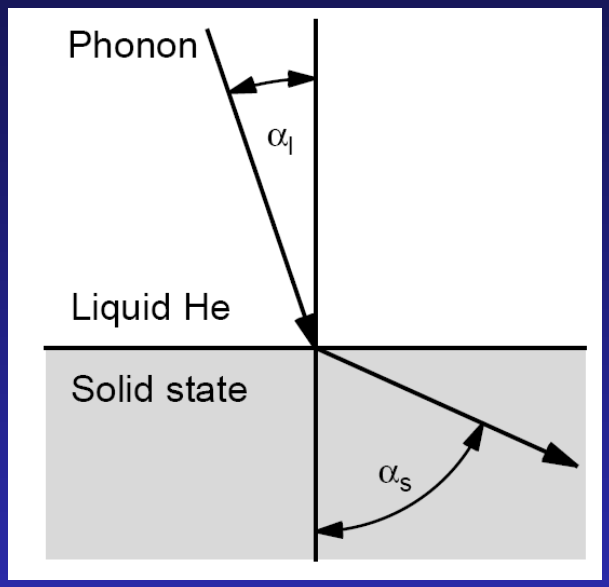
Heat sink of wires by using silver sinter and ^{3}He cells



Go to non contact measurements if possible



Thermal Boundary or Kapitza-Resistance



Kaptiza-Resistance occurs at any solid-solid, liquid-solid interface

Partciular problematic for liquid helium because of the low sound velocity

Snell's law of refraction

$$\frac{\sin \alpha_\ell}{\sin \alpha_s} = \frac{v_\ell}{v_s}$$

critical angle of total reflection

$$\alpha_\ell^c = \arcsin\left(\frac{v_\ell}{v_s}\right)$$

for lq. helium and copper $\alpha_\ell^c = 4°$

fraction of phonons incident within critical angle

$$f = \frac{1}{2}\sin^2 \alpha_\ell^c = \frac{1}{2}\left(\frac{v_\ell}{v_s}\right)^2 < 10^{-2}$$

Kapitza-Resistance

Transmission coefficient

$$t = \frac{4 Z_\ell Z_\mathrm{s}}{(Z_\ell + Z_\mathrm{s})^2} \simeq \frac{4 Z_\ell}{Z_\mathrm{s}} = \frac{4 \varrho_\ell v_\ell}{\varrho_\mathrm{s} v_\mathrm{s}}$$

Acoustic impedances

$$Z_\ell = \varrho_\ell v_\ell$$
$$Z_\mathrm{s} = \varrho_\mathrm{s} v_\mathrm{s}$$

fraction of phonons crossing the interface

$$\boxed{f t = \frac{2 \varrho_\ell v_\ell^3}{\varrho_\mathrm{s} v_\mathrm{s}^3}}$$

helium-copper
$f t < 10^{-5}$

Heat transfer (using Debye model)

$$\dot{Q} = \frac{\mathrm{d}\dot{\mathcal{Q}}}{\mathrm{d}T} \Delta T = \frac{2\pi^2 k_\mathrm{B}^4 \varrho_\ell v_\ell}{15 \hbar^3 \varrho_\mathrm{s} v_\mathrm{s}^3} A T^3 \Delta T$$

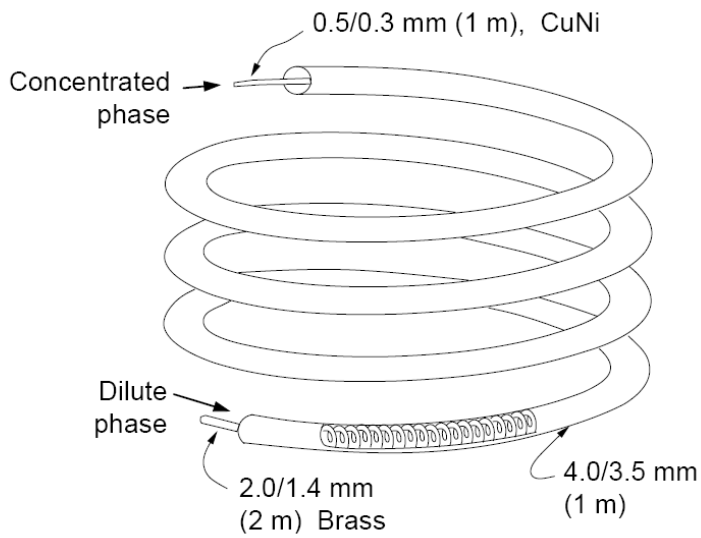
$$\dot{\mathcal{Q}} = \frac{1}{2} f t u v_\ell A = \frac{\pi^2 k_\mathrm{B}^4 \varrho_\ell v_\ell}{30 \hbar^3 \varrho_\mathrm{s} v_\mathrm{s}^3} A T^4$$

Kapitza Resistance

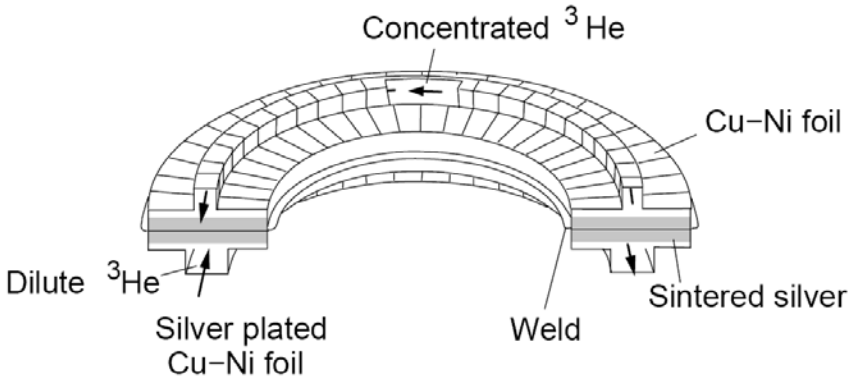
$$\boxed{R_\mathrm{K} = \frac{A \Delta T}{\dot{Q}} = \frac{15 \hbar^3 \varrho_\mathrm{s} v_\mathrm{s}^3}{2\pi^2 {k_\mathrm{B}}^4 \varrho_\ell v_\ell} \frac{1}{T^3}}$$

What can we do to fight the problem?

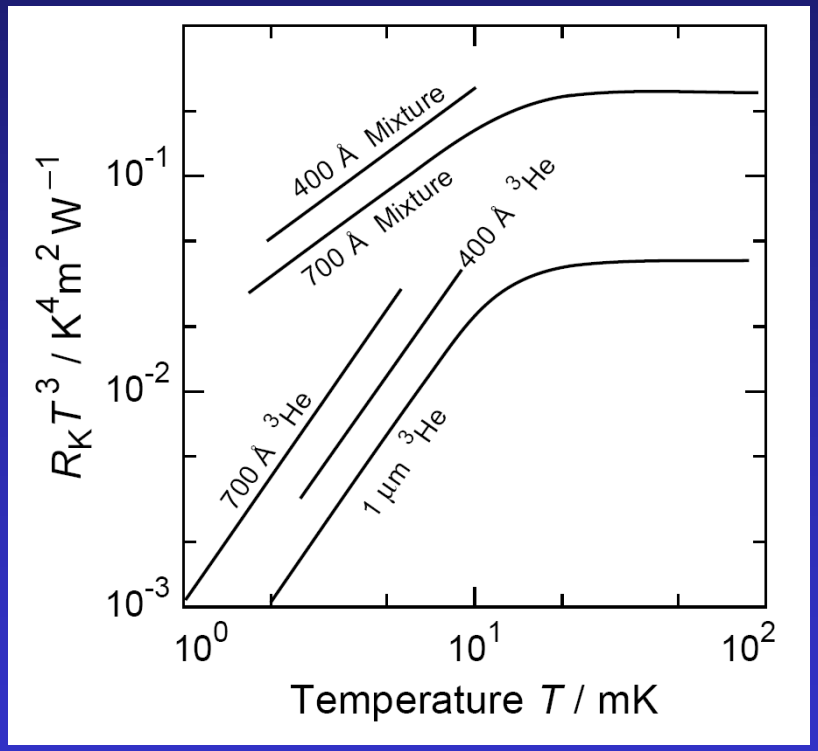
continuous heat exchanger



step heat exchanger

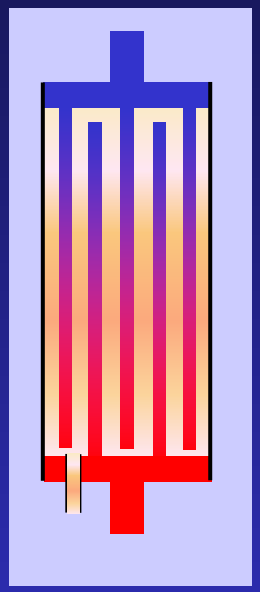


Data for Kapitza Resistance



Heat Switches

Gas heat switches



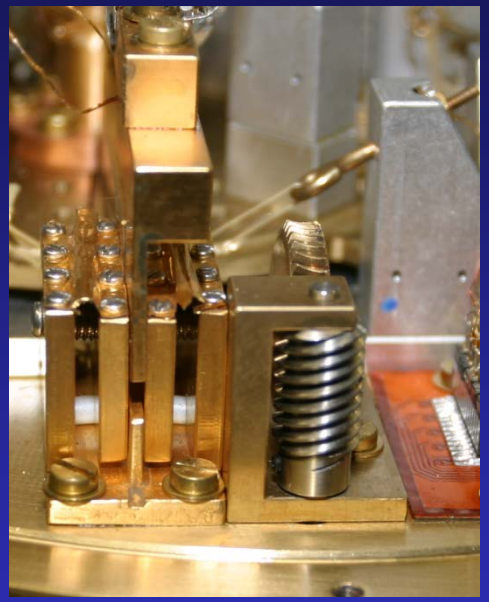
exchange gas → pumping

^{4}He superfluid layer
→ creep

H_2 ortho-para
conversion

^{3}He no exothermic reation
no creep
high vapour pressure

Mechanical heat switch

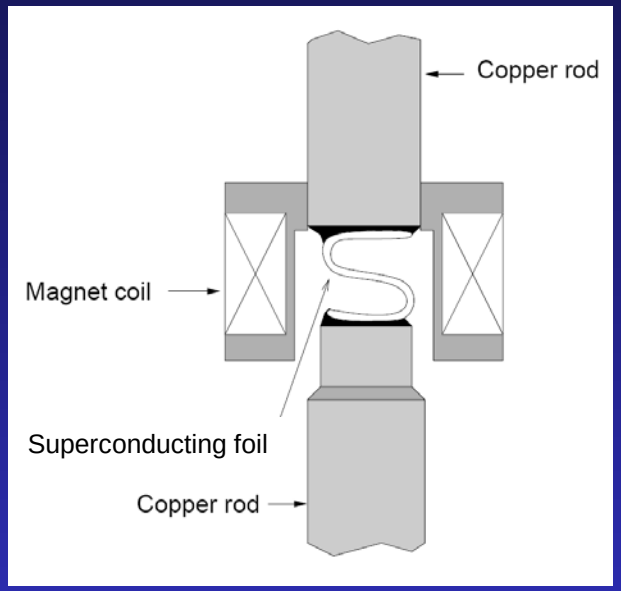


large force needed
typically 100 N

closed few mW/K
... 1 W/K @ 15K

heating on opening

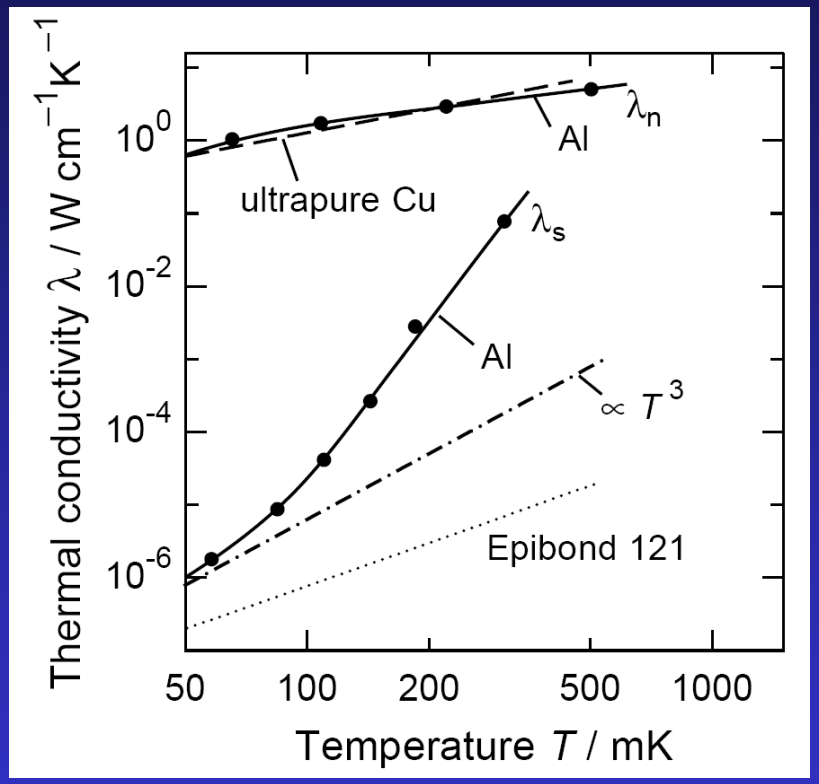
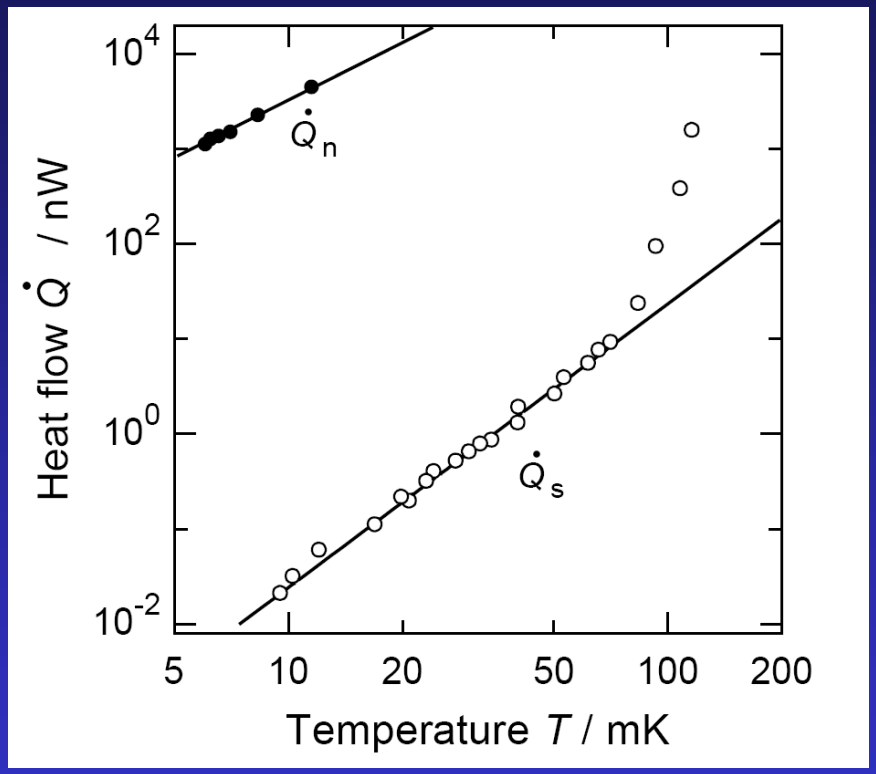
Superconducting heat switch



only well below T_c

open means low conductivity

eddy currents

flux trapping

Superconducting Heat Switch