THERMAL CONDUCTIVITY

Aims :

- understand the difference metals/insulators
- understand the importance for low temperature physics and techniques
- notions on the measurement

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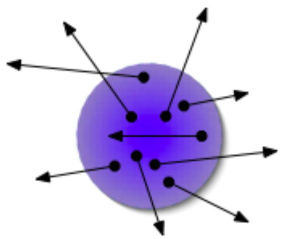
Outline :

1. Physical meaning
   - Transport of heat/Transport of charge
   - Kinetic formula
2. Various contributions, orders of magnitude
   - Insulators (crystals/amorphous materials)
   - Metals (Wiedemann-Franz law)
   - Superconductors
3. Measurement
   - One/Two thermometer-heaters
4. Use for low temperature technics.
   - Refrigerator design : case of large gradients
   - Thermal switch
5. Physics: Comparison Thermal Conductivity/Specific heat
   - Advantage/limitation of a thermodynamic probe
   - Advantage/limitation of a transport probe
   - Example

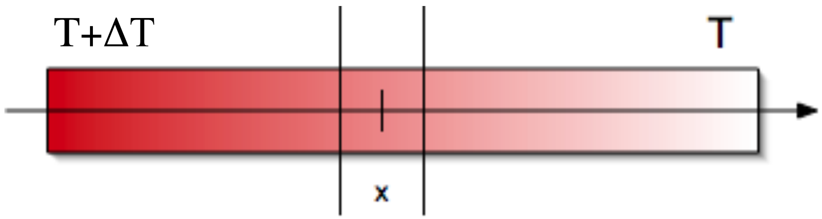
Transport of charge: all carriers have the same charge (to within a sign)



$$j_e = \frac{1}{dV}\sum_i ev_i = e\rho\bar{v} = -\sigma\nabla V$$

$$j_q = \frac{1}{dV}\sum_i \varepsilon_i v_i = -\kappa\nabla T$$

Transport of heat : in a material with a thermal gradient, the carriers have more "thermal energy" on one side than on the other. And if no mass flow, we may have <v> = 0 (Thermoelectric effects in a metal…)



$$j_q = \frac{1}{2}\rho\big(\varepsilon(x-l) - \varepsilon(x+l)\big)\bar{v}_x = -\frac{1}{2}\rho\left(\frac{\partial\varepsilon}{\partial T}\frac{\partial T}{\partial x}2l\right)\bar{v}_x, \text{ but :}$$

$$\frac{\partial\rho\varepsilon}{\partial T} = c_v, \text{ so:}$$

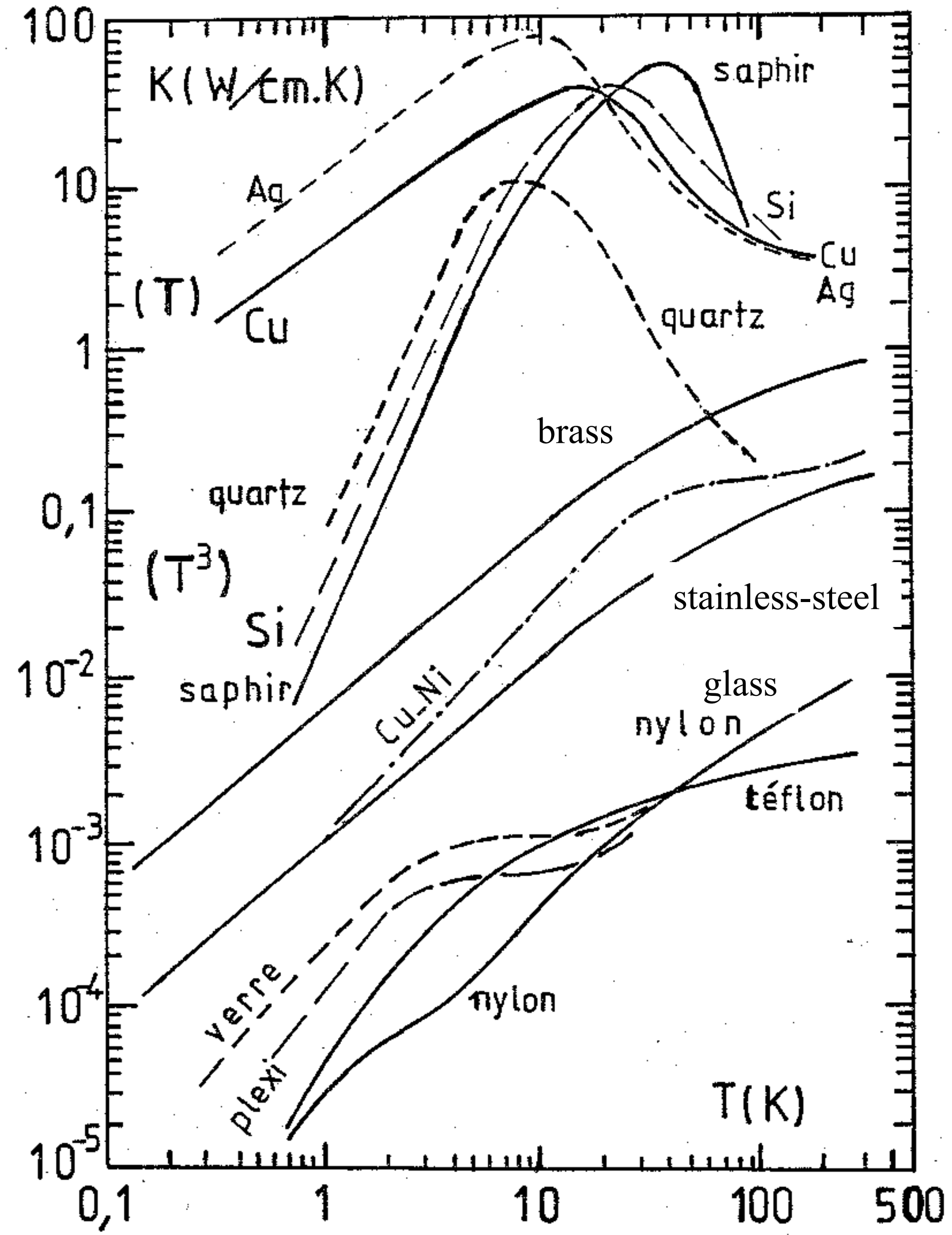
$$j_q = -c_v\bar{v}_x l\nabla T = -\kappa\nabla T, \text{ and in 3D, } \bar{v}_x = \frac{1}{3}\bar{v}$$

$$\kappa = \frac{1}{3}c_v\bar{v}l$$

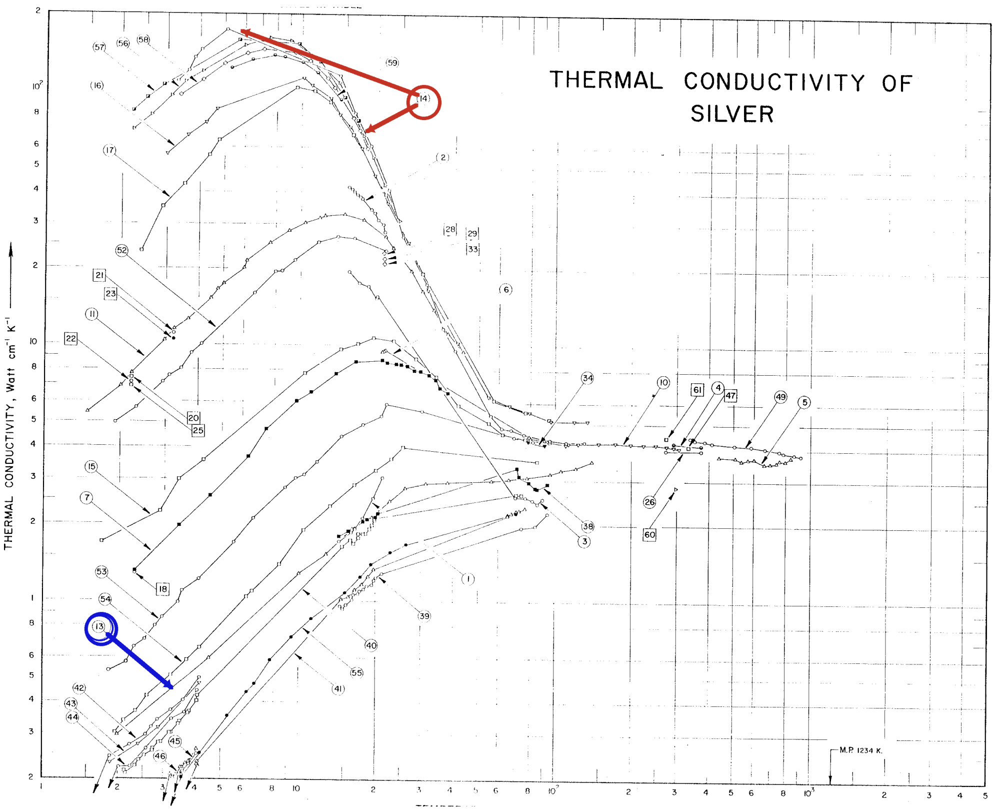
2. Various contributions, orders of magnitude:

$$\boxed{\kappa = \frac{1}{3} c_v \bar{v} l}$$

- metals conduct better than insulators (piece of wood/piece of cupper)
- pure metals conduct better than alloys
- annealing, amorphous/crystalline matters a lot…



2. Various contributions, orders of magnitude:



At low T, 3 orders of magnitude ≠for the same sample before/after annealing

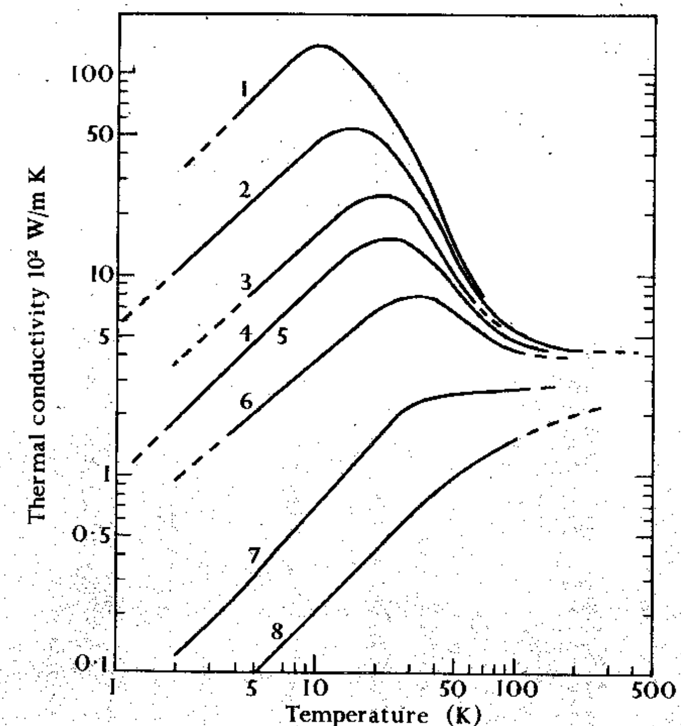
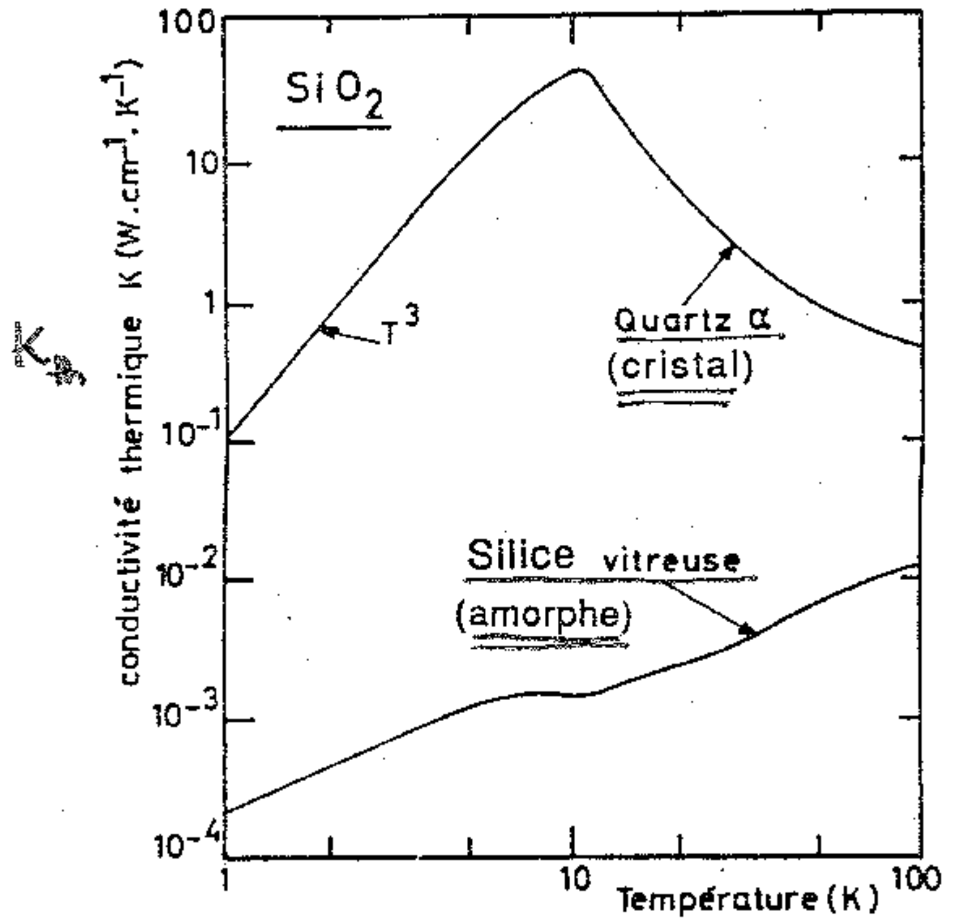


FIG. 11.4. Thermal conductivity of copper samples. 1, pure (99·999%, annealed, American Smelting and Refining); 2, pure (99·999%, annealed, Johnson Matthey); 3, coalesced (99·98%, oxygen-free annealed, Phelps Dodge); 4, pure (99·999%, cold worked, Johnson Matthey); 5, electrolytic tough pitch (99·9 +%, representing some tubes, much sheet and plate); 6, free-cutting tellurium (99% + 0·6% Te, representing machining rods and bar); 7, pure Cu + 0·056% Fe (annealed); 8, phosphorus deoxidized (99·8% + 0·1% P, representing some tubes, pipe, sheet, and plate). Curves 1, 3, 5, 6, and 8 from Powell, Rogers, and Roder (1957); curves 2 and 4 from White (1953), and curve 7 from White and Woods (1955).

2. Various contributions, orders of magnitude:

More than 4 orders of magnitude ≠ for SiO_2, whether crystalline or amorphous



2. Various contributions, orders of magnitude: Insulators

For specific heat: contribution of phonons, spins, cristal field… $\kappa = \frac{1}{3} c_v \bar{v} l$

For thermal conductivity: *transport* of heat => only phonons are thermal carriers. But beware of the collision time (or mean free path)…

• At high temperature, c_v is constant, <v> is of order the sound velocity (constant) but the mean free path becomes very short :

– high phonon density (number proportional to T)

– anharmonicity allows for Umklapp processes : strong phonon-phonon diffusion

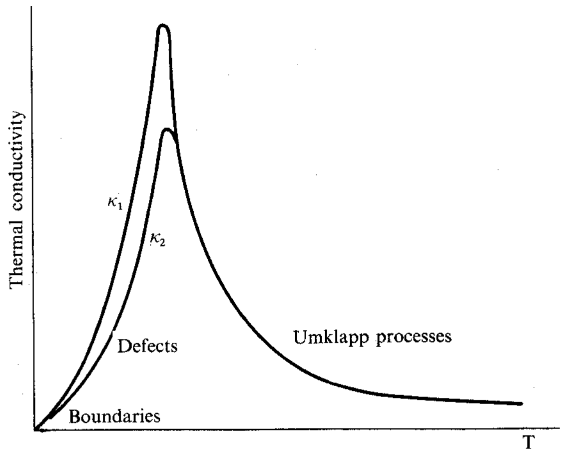
As a result $\kappa \sim T^{-x}$, with x in between 1 and 2

• At low temperature, the m.f.p is limited by defects, dislocations, surface… $\kappa \sim T^3$:

– it can be low for amorphous materials,

– huge and depend on the sample size for high purity-near perfect crystals…

• At some intermediate temperature, there is a maximum



Two parallel channels for heat conduction : phonons and electrons

$$\kappa = \kappa_e + \kappa_{ph}$$

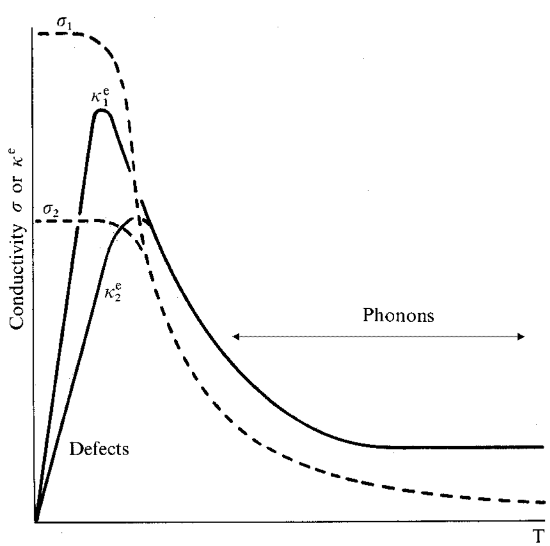
$$\boxed{\kappa = \frac{1}{3} c_v \bar{v} l}$$

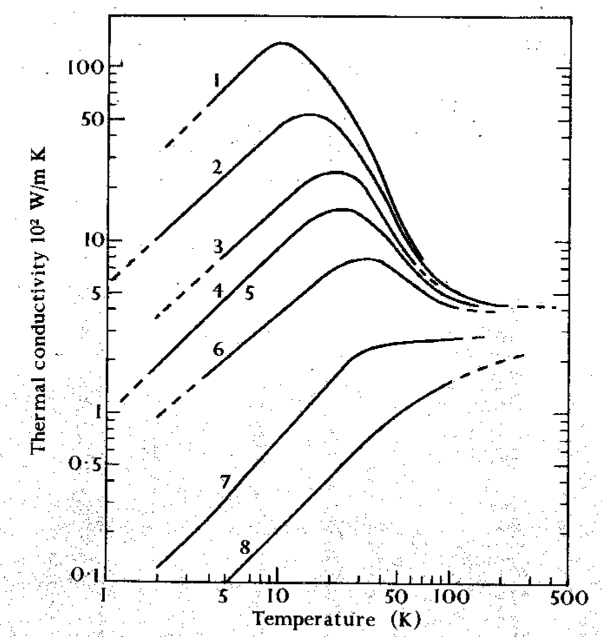
For the phonon contribution to thermal conductivity, electrons add a new scattering channel

- At high temperature : no change, still governed by phonon-phonon interactions
- At low T, controlled by collisions on electrons : $l \sim T^{-1}$, so $\kappa_{ph} \sim T^2$

For the electron contribution to thermal conductivity, $C_v = \gamma T$, v_F=cte:

- At high temperature : $l \sim T^{-1}$ governed by electron-phonon interactions =>$\kappa_e \sim$cte
- At low temperature, l is limited by defects, impurities… , l~cte, => $\kappa_e \sim T$
  - huge dependence on purity and annealing





In metals : $\kappa = \kappa_e + \kappa_{ph}$

$$j_e = \frac{1}{dV}\sum_i e v_i = e\rho\bar{v} = -\sigma\nabla V$$

$$j_q = \frac{1}{dV}\sum_i \varepsilon_i v_i = -\kappa\nabla T$$

$$\boxed{\kappa = \frac{1}{3}c_v\bar{v}l}$$

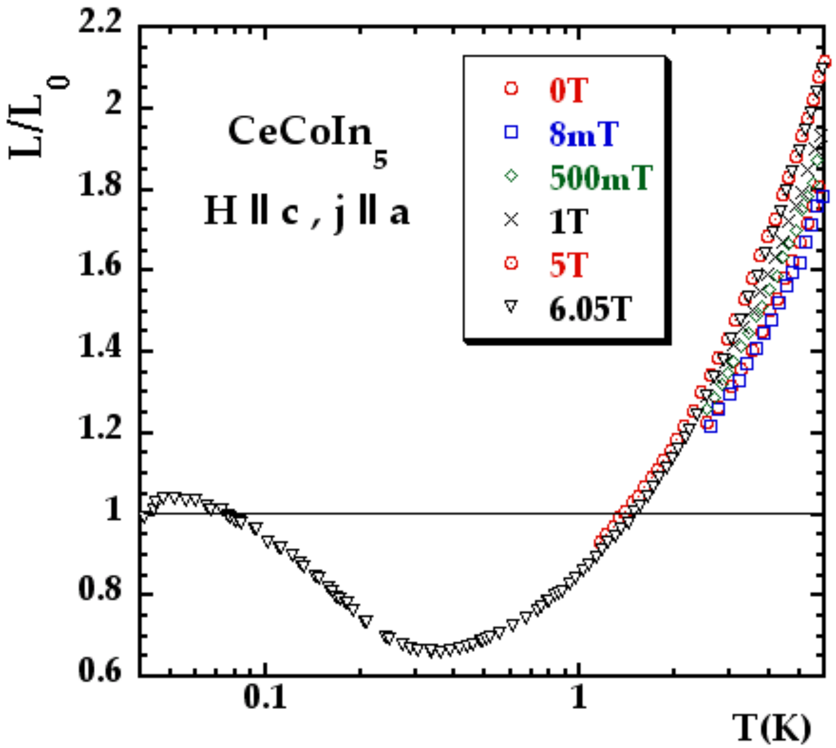
There is a simple relation between electric and thermal conductivity:

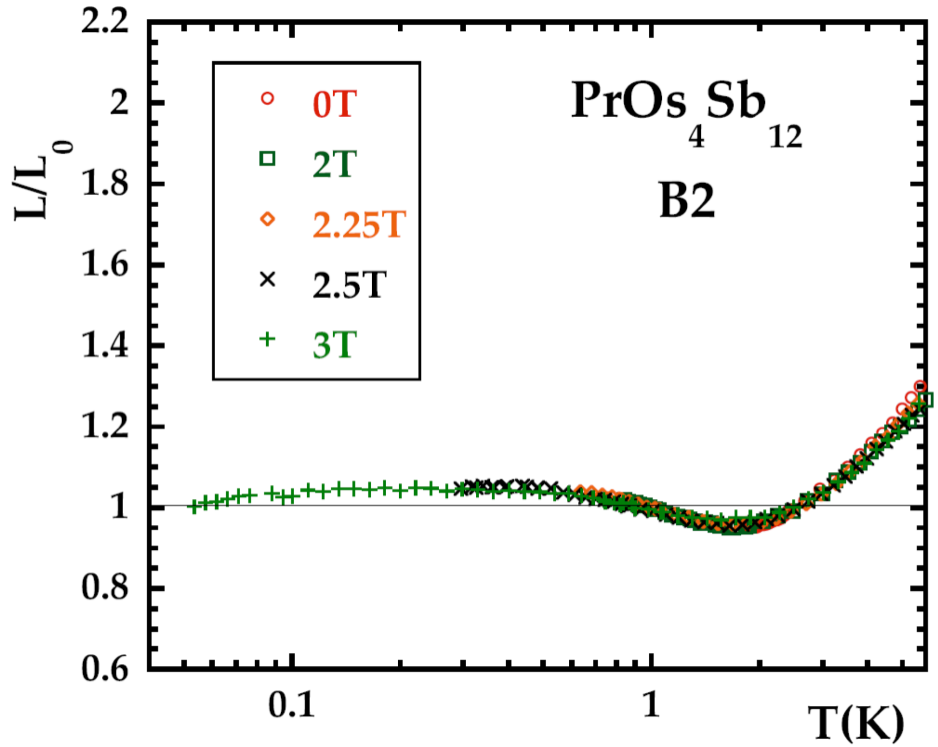
$$\left.\begin{aligned}
&\boldsymbol{\sigma} = \boldsymbol{\sigma}(\varepsilon_F),\ \boldsymbol{\kappa}_e = \boldsymbol{\kappa}_e(\varepsilon_F)\\
&\boldsymbol{\sigma}(\varepsilon) = e^2\tau(\varepsilon)\int \frac{d\mathbf{k}}{4\pi^3}\delta(\varepsilon-\varepsilon(\mathbf{k}))\mathbf{v}(\mathbf{k})\mathbf{v}(\mathbf{k})\\
&\boldsymbol{\kappa}_e(\varepsilon) = \frac{1}{e^2T}\int d\varepsilon\left(-\frac{\partial f}{\partial \varepsilon}\right)(\varepsilon-\mu)^2\boldsymbol{\sigma}(\varepsilon)
\end{aligned}\right\}\ \text{leading to}\ \left\{\begin{aligned}
&\boldsymbol{\kappa}_e = L_0T\boldsymbol{\sigma}\\
&\text{with } L_0 = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 = 2.44\times10^{-8}\ \text{W}\Omega/\text{K}^2
\end{aligned}\right.$$

L_0 is called the Lorentz number, and $\kappa_e = L_0 T\sigma$ is the Wiedemann-Franz law
The ratio $\kappa_e/(\sigma T)$ is often called L -> plots of L/L_0

It is valid and robust as long as the relaxation time is the same for $\boldsymbol{j_e}$ and $\boldsymbol{j_q}$...

Very useful to estimate κ_e of pure metals at very low temperature !





Wiedemann-Franz law: $\kappa_e = L_0 T\sigma$, but $L = (\kappa_e + \kappa_{ph} + \ldots)/(\sigma T)$

At high temperature, $L > L_0$ due to additional contributions to κ

At intermediate temperature, inelastic process may be more efficient to relax $\boldsymbol{j_q}$ than $\boldsymbol{j_e}$:

Wiedemann-Franz law: $\kappa_e = L_0 T \sigma$

Inelastic process may be more efficient to relax $\boldsymbol{j_q}$ than $\boldsymbol{j_e}$:

- at very low T, inelastic process are negligible compared to elastic process
- at high temperature, $\Delta\varepsilon$ during a collision may be negligible compared to kT
- at intermediate temperature, one may have $\kappa_e < L_0 T \sigma$

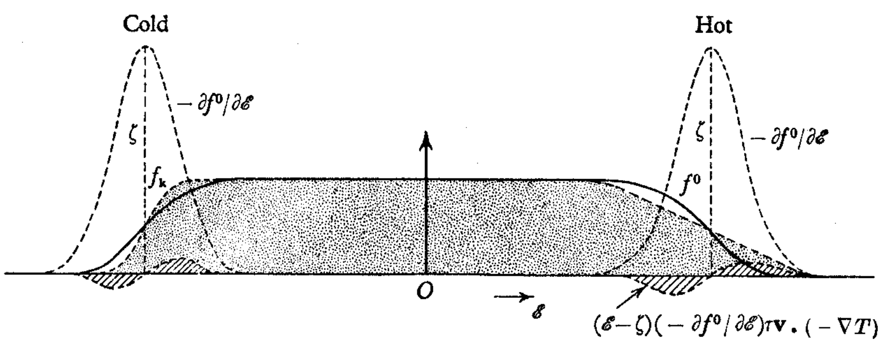


Fig. 126. Fermi distribution in thermal conductivity.

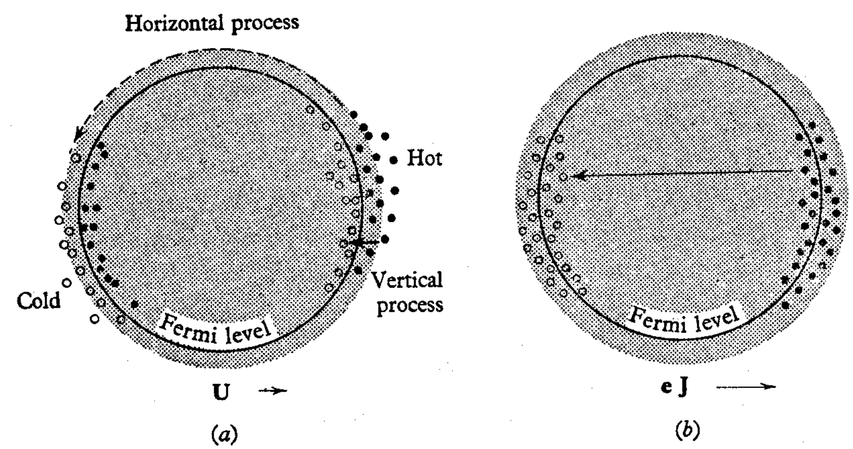


Fig. 127. Electron distributions, and scattering processes: (a) in thermal conduction; (b) in electrical conduction.

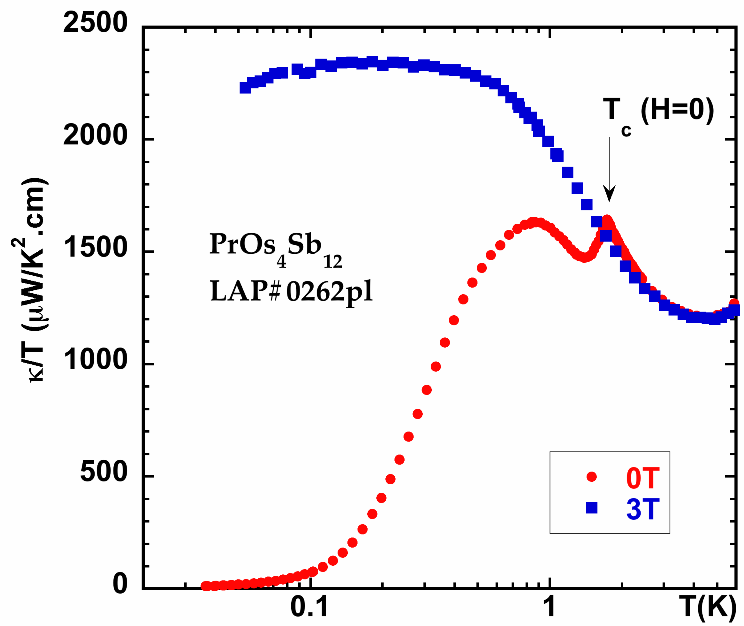
Wiedemann-Franz law: $\kappa_e = L_0 T \sigma$

For superconductors, below Tc:
- σ is infinite
- κ_e/T goes to zero as T goes to zero

complete violation of WF !

Superconductors, at temperatures << Tc behave like insulators ?

Reason : two fluid model.
– thermal conductivity: needs heat carriers => thermal excitations carrying entropy and heat (exist if T is defined)
– condensate of Cooper pairs with no entropy, short circuiting σ, no contribution to κ



3. Measurement: Two thermometers, one heater

$$\boxed{j_q = -\kappa \nabla T}$$

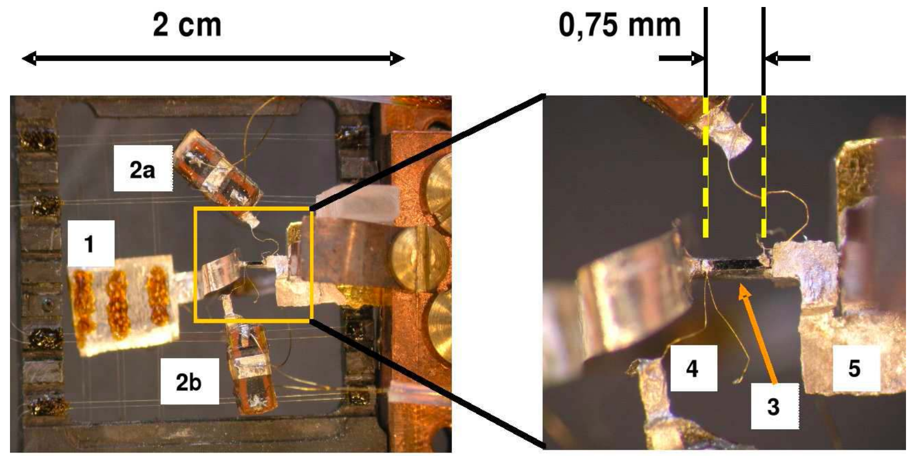
$$j_Q \times S = S \times \kappa \times \left(\frac{T_{hot} - T_{cold}}{l} \right)$$

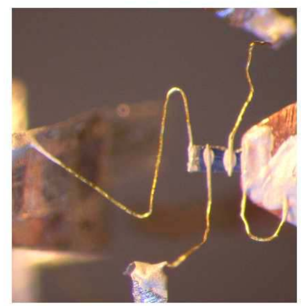
$$\boxed{\kappa = \left(\frac{l}{S} \right) \frac{P}{T_{hot} - T_{cold}}}$$

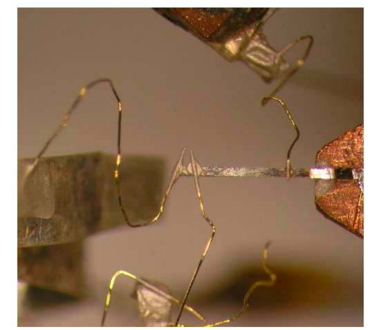
$$j_e \times S = S \times \sigma \times \left(\frac{V_1 - V_2}{l} \right)$$

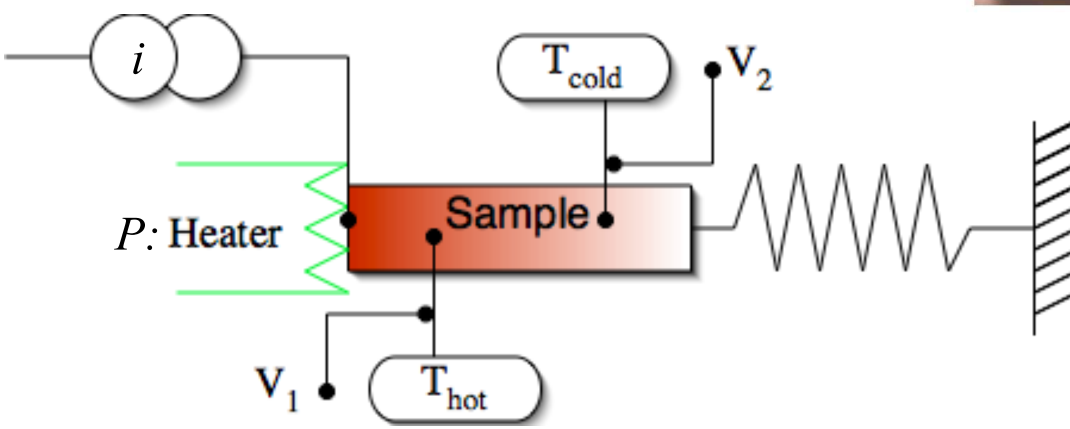
$$\sigma = \left(\frac{l}{S} \right) \frac{i}{V_1 - V_2}$$

$$\boxed{L = \frac{\kappa}{\sigma T} = \frac{P}{T_{hot} - T_{cold}} \times \frac{V_1 - V_2}{i} \times \frac{2}{T_{hot} + T_{cold}}}$$

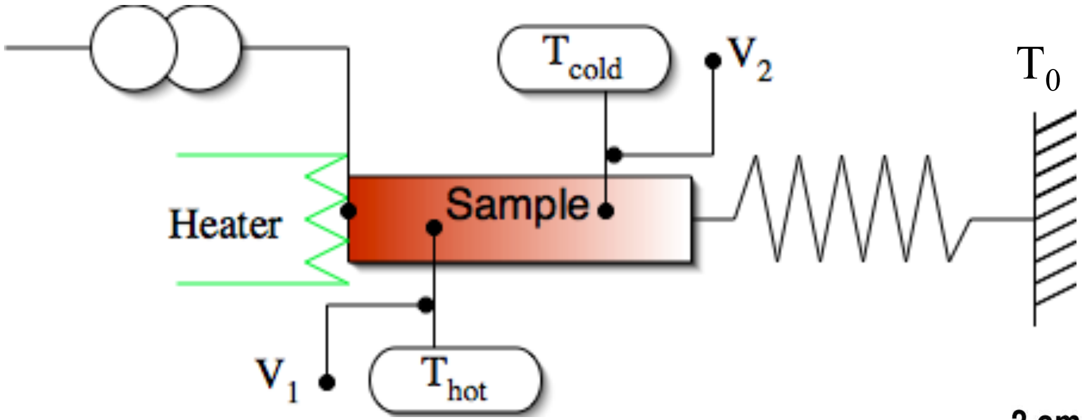




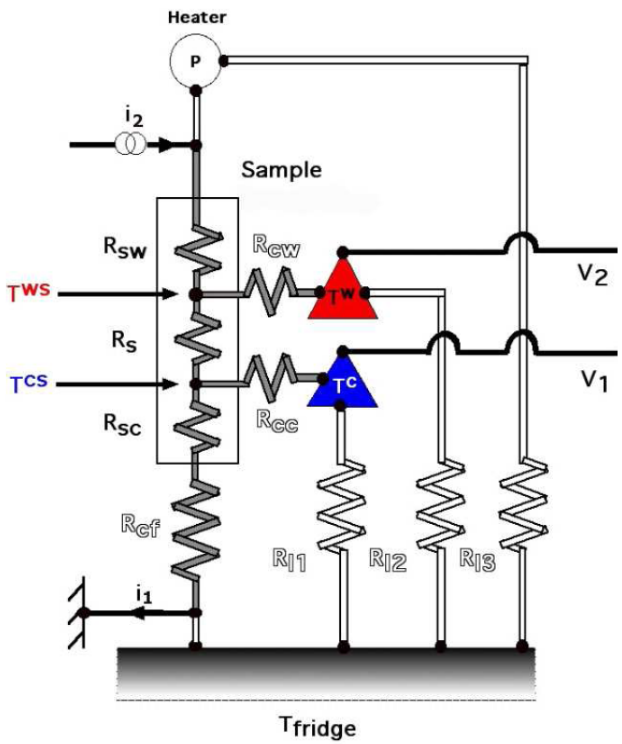


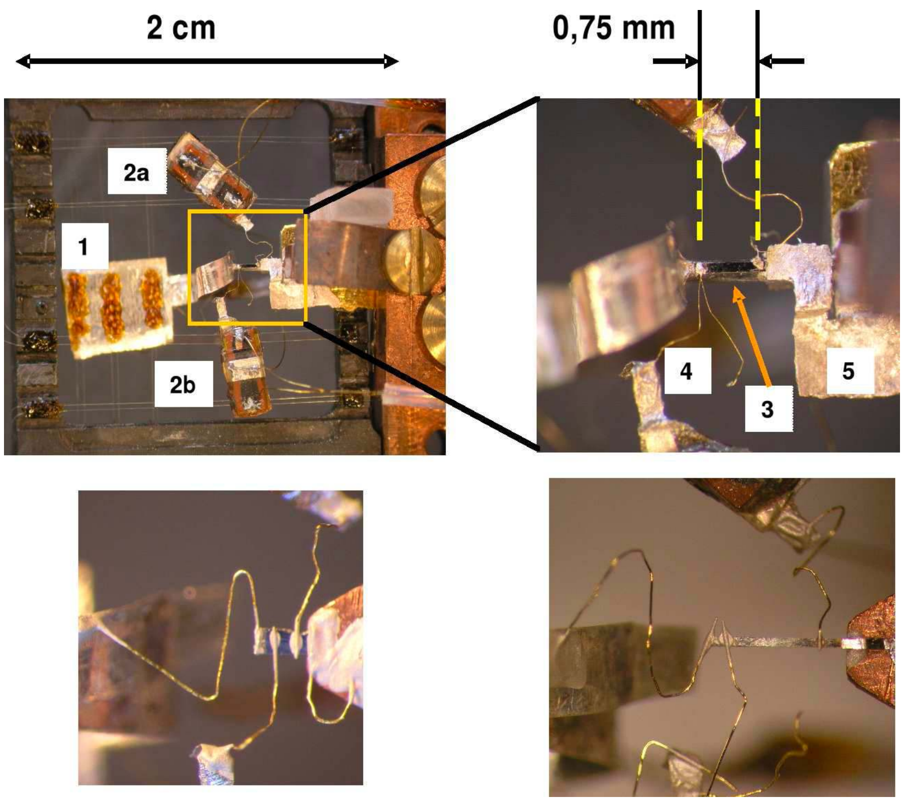


3. Measurement: Two thermometers, one heater: reality…



Problem if bad sample contact : difference of two big numbers (T_{hot}-T_0) and (T_{cold}-T_0)…



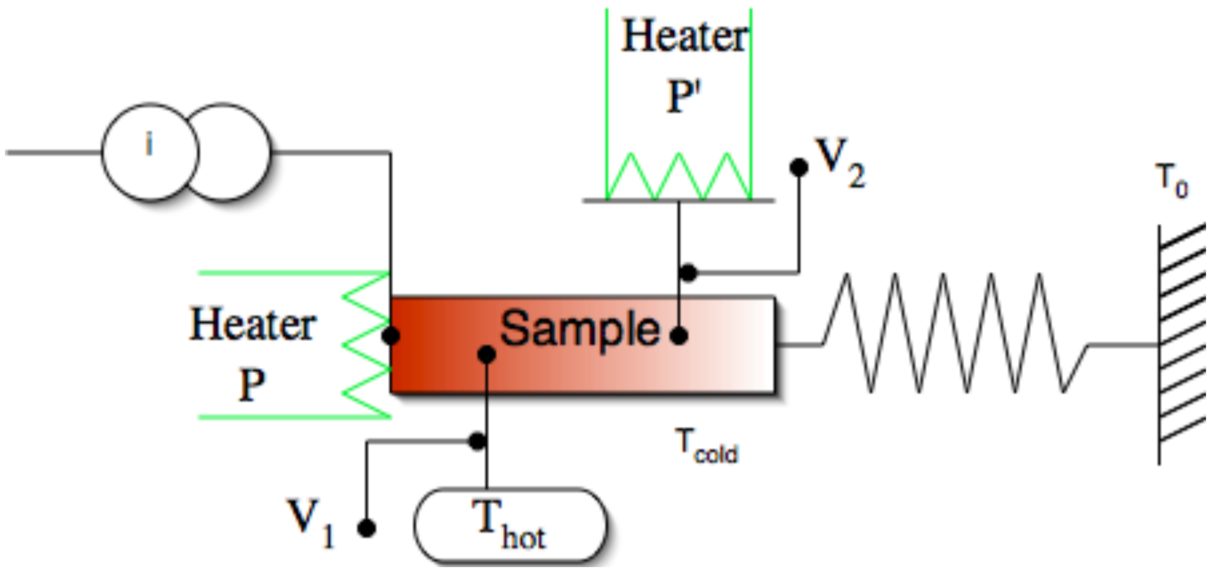


3. Measurement: Two thermometers, one heater

$$\Delta T_{hot} = \frac{P}{\kappa}\frac{l}{S} + P \times R_{contact}$$ where $R_{contact}$ is the thermal resistance between T_{cold} and T_0

$$\Delta T'_{hot} = P' \times R_{contact}$$ (send P' with P=0 to measure $R_{contact}$)

$$\Delta T_{hot} = \frac{P}{\kappa}\frac{l}{S} + P \times \frac{\Delta T'_{hot}}{P'}$$



In a refrigerator, there are large thermal gradients :
- between 300K and 4K
- between sample and cold part ?!...

Then, for an homogeneous, constant section material :

$$P = \frac{S}{l} \int_{T_{\min}}^{T_{\max}} \kappa(T)dT, \text{ so for } \kappa(T) = AT^{\alpha}$$

$$P = \frac{S}{l} \frac{A}{\alpha + 1} \left(T_{\max}^{\alpha+1} - T_{\min}^{\alpha+1}\right)$$

Handbooks give integrals of κ(T) to calculate the losses due to conduction between 300K and 4K : depends on material (for κ(T)), as well as geometrical factor (S/l) :
- difficult to make a short fridge out of copper !
- beware of losses in the bath due to magnet leads (often large to stand 100A, and in Cu)

At low temperature, parasitic losses limit the sample temperature, and it is easy to have a base temperature <5mK (T_{min}), and T_{sample}~30mK, 50mK…

In a refrigerator, one may need to isolate or couple thermally part of the fridge from the cold point : requires a thermal switch

- adiabatic specific heat measurements
- adiabatic demagnetization

It can be done mechanically. Pb :

- parasitic vibration power when opening the switch
- mechanics may imply conduction losses

It can be done with a superconductor (Aluminum, T_c~1K, lead T_c~7K)

- in zero field and T<<Tc, good insulator (beware of phonons in pure lead or niobium)
- under field above Hc, metal so a good conductor

Pb: need to apply a magnetic field.

4. Physics: Comparison Thermal Conductivity/Specific heat

Advantage of Cp :

- thermodynamic quantity, which can be "easily" calculated
- good probe of the "density of states"
- robust : little purity dependent, except for magnetic impurities (Schottky anomalies !)
- jump at phase transition :
  - good probe of sample homogeneity
  - good to explore phase diagrams

Problems :

- measures everything : also magnetic impurities, nuclear contributions… which can be dominant and mask the interesting physics.
- average everything : not sensitive to anisotropy of the sample, except if one adds magnetic field, or strain…
- needs rather "large samples" : the specific heat of the sample should be larger than that of the thermometer, heater… Otherwise,
  - Large addenda corrections.
  - Or only "relative measurements (ac Cp)

Advantage of κ :

- transport property: selects heat carriers, ie only electrons and phonons. Main advantage at low temperature: not sensitive to nuclei hyperfine contribution…
- sensitive to anisotropy: need thermal excitations with (more or less) wave vector in the heat current direction.
- can measure quantitatively small samples, even compared to thermometers, heaters… if one can make good contacts (using small wires)
- possible check at low T (for metals) with the Wiedemann-Franz law

Problems :

- very sensitive to the mean free path, to inelastic collisions: difficult to calculate, and so sometimes to interpret.
- not a thermodynamic quantity : may be insensitive to a phase transition, or have only a weak anomaly at the transition temperature (change of slope…)
- requires single crystals (sensitive to anisotropy)
- set-up takes much longer time (isolated sample, two contacts…)

Need both ! For UPt_3,
- phase diagram from C_p
- order parameter from κ