



SUPERCONDUCTIVITY

Aims :

- understand the behavior under magnetic field
- discuss basic notions on superconductors
- some use for low temperature physics

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Outline :

1. Type I and type II superconductors: general phenomena
 - Meissner effect, intermediate state (type I)
 - High critical fields, vortices (type II)
2. Main physical concepts
 - The 2 characteristic length (λ and ξ)
 - Two fluid model (condensate/excitations)->transport properties
 - order parameter: the phase
3. Some equations
 - London theory
 - Ginzburg-Landau Equations
 - Microscopic theory (BCS): the gap
4. Basic properties
 - zero resistance: magnets, low impedance measurements, Seebeck effect
 - field shielding/stabilization
 - quantum phenomena: SQUIDS, Josephson junctions...
5. Research area
 - High-T_c superconductors
 - unconventional superconductors, superfluid He³

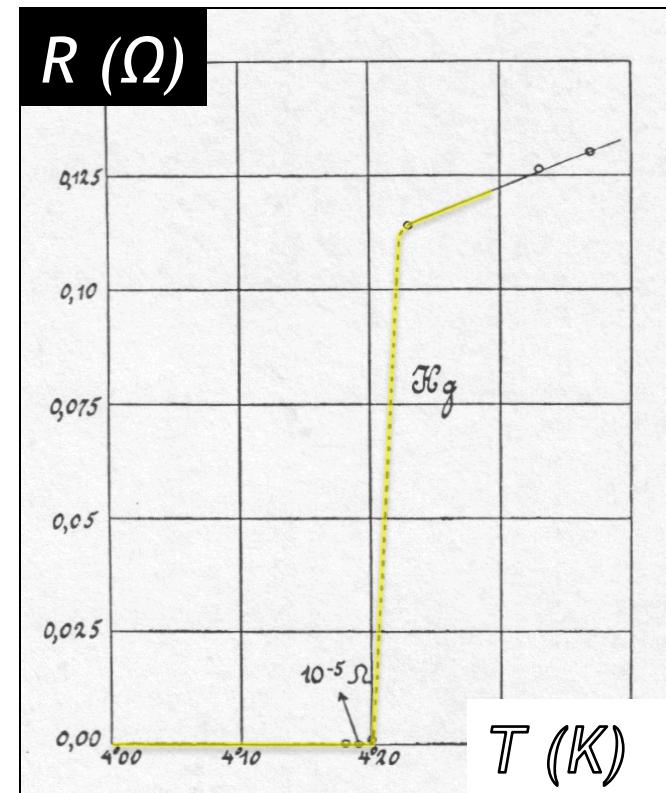
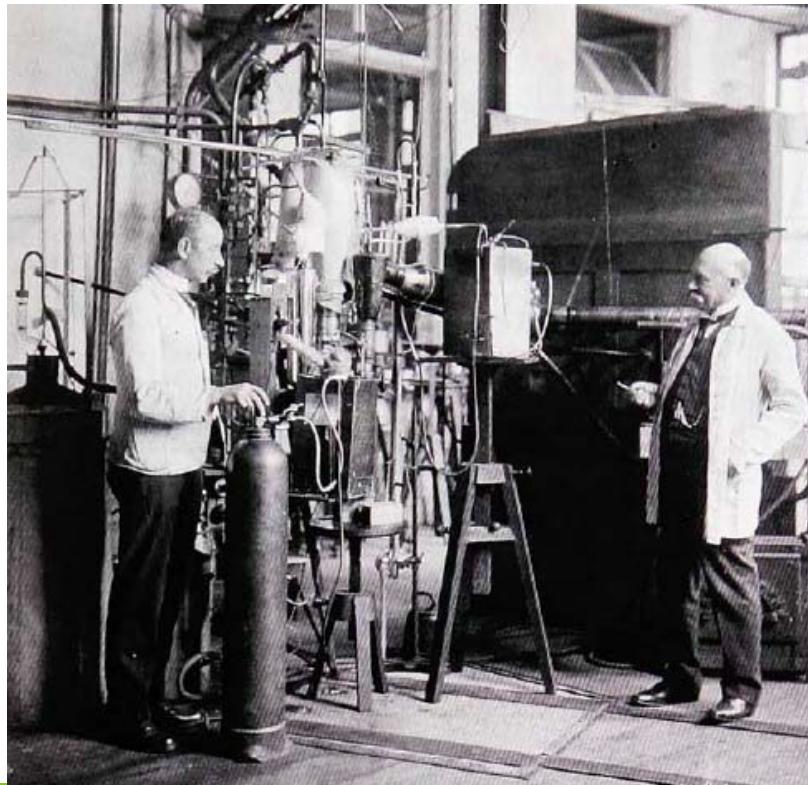
The main dates :

1908, Helium Liquefaction (Kammerling-Onnes)

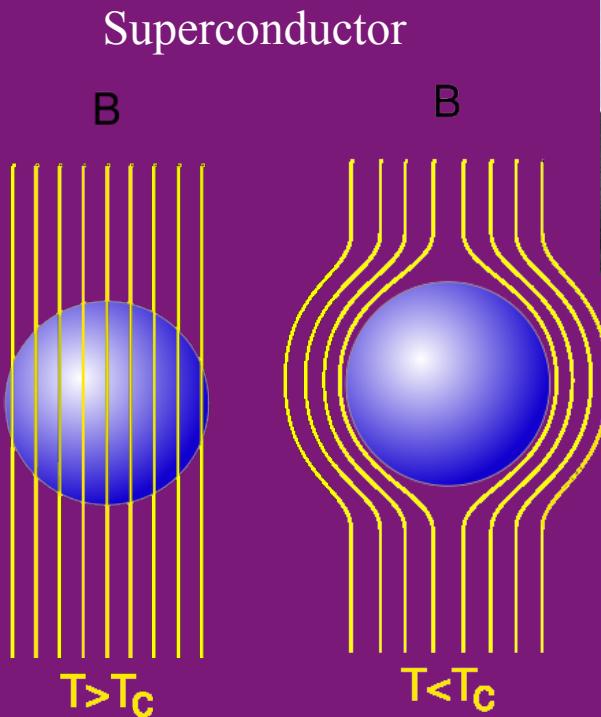
1911, electrons don't freeze : discovery of superconductivity in mercury: a new state of matter

1913, Kammerling-Onnes Nobel Prize...

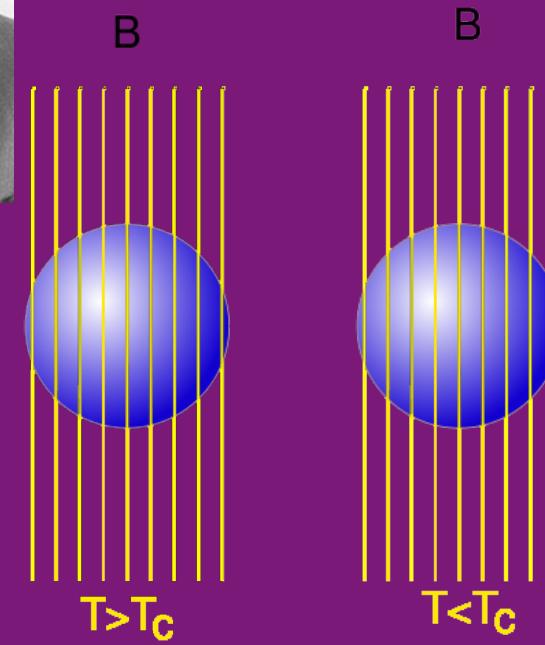
1933, The Meissner effect



The Meissner-Ochsenfeld effect



Perfect metal: Lenz law...



Meissner state: due to surface currents

$$\mu_0 \mathbf{j} = \nabla \times \mathbf{B}, \text{ donc}$$
$$\mu_0 \mathbf{j} \neq 0 \Rightarrow \mathbf{B} \neq 0$$

First “length” of the problem : λ , London penetration depth (10 – 500 nm)

$$\lambda^2 = \frac{m^*}{\mu_0 n_s e^2} \text{ for } T=0$$

Note that λ does NOT depend on T_c ...

1. Type I superconductors: Meissner effect

Meissner effect

- valid for type I superconductors:
usually pure metals, Hg, Al, In, Pb, Nb (if annealed)...
- up to a critical field H_c , for which:

$$\frac{H_c^2(T)}{2\mu_0} = f_n(T) - f_s(T)$$

H_c is usually not so large: of order 10^{-2} - 10^{-1} Teslas

- depending on the geometry, B will penetrate in the sample for $H_i < H < H_c$: "intermediate state"

Conceptually:

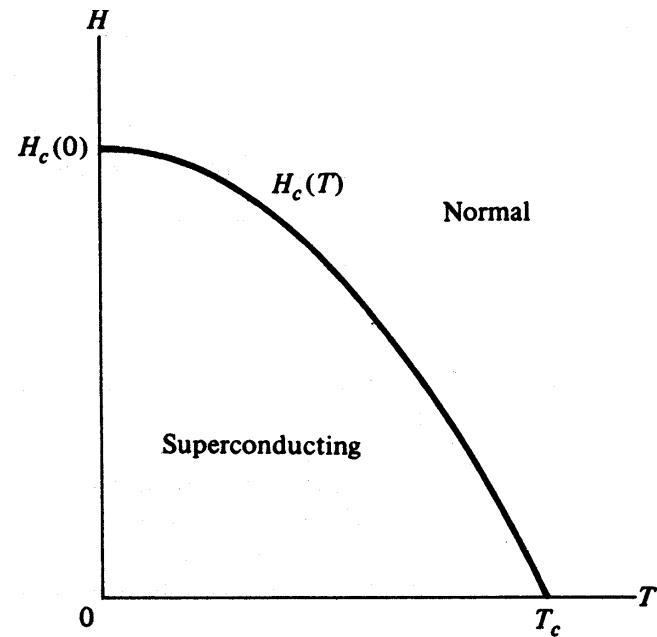
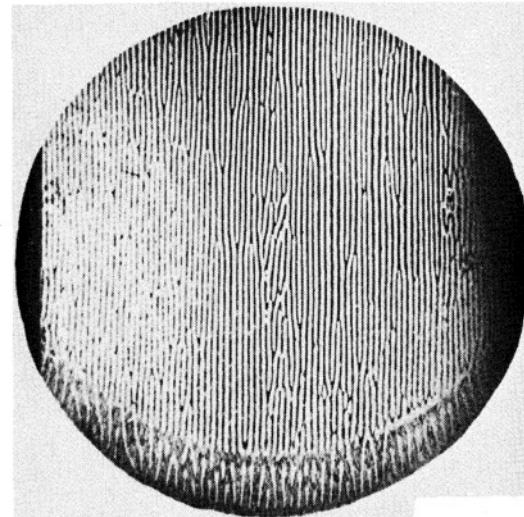
very important.

Practically:

Difficult to observe

Useless for applications

NOT at the origin of most **levitation** experiments.



Part of "100% conducteurs : les supraconducteurs »

A movie from : Alain MONCLIN

Production : INP, Université Paris-Diderot, CNRS Images

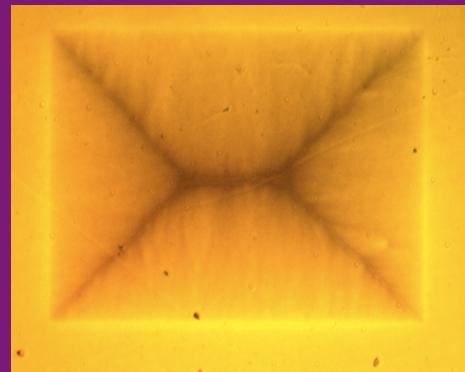
L'institut de physique du CNRS

CNRS Images et l'Université Paris-Diderot

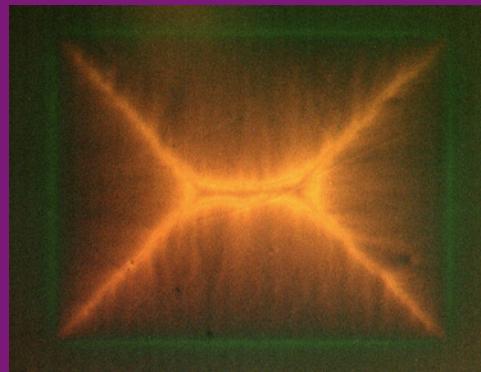
présentent

Flux trapped in superconductors: magneto-optics imaging...

$\text{YBa}_2\text{Cu}_3\text{O}_7$ thin film on YSZ substrate; $T = 11 \text{ K}$

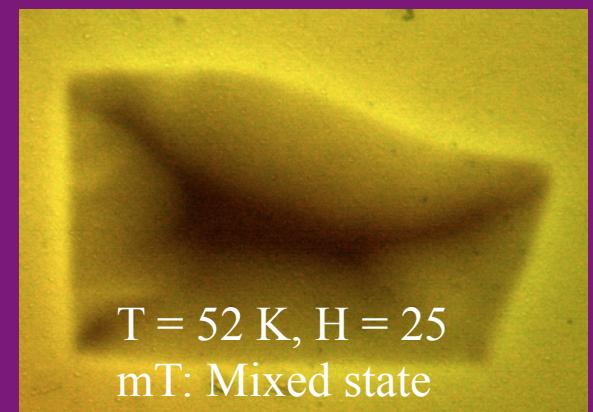
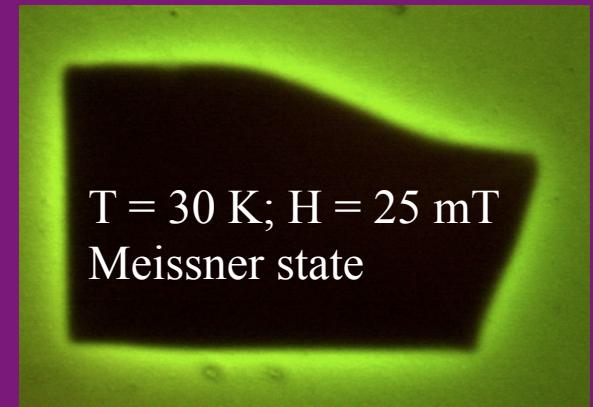


30 mT,



puis OT: flux piégé

$\text{YBa}_2\text{Cu}_3\text{O}_7$ crystal



Kees van der Beek et al.



Vortex entry hindered by *flux pinning* : critical state

1. Type II superconductors: general phenomena-Vortices

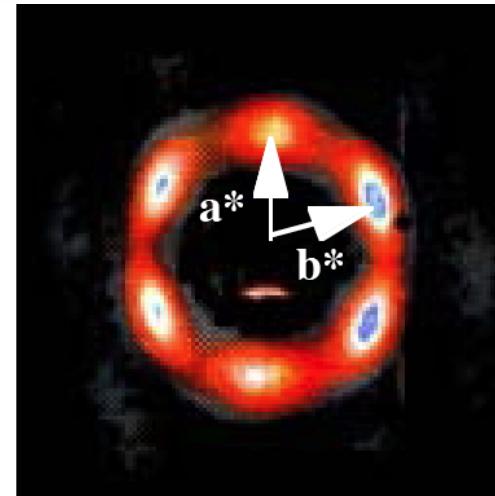
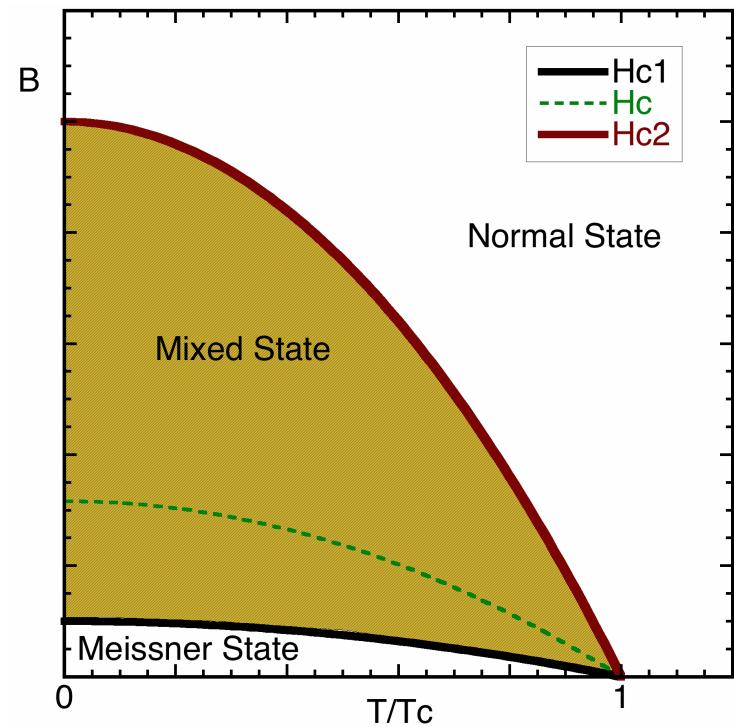
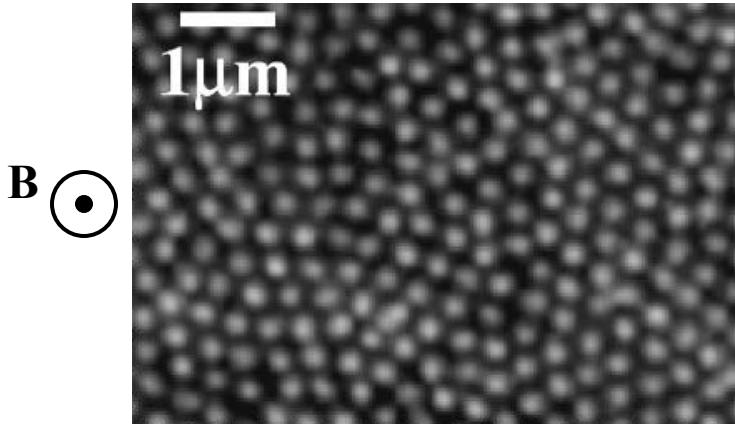
Many superconductors support large fields
CERN-LHC, $H > 8\text{T}$ over kms...

Nb, (pure), $T_c \sim 9\text{K}$ and $H_c \sim 0.1\text{T}$
NbTi alloy, $T_c \sim 9\text{K}$ and $H_{c2} > 10\text{T}$

CeCoIn₅, $T_c \sim 2\text{K}$, $H_{c2} \sim 12\text{T}$...

But they show very little Meissner effect !
Type II superconductors

In the mixed state, B penetrates as flux lines,
forming an ordered lattice, seen by decoration,
neutrons, STM...

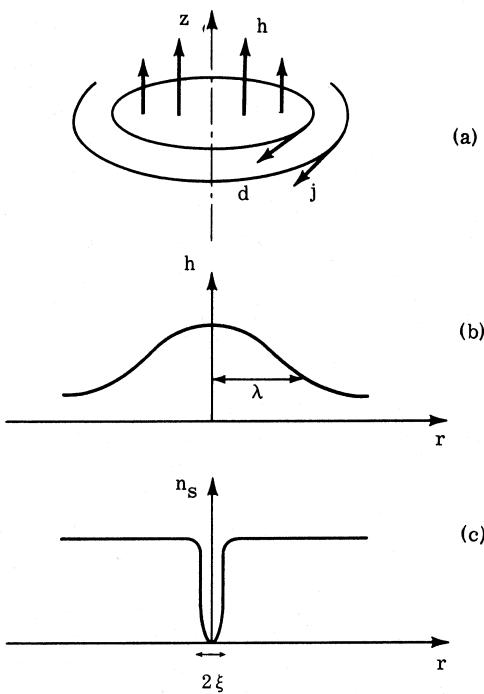


1. Type II superconductors: general phenomena-Vortices

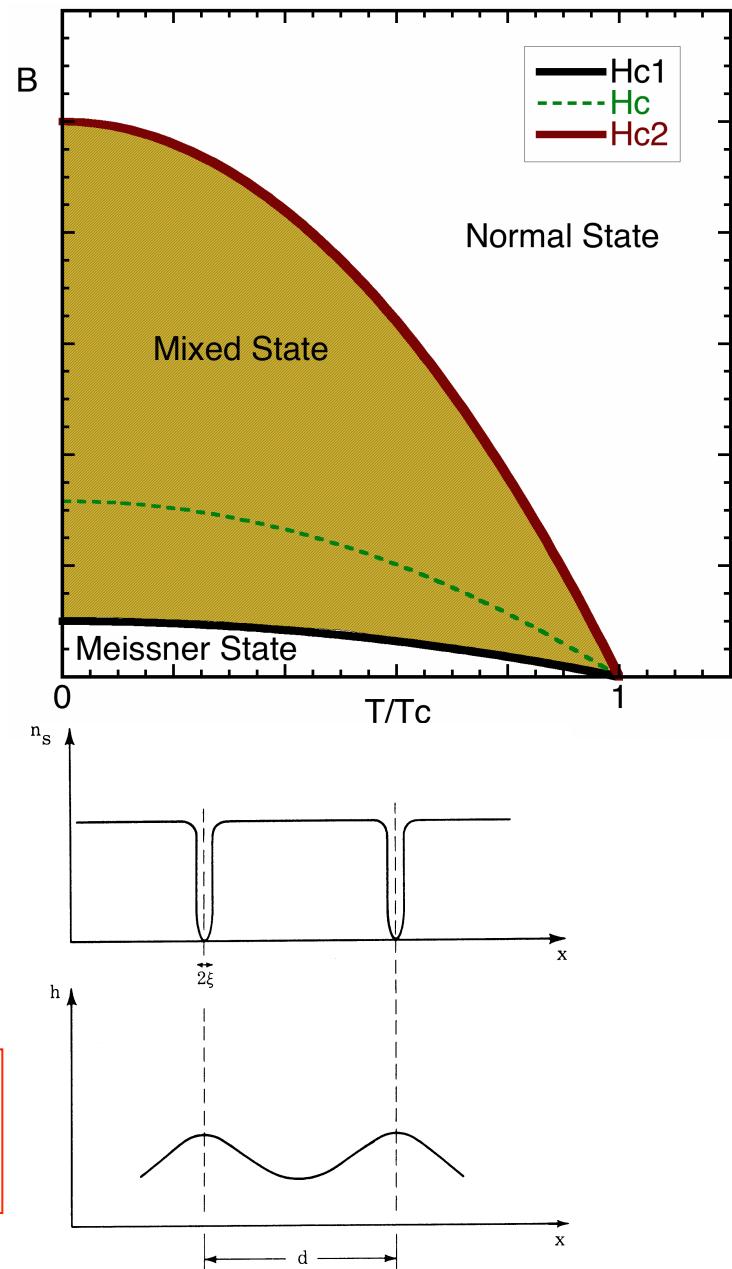
In the mixed state, B penetrates as flux lines, and in strong type II, $B \sim B_{\text{ext}}$ ($B_{\text{ext}} \gg B_{c1}$) !

Diameter of the flux “tubes” $\sim \lambda$ (created by supercurrents)

The superconducting state is destroyed in the vortex cores, of size $\sim \xi \ll \lambda$



$$\left. \begin{aligned} B \cdot d^2 &\sim \Phi_0 \\ B_{c1} \cdot \lambda^2 &\sim \Phi_0 \end{aligned} \right\} \frac{d}{\lambda} \sim \sqrt{\frac{B_{c1}}{B}}$$



2. Main physical concepts: two length scales

Both type I and type II superconductors are characterized by these two length scales:

The *London penetration depth* λ , the scale for current flow \Leftrightarrow magnetic field decay :

The "usual" coherence length ξ , the rigidity of the order parameter, the length over which it can deviate from equilibrium value...

In type I superconductors, $\lambda < \xi$

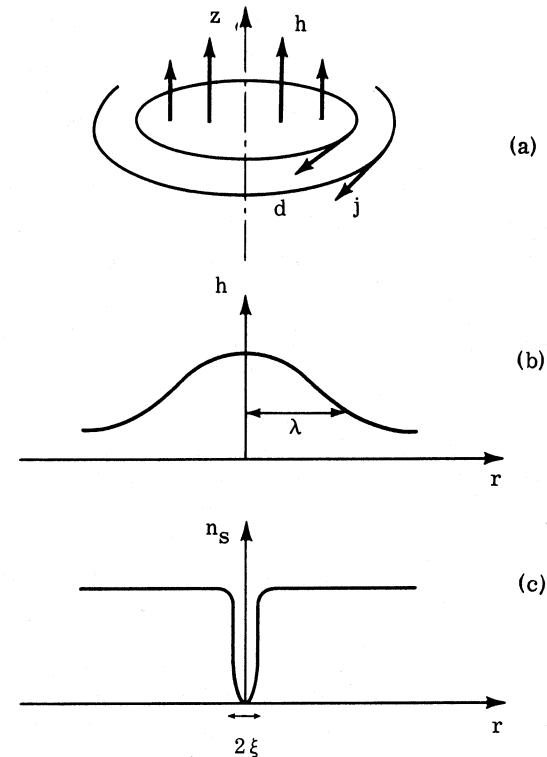
In type II superconductors, $\xi < \lambda$

Defines κ (Maki parameter) : $\kappa = \lambda/\xi$, limit for $\kappa = \frac{1}{\sqrt{2}}$

$$\lambda^2 = \frac{m^*}{\mu_0 n_s e^2} \text{ for } T = 0$$

$$\xi_0 = 0.18 \frac{\hbar v_F}{k_B T_c}$$

but ξ is also reduced for small mean free path, when $l < \xi_0$



2. Main physical concepts: two fluids model

Superconductors sustain current without dissipation: zero resistance (see also Meissner state, vortices...), persistent current in a ring...

Simple idea:

- The electrons are “locked” together to build the current flow =>
- Condensate of electrons, like the coherent condensate of photons in a laser, or the condensate of He⁴ atoms in superfluid He⁴...
but electrons are fermions =>
- **Condensate of electron pairs**, so called **Cooper pairs** -> **builds the first fluid**
Electron pairs are all in the same state => no entropy, no temperature. But properties of a superconductor change with T =>
- There also exist **thermal excitations**: “normal electrons”, they build the **second fluid**

Consequence on transport properties:

- **resistivity**: zero resistance because the Cooper pair condensate short-circuits thermal excitations ($\sigma = \sigma_{\text{condensate}} + \sigma_{\text{thermal}}$)
- **thermal conductivity**: bad thermal conductor, as $\kappa = \kappa_{\text{condensate}} + \kappa_{\text{thermal}}$,
but $\kappa_{\text{condensate}} = 0$ (no entropy, no energy to exchange)
 κ_{thermal} disappears with the number of thermal excitations when $T > 0$

2. Main physical concepts: the order parameter (London 1933-Ginzburg-Landau 1950)

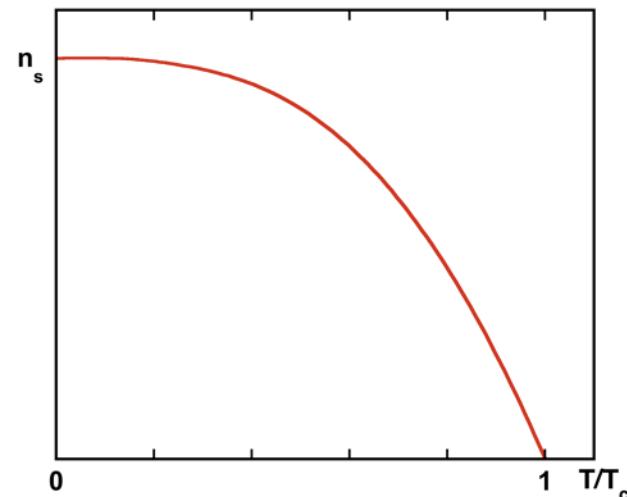
The change that appear in a superconductor below T_c is the condensate of Cooper pairs:
the **order parameter** could be n_s , the **superfluid density**

It is more than that ! Condensate described by a
complex wave function, Ψ ,

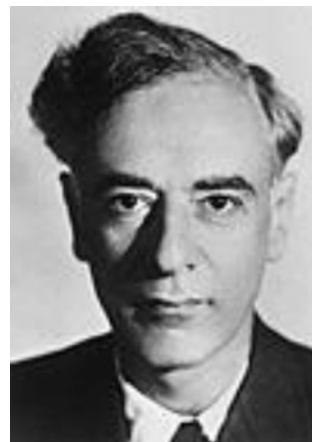
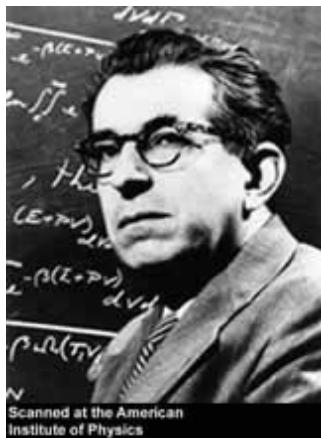
Wave function of a macroscopic object:

the condensate of all electrons...

$$|\Psi|^2 = n_s, \text{ and } \Psi = \sqrt{n_s} e^{i\varphi}$$



The phase is controlled by the current (quantum mechanics)



3. Some Equations : the London Equation & flux quantization

Based on the description of the superconductor as a macroscopic wave function : describe the Meissner effect

$$|\Psi|^2 = n_s, \text{ so } \mathbf{J} = e^* |\Psi|^2 \mathbf{v}$$

$$\mathbf{J} = -\frac{i\hbar e^*}{2m^*} \left[\Psi^* (\nabla - \frac{ie^*}{\hbar} \mathbf{A}) \Psi - c.c \right]$$

$$\mathbf{J} = \frac{\hbar e^*}{m^*} |\Psi|^2 \left(\nabla \varphi - \frac{e^*}{\hbar} \mathbf{A} \right), \text{ with in addition (London)}$$

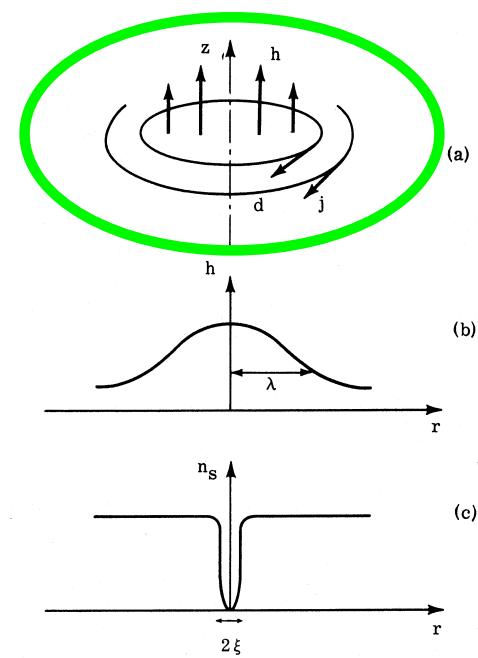
- Meissner state: rigid wave function, $\nabla \varphi = 0$ in the bulk. With $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$:

$$\mathbf{B} - \lambda^2 \Delta \mathbf{B} = 0, \text{ and } \lambda^2 = \frac{m^*}{\mu_0 |\Psi|^2 (e^*)^2}$$

- Vortex state: away from vortex core, $\mathbf{v}=0 \Rightarrow \nabla \varphi = \frac{e^*}{\hbar} \mathbf{A}$

$$\oint d\varphi = 2\pi.n = \frac{e^*}{\hbar} \oint \mathbf{A} \cdot d\mathbf{x} = \frac{e^*}{\hbar} \int B \cdot dS$$

$$\Phi = n\Phi_0, \text{ where } \Phi_0 = \frac{h}{2e} = 2.10^{-15} Wb, \text{ is the flux quantum}$$



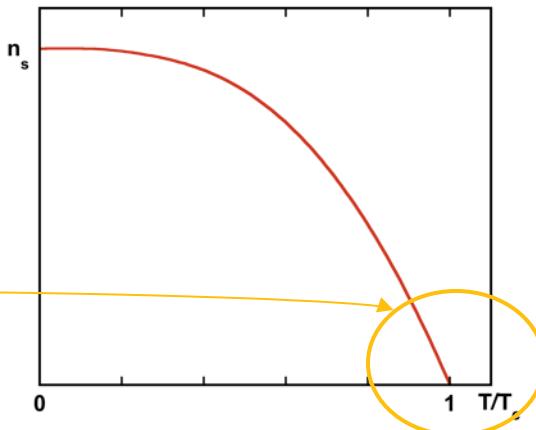
3. Some Equations : the Ginzburg-Landau Equations

- Expression of the free energy F versus the order parameter Ψ :

$$F(T, \Psi) = F_0(T) + a(T - T_c) |\Psi|^2 + \frac{b}{2} |\Psi|^4$$

$$\Rightarrow \text{for } T < T_c, |\Psi|^2 = \frac{a}{b} (T_c - T),$$

$$\begin{cases} \Psi = \sqrt{n_s} e^{i\varphi} \\ n_s = \frac{a}{b} (T_c - T) \end{cases}$$



- Under magnetic field, add kinetic energy and magnetic field energy:

$$F = \int d^3x \mathcal{F}(T, \Psi, \mathbf{A}), \text{ with}$$

$$\mathcal{F}_0(T, \Psi, \mathbf{A}) = \mathcal{F}_0(T) + a(T - T_c) |\Psi|^2 + \frac{b}{2} |\Psi|^4 + \frac{1}{2m^*} \left| \left(\frac{\hbar}{i} \nabla - e^* \mathbf{A} \right) \Psi \right|^2 + \frac{B^2}{2\mu_0} - \mathbf{H}_0 \cdot \mathbf{B}$$

the coherence length is the coefficient in front of $\nabla \Psi$:

$$\xi^2(T) = \frac{\hbar^2}{2m^* a(T_c - T)}$$

Minimizing F , one derives the G.L. equations, and ($\kappa = \frac{\lambda}{\xi}$):

$$B_{c2}(T) = \frac{\Phi_0}{2\pi\xi^2(T)}, \quad B_{c1}(T) = \frac{\Phi_0}{2\pi\lambda^2(T)} \ln \kappa, \quad B_{th}(T) = \frac{\Phi_0}{2\pi\sqrt{2}\lambda\xi}$$

From Ginzburg Landau...to Abrikosov !



© A.I. Buzdin...

Nobel Prize 2003:

- Discovery of type II superconductivity: most superconductors (NbTi, NbSn, cuprates, pnictides...)
- Discovery of the mixed (vortex) state. At the heart of all large current-high field applications

1957 : BCS microscopic theory

In 1972...



John Bardeen



Leon N. Cooper

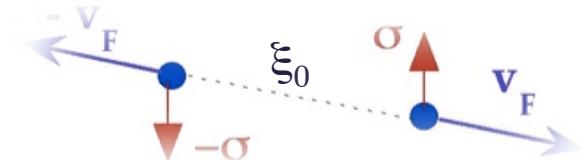


John R. Schrieffer

Ingredients of the microscopic BCS description

- electron-phonon interaction => effective attractive electron-electron interaction
- Presence of the Fermio sea : bound state = « Cooper pair »

$$\psi^+ = \sum_{\mathbf{k},\sigma} \phi(\mathbf{k}) a_{\mathbf{k},\sigma}^+ a_{-\mathbf{k},-\sigma}^+ : \text{opposite wave vectors } (\mathbf{k}) \text{ and spins } (\sigma)$$



- Superconductor = Cooper pair condensate

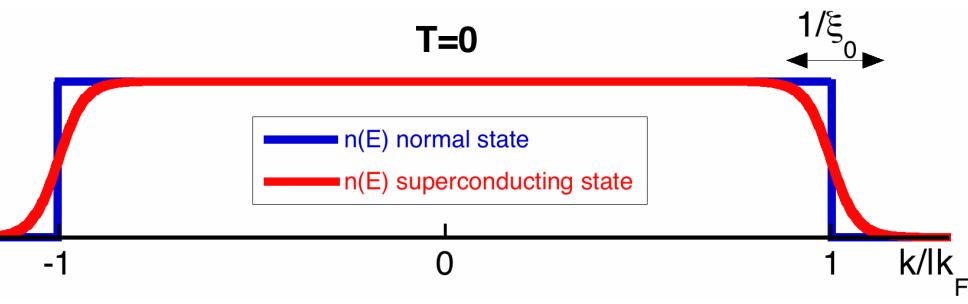
$$\begin{aligned} |\Psi_{\text{BCS}}\rangle &= A \exp(\psi^+) |0\rangle = A \sum_n 1/n! (\psi^+)^n |0\rangle \\ &= \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} a_{\mathbf{k},\sigma}^+ a_{-\mathbf{k},-\sigma}^+) |0\rangle \end{aligned}$$

- Quantitative predictions on the energy spectrum & wave function (coherence factors)!...

Binding energy $\Delta \approx \hbar \omega_D \exp\left(-\frac{2}{\rho_d |V_0|}\right)$, with extension $\xi_0 \approx \frac{1}{(\Delta k)} \approx \frac{1}{(\Delta \epsilon)} \left(\frac{\partial \epsilon}{\partial k}\right) \approx \frac{\hbar v_F}{\Delta}$

$$k_B T_c \approx \hbar \omega_D \exp\left(-\frac{1}{\rho_d |V_0|}\right)$$

Excitations $E_k = \sqrt{\xi_k^2 + \Delta_k^2}$, and $\xi_k = \frac{\hbar^2 k^2}{2m}$



3. Some Equations : the microscopic BCS theory

Gap in the excitation spectrum : $E_k = \sqrt{\xi_k^2 + \Delta_k^2}$, and $\xi_k = \frac{\hbar^2 k^2}{2m} - \mu$

- exponential number of thermal excitations
- direct observation of the density of states by tunnel spectroscopy, Raman, ARPES...

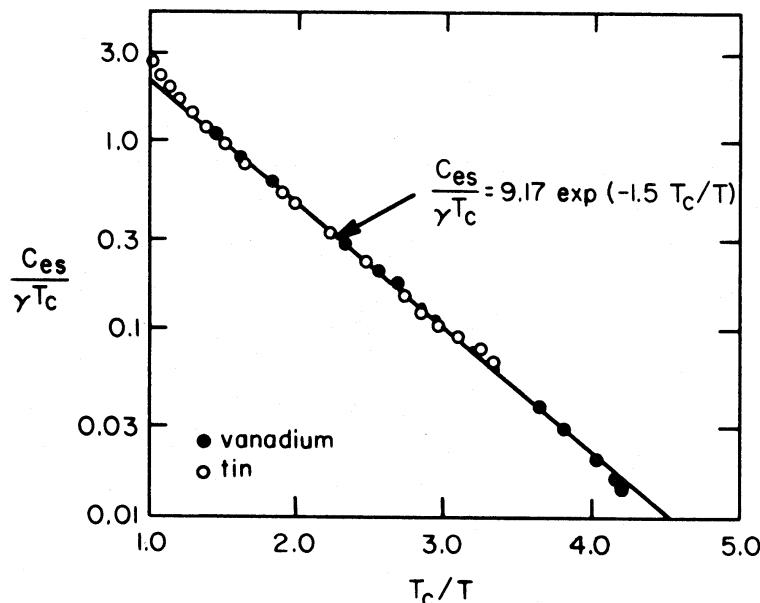
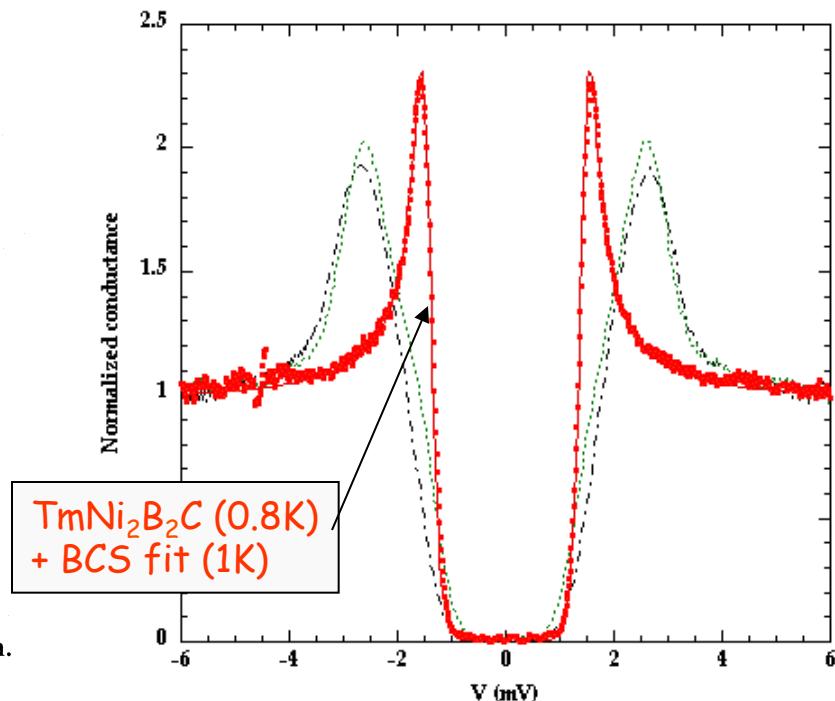


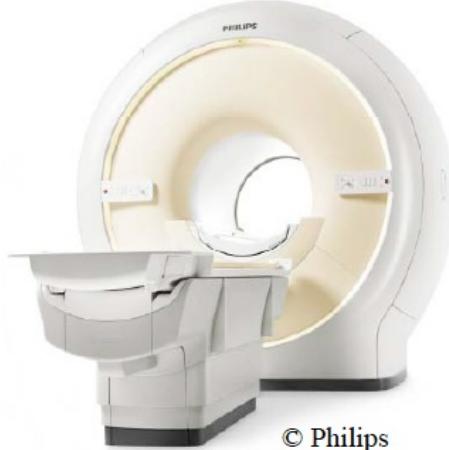
Fig. 22. Reduced electronic specific heat in superconducting vanadium and tin.
[From Biondi et al., (150).]



Martinez Samper et al.
Phys. Rev. B **67** 014526 (2003)

4. Basic properties : zero resistance

- magnets: large scale application of superconductivity, high field production (several Telas). Used for IRM, particles accelerators (CERN-LHC). See P. Tixador.



- low impedance measurements: large use at very low T, for measurements wire at low currents. Low impedance measurements (avoiding thermal noise), thermal decoupling, control of dissipation in the heater.
- Seebeck effect: often used as a reference in a thermocouple, because there is no Seebeck effect in a superconductor (no voltage across a superconductor).

PRECISION: **zero resistance is valid when no magnetic field is present:** when vortices enter the superconductor, induced voltage develop if they move due to coupling between current and field. Then a voltage may appear (flux flow resistance, Nernst effect...)

4. Basic properties : field shielding/stabilization

- Meissner effect: use of superconductor to shield a DC magnetic field ? **NO !**
 - Superconductor exclude field from its interior
 - Quantize flux in a ring...
 - Does not exclude the field from the shield
- Very different from a ferromagnetic material (flux line concentration inside the material).
- But it is very efficient for shielding from ac fields (as long as $\omega < \Delta$)
 - Perfect diamagnetism on a scale $> \lambda$
 - Very efficient for 50Hz shielding...
 - Very low impedance for RF cavities: trapping photons, accelerating ions, electrons...
- It is very efficient to trap a very stable magnetic field
 - With type I superconductors, up to 0.1T
 - With type II superconductors, need to maintain the external field + thick material...

Detection of very low fields by quantum interferences

Applying a magnetic field forces a phase shift between the two arms.

Introduce two “weak-links” inside the arms for the detection (Josephson junctions):

$$I = I_c \sin \Delta\varphi$$

Inside the superconductor, except in the junctions, $\mathbf{p} = 0$

$$\int_1^2 d\varphi + \Delta_1 + \int_2^r d\varphi - \Delta_2 = \phi \frac{1}{\hbar} (-2e\mathbf{A}) \cdot \mathbf{dx} + \Delta_1 - \Delta_2 = 2\pi \cdot n$$

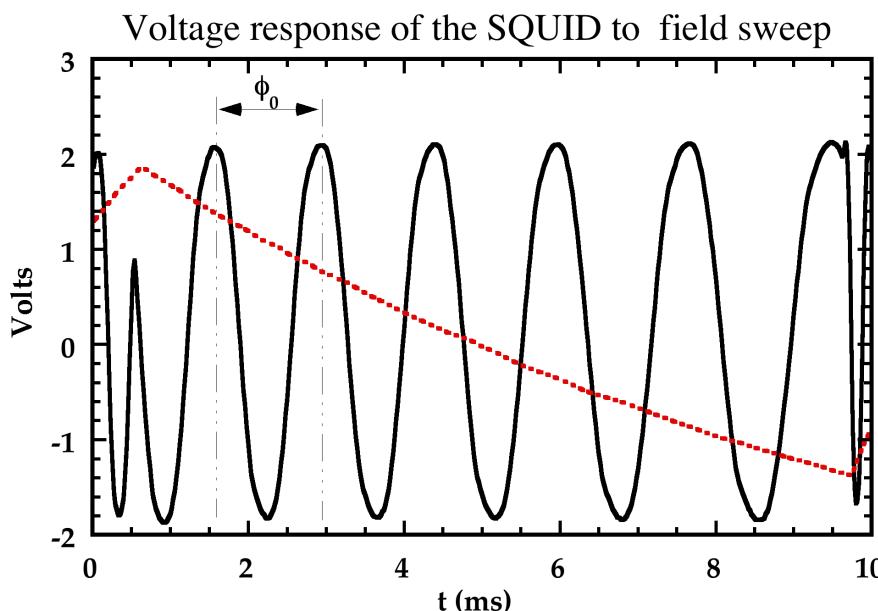
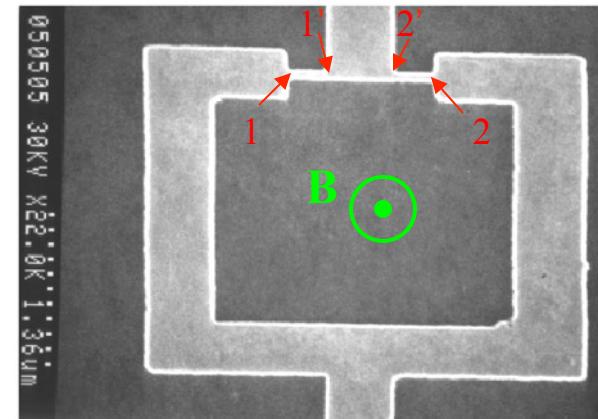
$$\Delta_1 - \Delta_2 = 2\pi \cdot n - \frac{2e}{\hbar} \int B \cdot dS = 2\pi \left(n - \frac{\Phi}{\Phi_0} \right)$$

Then, the critical current across the SQUID behaves like:

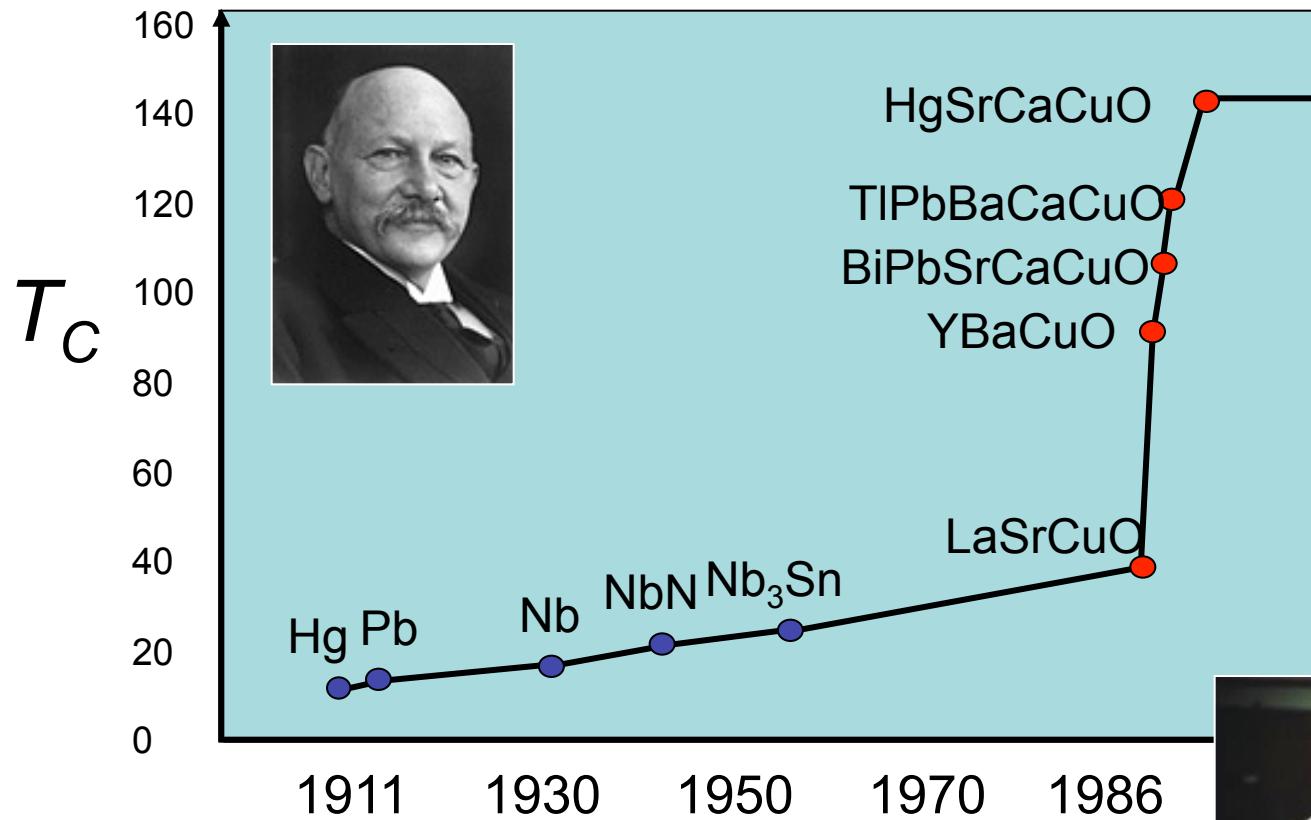
$$I = I_c \sin \Delta_1 + I_c \sin \Delta_2, \text{ maximum at}$$

$$I_{\max} = 2I_c \left| \cos \left(\pi \frac{\Phi}{\Phi_0} \right) \right|$$

Periodic function of Φ / Φ_0 , widely used for very low fields, current, voltage... in many areas



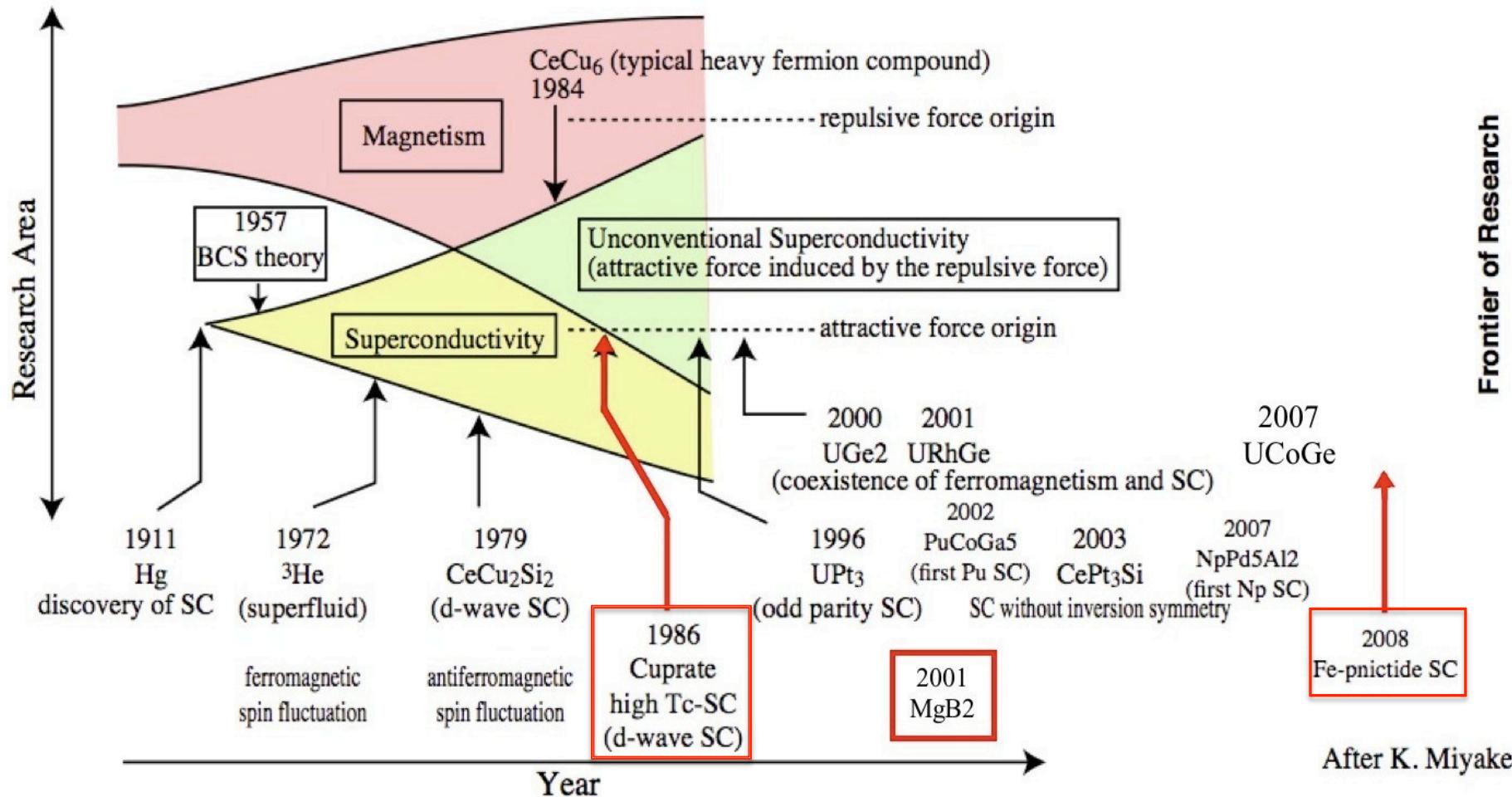
1986 : Revolution of the cuprates...



© AIP



Magnetism & Superconductivity

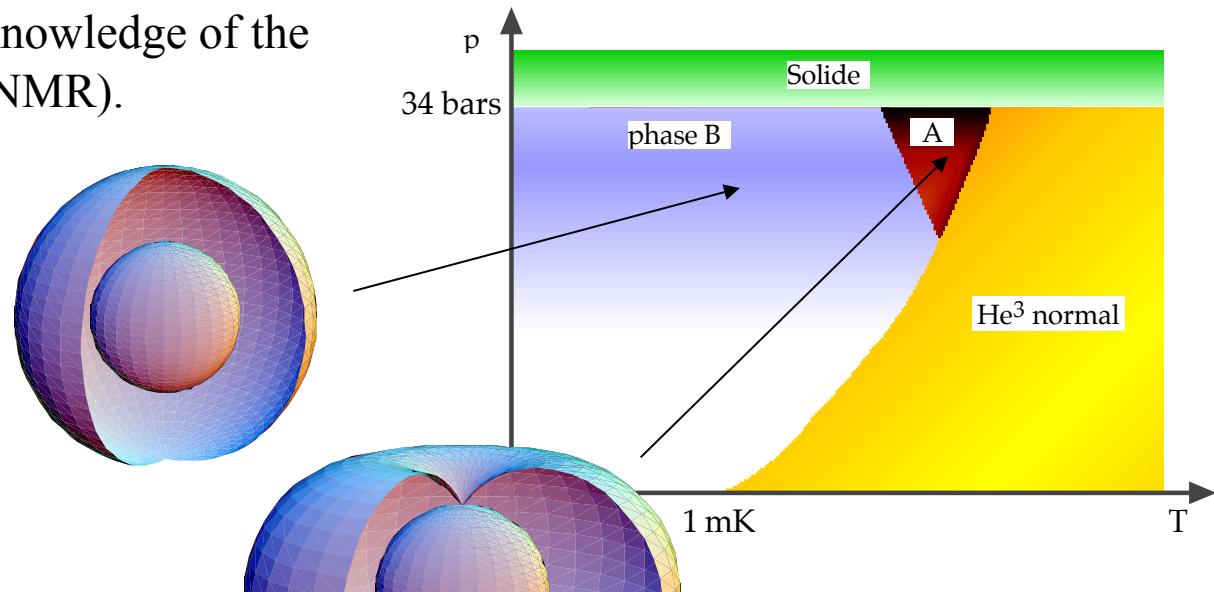
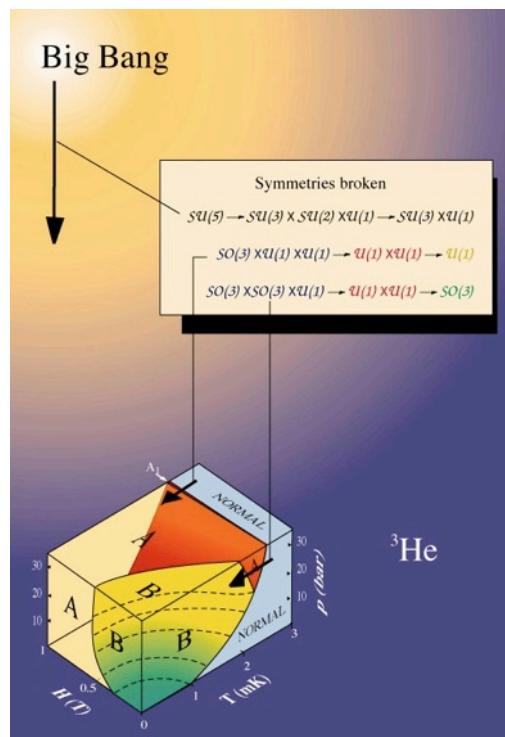


5. Research area: superfluid He³

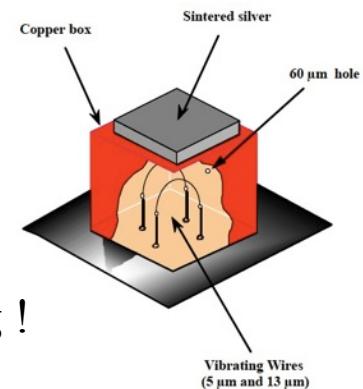
p-wave superfluid with $T_c \sim 2\text{mK}$, discovered in 1972

Purest material... extensively studied, used now as bolometer or dark matter detector
(ask Henri !)

Most precise quantitative knowledge of the
superfluid wave function (NMR).

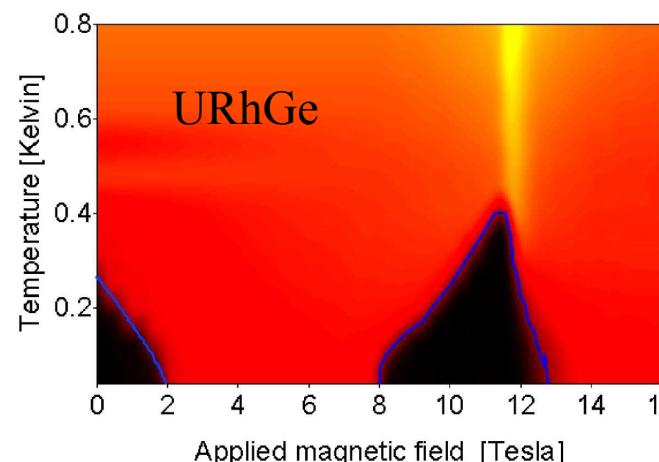
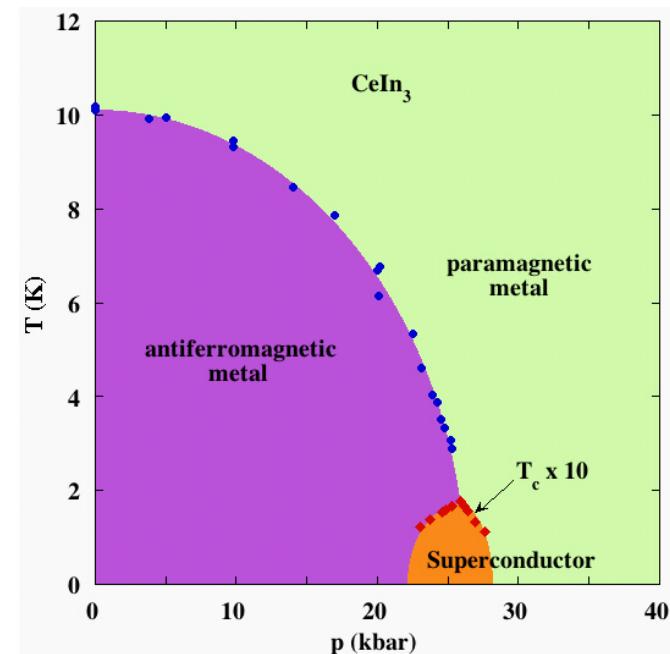
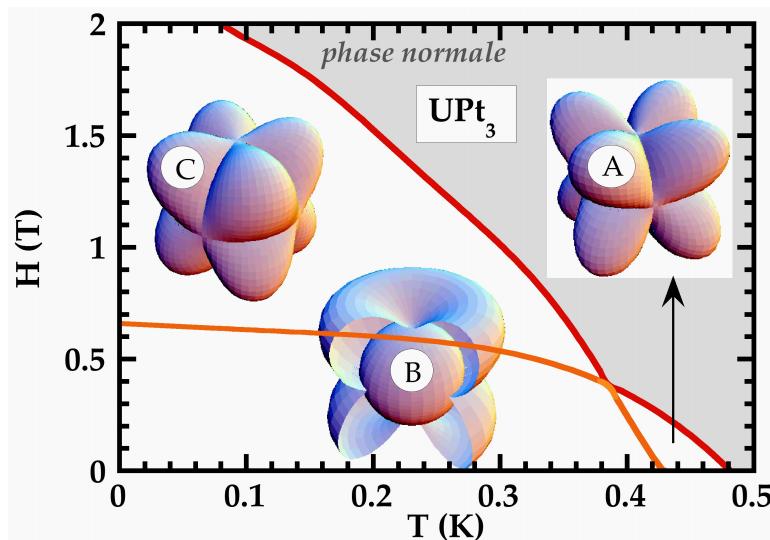


Modeling the universe... and measuring !



5. Research area: Heavy fermion superconductors

- CeCu₂Si₂ (1979), UBe₁₃ (1983), UPt₃ (1984), CeCoIn₅ (2002), URhGe (2001), UCoGe (2008)..
- "p,d,f wave"
- 3D anisotropic (tetragonal, cubic, hexagonal)
- f electrons=> hard core + close to AF instability
- Coexistence of ferromagnetic and superconductivity



5. Research area: High-Tc cuprates

High-Tc cuprates (1986...)

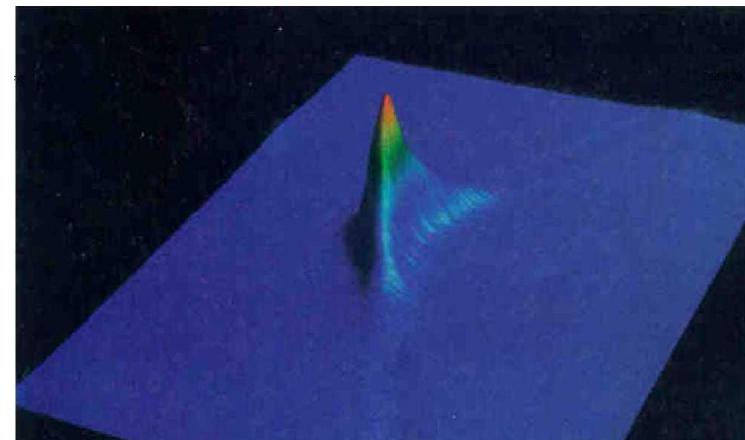
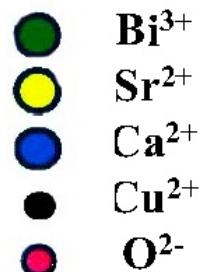
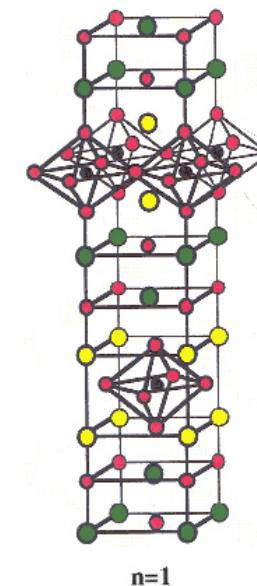
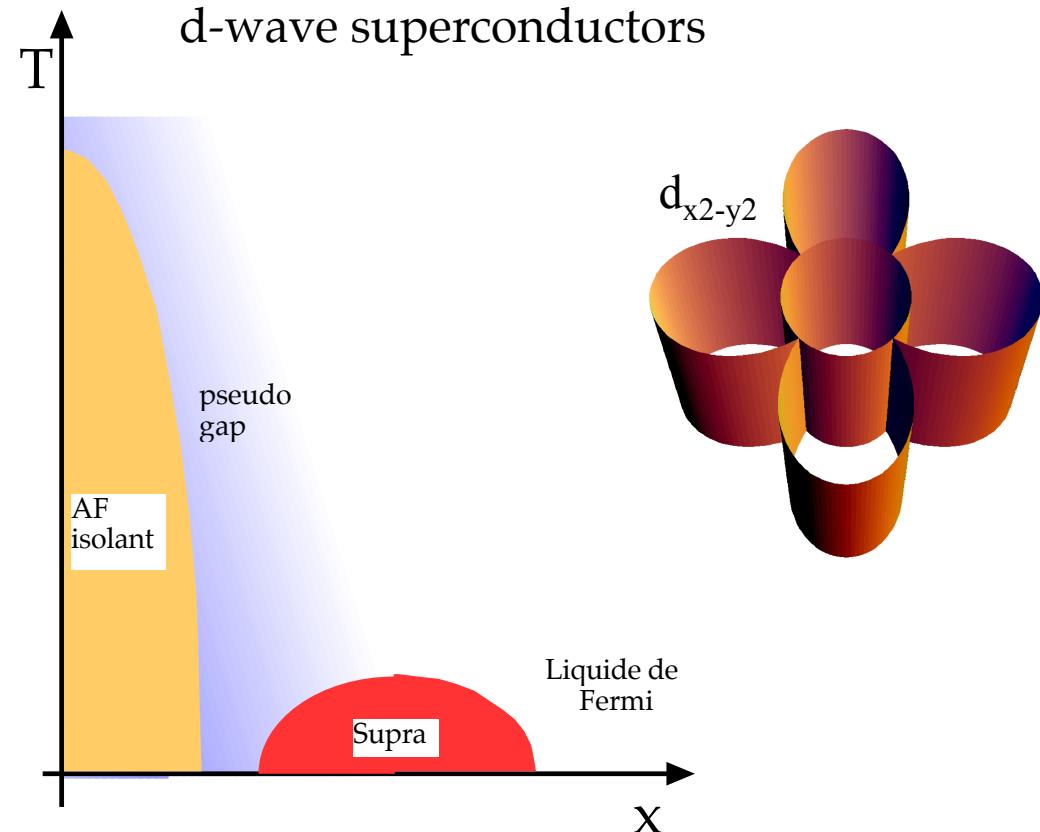
$\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$ ($T_c \sim 30\text{K}$), $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (90K),

$\text{HgBa}_2\text{Ca}_2\text{Cu}_3\text{O}_{8+\delta}$ (160K à 300kbar)...

orthorombic or tetragonal, 2D

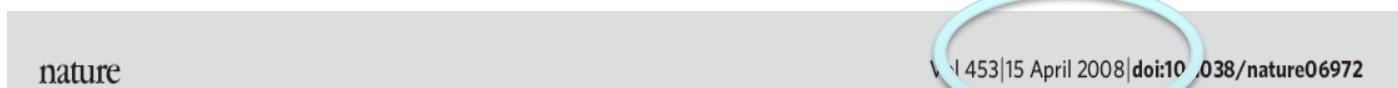
close to a Mott metal-insulator transition

d-wave superconductors



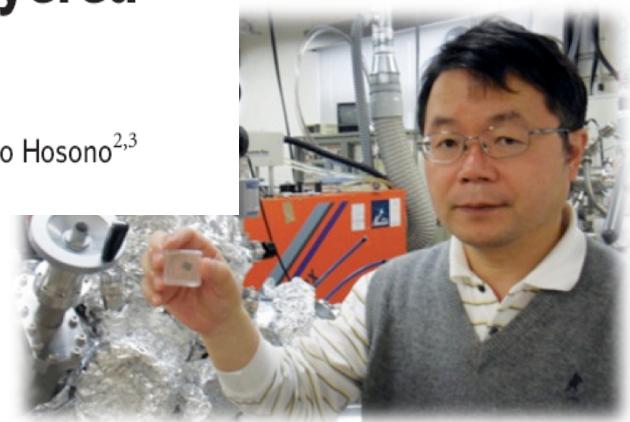
Tsuei et al.

In Japan...

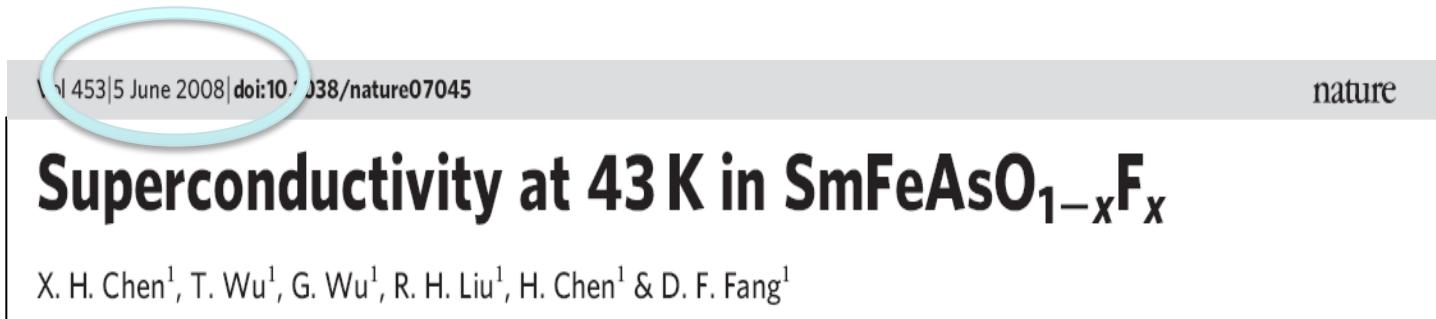


Superconductivity at 43 K in an iron-based layered compound $\text{LaO}_{1-x}\text{F}_x\text{FeAs}$

Hiroki Takahashi¹, Kazumi Igawa¹, Kazunobu Arai¹, Yoichi Kamihara², Masahiro Hirano^{2,3} & Hideo Hosono^{2,3}



... and in China !

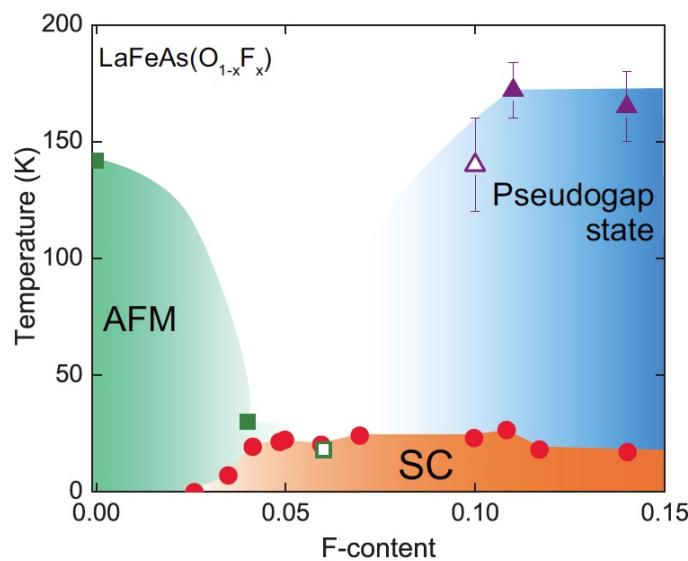
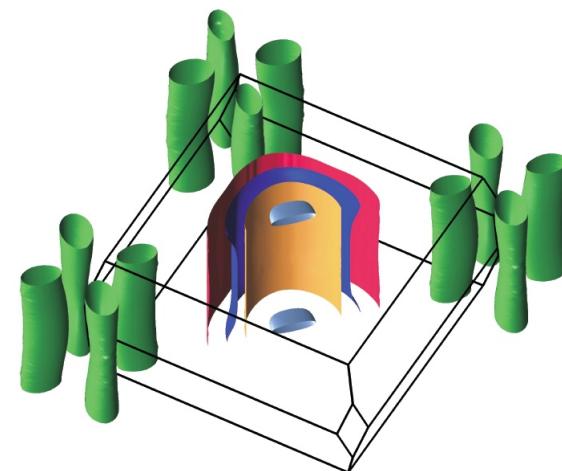
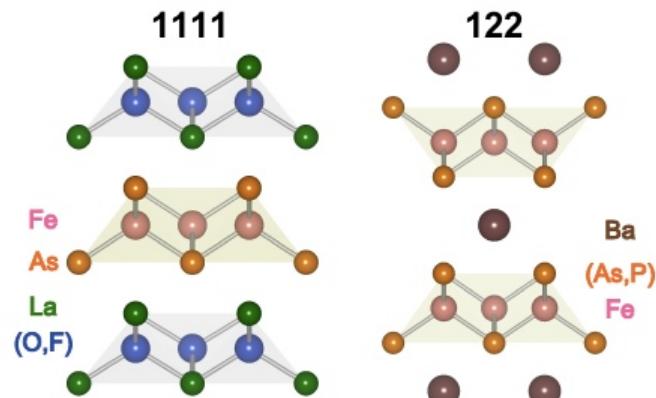


Superconductivity at 43 K in $\text{SmFeAsO}_{1-x}\text{F}_x$

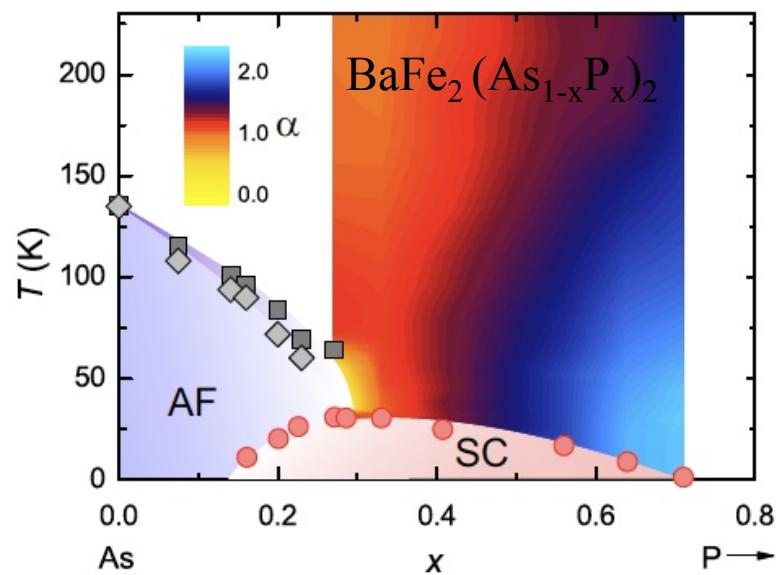
X. H. Chen¹, T. Wu¹, G. Wu¹, R. H. Liu¹, H. Chen¹ & D. F. Fang¹

5. Research area-latest revolution : the iron based pnictides...

Iron pnictides: multigap, s⁺/⁻ ?...



Y. Nakai et al., New J.
Phys. 11 (2009) 045004.



Y. Nakai et al., PRL 105 (2010) 107003

5. Research area-a big surprise, MgB₂

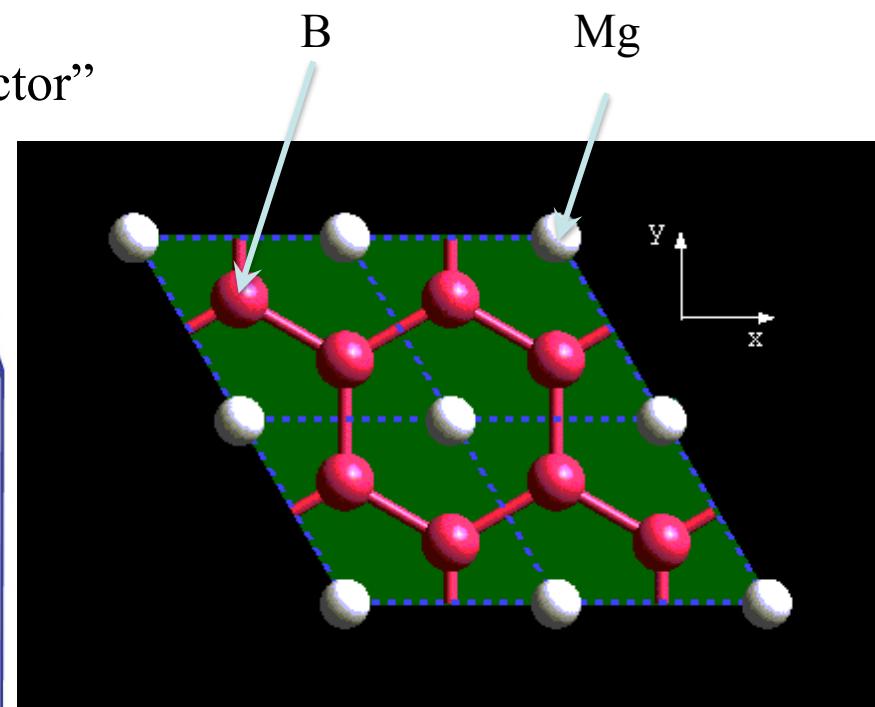
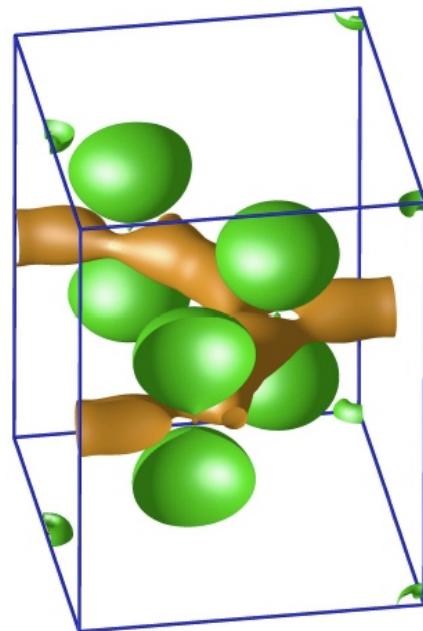
In 2001, MgB₂ superconducting at 39K ?...

Very wsimple system, available in any chemist grocery store

- electron-phonon coupling !
- T_c higher than LaSrCuO !...
- Paradigm of “multigap superconductor”



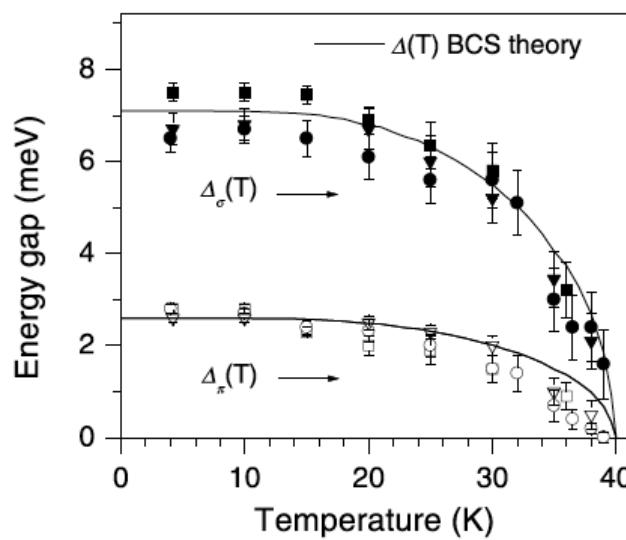
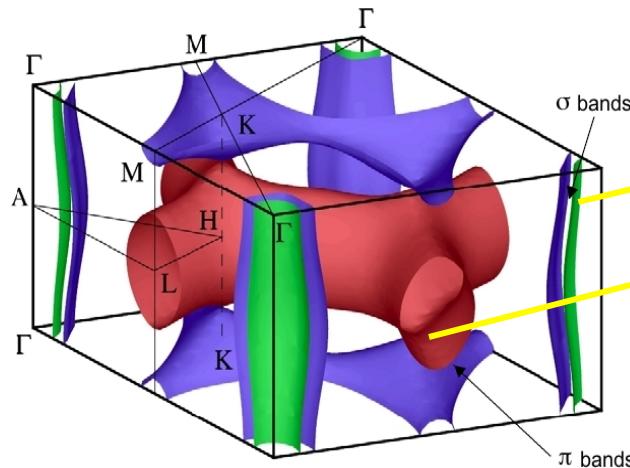
J Akimitsu



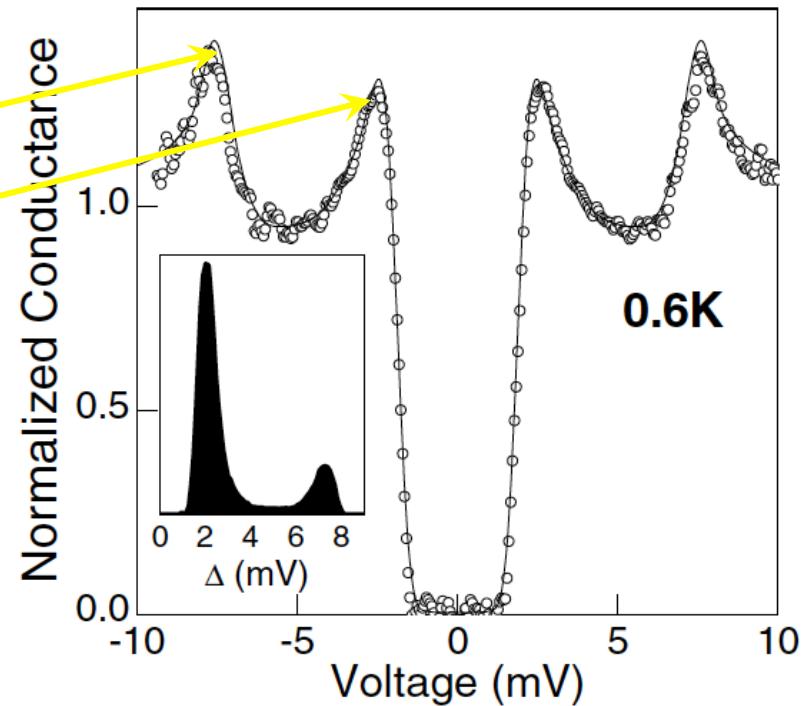
Nagamatsu et al., Nature 2001,
More than 3000 citations !

5. Research area-a big surprise, MgB₂

MgB₂



Samuely et al., Physica C 385 (2003) 244



P. Martinez-Samper et al., Physica C 385 (2003) 233

Giubileo et al. Europhys. Lett., 58, (2002) 764

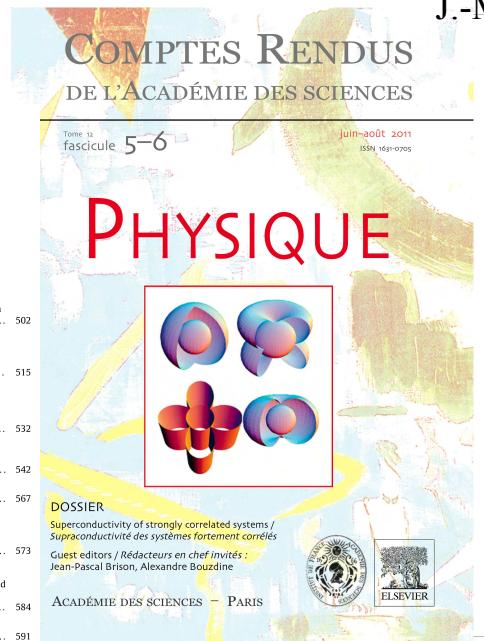


COMPTES RENDUS
DE L'ACADEMIE DES SCIENCES
PHYSIQUE

ACADEMIE DES SCIENCES, PARIS

2011 - Tome 12 - N° 5-6

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Séminaire Daniel Dautreppe
Les défis actuels de la supraconductivité
SFP-Alpes
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SUPRA 2011 SUPRA 2011
Centre national de la recherche scientifique

LA SUPRA CONDUCTIVITÉ 1911 > 2011, 100 ANS DE DÉCOUVERTES

Evénements
Du 14 au 19 février
Paris-Montagne : Science Académie
Paris

Science Académie Association
Paris-Montagne, École Normale Supérieure, 45 rue d'Ulm, 75005 Paris

DU 1er mars au 30 juin
ENSCI : Usage de la lévitation
Paris

ENSCI - Les Ateliers, 48 rue St Sabin, 75011 Paris

SUPRA 2011
Le CNRS se mobilise pour fêter la supraconductivité : bienvenue sur son site événementiel !

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Ce site a pour vocation de promouvoir les opérations organisées dans le cadre des 100 ans de la supraconductivité, organisées par ou auxquelles participent les personnels des laboratoires du CNRS et de ses partenaires, durant l'année 2011 et dans toute la France.

3. Some Equations : the microscopic BCS theory

Cooper pairs: a weak interaction between electrons makes leads to a bounding state, of binding energy

$$\Delta \approx \hbar\omega_d \exp\left(-\frac{2}{\rho_d |V_0|}\right), \text{ with extension } \xi_0 \approx \frac{1}{(\Delta k)} \approx \frac{1}{(\Delta \varepsilon)} \left(\frac{\partial \varepsilon}{\partial k}\right) \approx \frac{\hbar v_F}{\Delta}$$

Mechanism for the interaction:

electron-phonon interaction, or
“polarizability of the lattice”->
favored for “time reversed states”

Bound state: classified by its “orbital momentum”:

$$\psi(1,2) = -\psi(2,1), \text{ and } \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$$

$$= \varphi(\mathbf{r}) \chi(\sigma_1, \sigma_2), \text{ with } \chi(\sigma_1, \sigma_2) = |S=0\rangle \text{ or } |S=1\rangle$$

$$\varphi(\mathbf{r}) = f(r) Y_l^m\left(\frac{\mathbf{r}}{r}\right), \text{ if } l=0, 2\dots \text{ then } S=0 \text{ (singlet)}$$

$$\text{if } l=1, 3\dots \text{ then } S=1 \text{ (triplet)}$$

if $l=0$: s - wave superconductivity

if $l>0$, unconventional superconductivity

$l=1$: p - wave (superfluid He³), $l=2$: d - wave (high - Tc)...

